A note on "A Note on Constraining AR(2) Parameters in Estimation"

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The following is a more detailed explanation of the result in Morley (1999):

$$b^2 = (1 - |a|)\, b_{uc}^2 + |a| - a^2, \tag{1}$$

where b_{uc}^2 is constrained to be between -1 and 1.

As reflected in the "triangle" diagram in Hamilton (1994), for the eigenvalues of the AR(2) to remain inside the unit circle, and hence imply stationary dynamics, ϕ_2 must satisfy for a given ϕ_1:

$$-1 \leq \phi_2 \leq 1 - \phi_1, \qquad \text{if } \phi_1 \geq 0, \tag{2}$$

$$-1 \leq \phi_2 \leq 1 + \phi_1, \qquad \text{if } \phi_1 < 0. \tag{3}$$

Since the triangle is symmetric about $\phi_1 = 0$, the bounds for ϕ_2 do not depend on the *sign* of ϕ_1. Therefore, if we consider the absolute value of ϕ_1 instead, the only constraints on ϕ_2 are: $-1 \leq \phi_2 \leq 1 - |\phi_1|$.

Then, using $\phi_1 = 2a$ and $\phi_2 = -(a^2 + b^2)$, it can be easily shown that

$$-1 + 2|a| - a^2 \leq b^2 \leq 1 - a^2. \tag{4}$$

Morley (1999) states that equation (1) satisfy the constraints. To see this, note that as $b_{uc}^2 \to -1$, $b^2 \to -1 + 2|a| - a^2$, and as $b_{uc}^2 \to 1$, $b^2 \to 1 - a^2$. In addition, note that equation (1) is linear function of b_{uc}^2 for a given a.