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The Digital Economy, New Products and Consumer Welfare

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Background

Much recent interest in measurement and the Digital Economy:

- **Feldstein (2017)**
- **Reinsdorf and Schreyer (2017)**
- **Syverson (2017)**
- **Groshen, Moyer, Aizcorbe, Bradley and Friedman (2017)**
- **Ahmad, Ribarsky and Reinsdorf (2017)**
- **Hulten and Nakamura (2017)**
- **Ahmad and Schreyer (2016)**
- **Byrne, Fernald and Reinsdorf (2016)**
- **Brynjolfsson, Diewert, Eggers, Fox and Gannamaneni (2018)**

Background

- Benefits of the Digital Economy are evident in everyday life, but are they reflected appropriately in official statistics?
- Many new products, and many disappearing products.
- The measurement of the net benefits of new and disappearing products depends on what type of index the NSO is using to deflate final demand aggregates.
- Derive expressions for quantifying biases in e.g. GDP from standard NSO practices.

Background

- Statistical agencies typically use a "matched model" approach to construct price indexes → **maximum overlap index**

- These are used to deflate value aggregates.

- From the economic approach to index numbers, **reservation prices** for the missing products should be matched with the zero quantities for the missing products in each period

- The reservation price for a missing product is the price which would induce a utility maximizing potential purchaser of product to demand zero units of it (Hicks 1940; Hofsten 1952; Hausman 1996).

Background

- If reservation prices are estimated, elicited from surveys, online experiments, or guessed, then the "true" price index can be calculated and compared to its maximum overlap counterpart.

- An estimate of the bias in the deflator can be formed. This bias in the deflator translates into a corresponding bias in the real output aggregate.

- The context we consider is one in which transaction level data are available so that indexes can be calculated from the elementary level.

True Share and Maximum Overlap Shares

Group 1 Products: Present in both periods

$$p_1^t \equiv [p_{11}^t,\ldots,p_{1N}^t] >> 0_N \text{ and } q_1^t \equiv [q_{11}^t,\ldots,q_{1N}^t] > 0_N \text{ for } t = 0,1.$$

Group 2 Products: New goods only available from period 1

Period 0: $p_2^{0*} \equiv [p_{21}^{0*},\ldots,p_{2K}^{0*}] >> 0_K$ and $q_2^0 \equiv [q_{11}^0,\ldots,q_{1K}^0] = 0_K$.

NB: p_2^{0*} are the positive reservation prices

Period 1: $p_2^1 \equiv [p_{21}^1,\ldots,p_{2K}^1] >> 0_K$ and $q_2^1 \equiv [q_{21}^1,\ldots,q_{2K}^1] > 0_K$

True Share and Maximum Overlap Shares

Group 3 Products: Disappearing goods, only available in period 0

Period 0: $p_3^0 \equiv [p_{31}^0,\ldots,p_{3M}^0] \gg 0_M$ and $q_3^0 \equiv [q_{31}^0,\ldots,q_{3M}^0] > 0_M$.

Period 1: $p_3^{1*} \equiv [p_{31}^{1*},\ldots,p_{3M}^{1*}] \gg 0_M$ and $q_3^1 \equiv [q_{31}^1,\ldots,q_{3M}^1] = 0_M$.

NB: p_3^{1*} are the positive reservation prices

True Share and Maximum Overlap Shares

	Period 0	Period 1
Group 1 Continuing	✓	✓
Group 2 New	X	✓
Group 3 Disappearing	✓	X

True Share and Maximum Overlap Shares

Group 1 True expenditure shares (continuing goods):

$s_{1n}^0 \equiv p_{1n}^0 q_{1n}^0/[p_1^0 \cdot q_1^0 + p_2^{0*} \cdot q_2^0 + p_3^0 \cdot q_3^0]$; $n = 1,\ldots,N$;

$= p_{1n}^0 q_{1n}^0/[p_1^0 \cdot q_1^0 + p_3^0 \cdot q_3^0]$ since $q_2^0 = 0_K$;

$s_{1n}^1 \equiv p_{1n}^1 q_{1n}^1/[p_1^1 \cdot q_1^0 + p_2^1 \cdot q_2^1 + p_3^{1*} \cdot q_3^1]$; $n = 1,\ldots,N$;

$= p_{1n}^1 q_{1n}^1/[p_1^1 \cdot q_1^1 + p_2^1 \cdot q_2^1]$ since $q_3^1 = 0_M$.

Can be calculated using observable data.

True Share and Maximum Overlap Shares

Group 2 True expenditure shares (new goods):

$s_{2k}^0 \equiv 0$ since $q_2^0 = 0_K$;

$s_{2k}^1 \equiv p_{2k}^1 q_{2k}^1/[p_1^1 \cdot q_1^1 + p_2^1 \cdot q_2^1]$ since $q_3^1 = 0_M$.

Group 3 True expenditure shares (disappearing goods):

$s_{3m}^0 \equiv p_{3m}^0 q_{3m}^0/[p_1^0 \cdot q_1^0 + p_3^0 \cdot q_3^0]$ since $q_2^0 = 0_K$;

$s_{3m}^1 \equiv 0$ since $q_3^1 = 0_M$

True Share and Maximum Overlap Shares

Maximum overlap share for product n in period t:

$s_{1nO}{}^t \equiv p_{1n}{}^t q_{1n}{}^t / p_1{}^t \cdot q_1{}^t$; $t = 0,1;\ n = 1,\ldots,N.$

Relationships between the true Group 1 shares and the maximum overlap Group 1 shares:

$s_{1n}{}^0 = s_{1nO}{}^0[1 - \Sigma_{m=1}\ s_{3m}{}^0]$; $n = 1,\ldots,N;$

$s_{1n}{}^1 = s_{1nO}{}^1[1 - \Sigma_{k=1}\ s_{2k}{}^1]$. $n = 1,\ldots,N;$

(de Haan and Krisnich 2012)

Törnqvist Price Index

Törnqvist index is the target index for the US CPI.

Log of the Törnqvist maximum overlap index:

$$\ln P_{TO} \equiv \Sigma_{n=1} (1/2)(s_{1nO}{}^0 + s_{1nO}{}^1)\ln(p_{1n}{}^1/p_{1n}{}^0)$$

Log of the true Törnqvist maximum overlap index:

$$\ln P_T \equiv \Sigma_{n=1}(1/2)(s_{1n}{}^0 + s_{1n}{}^1)\ln(p_{1n}{}^1/p_{1n}{}^0) + \Sigma_{k=1} (1/2)(s_{2k}{}^0 + s_{2k}{}^1)\ln(p_{2k}{}^1/p_{2k}{}^{0*})$$

$$+ \Sigma_{m=1}(1/2)(s_{3m}{}^0 + s_{3m}{}^1)\ln(p_{3m}{}^{1*}/p_{3m}{}^0)$$

$$= \ln P_{TO} + \ln\kappa + \ln\mu$$

Törnqvist Price Index

$P_T = P_{TO} \times \kappa \times \mu$

κ can be regarded as a measure of the ***reduction*** in the true cost of living due to the introduction of new products. Thus **κ is likely to be less than 1.**

μ can be regarded as a measure of the ***increase*** in the true cost of living due to the disappearance of existing products. Thus **μ is likely to be greater than 1.**

Törnqvist Price Index

In case you're wondering…..

$$\ln\kappa \equiv (1/2)\Sigma_{k=1}\, s_{2k}^{1}[\ln(p_{2k}^{1}/p_{2k}^{0*}) - \ln P_{JO}^{1}];$$

$$\ln\mu \equiv (1/2)\Sigma_{m=1}\, s_{3m}^{0}[\ln(p_{3m}^{1*}/p_{3m}^{0}) - \ln P_{JO}^{0}],$$

where:

$$\ln P_{JO}^{1} \equiv \Sigma_{n=1}\, s_{1nO}^{1} \ln(p_{1n}^{1}/p_{1n}^{0});$$

$$\ln P_{JO}^{0} \equiv \Sigma_{n=1}\, s_{1nO}^{0} \ln(p_{1n}^{1}/p_{1n}^{0}).$$

Törnqvist Price Index

Imputed carry backward prices:

$$p_{2kb}^{0} \equiv p_{2k}^{1}/P_{JO}^{1}$$

Imputed carry forward prices:

$$p_{3mf}^{1} \equiv p_{3m}^{0}P_{JO}^{0}$$

Economic theory suggests that the reservation prices will be greater than their inflation adjusted carry forward or backward prices.

$$1 + \kappa_k \equiv p_{2k}^{0*}/p_{2kb}^{0}$$

$$1 + \mu_m \equiv p_{3m}^{1*}/p_{3mf}^{1}$$

Törnqvist Price Index

Exact relationship **between the true Törnqvist index P_T and its maximum overlap counterpart P_{TO}:**

$$\ln(P_T/P_{TO}) = \Sigma_{m=1}(1/2)s_{3m}^{0}\ln(1 + \mu_m) - \Sigma_{k=1}(1/2)s_{2k}^{1}\ln(1 + \kappa_k)$$

Using a first order Taylor's series approximation:

$$(P_T/P_{TO}) - 1 \approx \Sigma_{m=1}(1/2)s_{3m}^{0}\mu_m - \Sigma_{k=1}(1/2)s_{2k}^{1}\kappa_k$$

Törnqvist Price Index

Value aggregates for the goods and services in the group of N + K + M commodities under consideration, v^0 and v^1:

$v^0 \equiv p_1^0 \cdot q_1^0 + p_3^0 \cdot q_3^0$; $v^1 \equiv p_1^1 \cdot q_1^1 + p_2^1 \cdot q_2^1$

True implicit Törnqvist quantity index:

$$Q_T \equiv [v^1/v^0]/P_T$$

Maximum overlap Törnqvist quantity index:

$$Q_{TO} \equiv [v^1/v^0]/P_{TO}$$

Bias in Q_{TO} relative to Q_T:

$$Q_T/Q_{TO} = P_{TO}/P_T$$

Törnqvist Price Index

First order approximation:

$$(Q_T/Q_{TO}) - 1 \approx \Sigma_{k=1}(1/2)s_{2k}{}^1\kappa_k - \Sigma_{m=1}(1/2)s_{3m}{}^0\,\mu_m.$$

If there are no disappearing goods, the right hand side of becomes:

$\Sigma_{k=1}(1/2)s_{2k}{}^1\kappa_k \rightarrow$ **the downward bias in the maximum overlap Törnqvist quantity index for the value aggregate in percentage points.**

That is, the downward bias in welfare from ignoring new goods and services.

Paasche Price Index

We derive similar results for Laspeyres, Paasche and Fisher indexes. Fisher result is very similar to that of the Törnqvist index.

Here we consider the Paasche price index, as it corresponds to a Laspeyres quantity index, which is used by many countries to construct GDP.

Maximum overlap Paasche price index:

$$P_{PO} \equiv p_1^1 \cdot q_1^1 / p_1^0 \cdot q_1^1 = [\Sigma_{n=1}\, s_{1nO}^1\, (p_{1n}^1/p_{1n}^0)^{-1}]^{-1}$$

True Paasche price index:

$$P_P \equiv [\Sigma_{n=1} s_{1n}^1\, (p_{1n}^1/p_{1n}^0)^{-1} + \Sigma_{k=1}\, s_{2k}^1\, (p_{2k}^1/p_{2k}^{0*})^{-1}]^{-1}$$

Paasche Price Index

Going through similar steps as before, we have:

$(P_{PO}/P_P) - 1 = \Sigma_{k=1}\ s_{2k}^{1}\ \beta_k.$

where P_P is the true Paasche index and P_{PO} is the maximum overlap Paasche index, and

β_k expresses how much higher each reservation price is from its Paasche inflation adjusted carry backward price counterpart:

$1 + \beta_k \equiv p_{2k}^{0*}/p_{2kb}^{0}$

Thus, expect Paasche maximum overlap index to have upward bias if there are new products in period 1.

Paasche Price Index

True and maximum overlap Laspeyres indexes:

$$Q_L \equiv [v^1/v^0]/P_P \qquad Q_{LO} \equiv [v^1/v^0]/P_{PO}.$$

The bias in Q_{LO}, the maximum overlap Laspeyres index, relative to its true counterpart Q_L can be measured by the ratio Q_L/Q_{LO}:

$$(Q_L/Q_{LO}) - 1 = (P_{PO}/P_P) - 1 = \Sigma_{k=1}\, s_{2k}^{1}\, \beta_k$$

Thus the upward bias in the maximum overlap Paasche price index P_{PO} translates into a downward bias in the companion maximum overlap Laspeyres quantity index, Q_{LO}.

Conclusions

- **NSOs often use a maximum overlap index to deflate a value aggregate to construct estimate of e.g. real consumption.**
- **Only products that exist in both periods being compared are then considered.**
- **Derive expressions which arise from the use of maximum overlap indexes for the Törnqvist, Laspeyres, Paasche and Fisher price and quantity index formulae.**
- **Simple expressions, but require transaction level data and Hicksian reservation prices for the missing products in both periods.**
- **Also consider bias formulae for replacement samples (*à la* Triplett 2004)**

Diewert, W.E., K.J. Fox and P. Schreyer (2017), "The Digital Economy, New Products and Consumer Welfare," Vancouver School of Economics Discussion Paper 17-09, University of British Columbia