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Decomposing Value Added Growth into Explanatory Factors

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Abstract

A method for decomposing nominal value added growth is presented, which identifies the contributions from efficiency change, growth of primary inputs, changes in output and input prices, technical progress and returns to scale. In order to implement the decomposition, an estimate of the relevant cost constrained value added function for the two periods under consideration is required. This is taken to be the free disposal hull of past observations. Aggregation over sectors is also considered. The methodology is illustrated using U.S. data for two sectors over the years 1960-2014.

JEL Classification Numbers: C43, D24, D61, E23, H44, O47

Key Words: Measurement of output, input and productivity, value added functions, revenue functions, variable profit functions, duality theory, economic price and quantity indexes, technical progress, total factor productivity, revenue efficiency, aggregation over sectors.

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1. Introduction

Understanding sources of economic growth has long been of interest to academics and policy makers. A better understanding of the determinants of value added growth can provide insights into the potential for policies to address inefficiencies and a deeper understanding of the drivers of productivity, a topic of heightened recent interest given the slowdown in productivity growth across many developed countries; see e.g. Gordon (2016), Mokyr, Vickers and Ziebarth (2015), Byrne, Fernald and Reinsdorf (2016), and Syverson (2016).

While there has been much attention to growth at the aggregate economy level, there has less at the sectoral level. To address this, we derive exact decompositions of nominal value added growth for sectors of an economy into explanatory factors, and illustrate these using data for the US Corporate Nonfinancial and the Noncorporate Nonfinancial sectors, 1960 to 2014.

We take the explanatory factors of value added growth in a sector to be as follows:

- efficiency changes,
- changes in output prices,
- changes in primary inputs,
- changes in input prices,
- technical progress, and
- returns to scale.

We start by decomposing value added growth in a single production sector into these components, before considering the relationship with aggregate (across sector) value added growth. In order to implement our decomposition, an estimate of the sector's best practice technology for the two periods under consideration is required. This could be obtained using econometric techniques or nonparametric frontier modelling, such as Data Envelopment Analysis (DEA) type techniques; see e.g. Charnes and Cooper (1985) and Färe, Grosskopf and Lovell (1985). We do not make any of the convexity assumptions that are typical in this literature, and instead use the Free Disposal Hull (FDH) approach of Tulkens (1986)(1993) and his co-authors; see also Diewert and Fox (2014)(2016a).

During recessions, it seems unlikely that production units are operating on their production frontiers (fixed capital stock components cannot be readily reduced in the light of reduced output demands) and thus it is important for a growth accounting methodology to allow for technical and allocative inefficiency. Our methodological approach does this. It has the advantages that it does not involve any econometric estimation, and involves only observable data on input and output prices and quantities for the sector. Thus it is simple enough to be implemented by statistical agencies.

Another positive feature of our approach is that it rules out technical regress, which is a problematic concept for a broad range of economic models; see e.g. Aiyar, Dalgaard and Moav (2008) and Diewert and Fox (2016b). A consequence of ruling out technical regress is that when there is a recession, for example, the loss of efficiency is gross loss of efficiency less any technical progress that occurs during recession years. Hence, in this case estimates of efficiency losses may be offset by technical progress, and what is measured as efficiency change is the net effect.

The rest of the chapter is organized as follows. The core methodology is explained in the following section, where we introduce the *cost constrained value added function* which is used throughout. In section 3, the method for decomposing value added growth into our six components for each sector is derived. Section 4 describes our nonparametric approach to obtaining empirical estimates for the best practice cost constrained value added functions, which allows us to decompose TFP growth for a sector into explanatory components. Using our data on two major sectors of the U.S. economy, sections 5 and 6 provide empirical applications of the approach, with the results shedding light on sources of value added and productivity growth for the U.S. over a 55 year period. Section 7 presents results from one solution to the problem of aggregating over sectors, drawing on the results of Diewert and Fox (2016c), and section 8 concludes.

2. The Cost Constrained Value Added Function for a Sector

Suppose that a sector produces M net outputs,[1] $y \equiv [y_1,...,y_M]$, using N primary inputs $x \equiv [x_1,...,x_N] \geq 0_N$, while facing the strictly positive vector of net output prices $p \equiv [p_1,...,p_M] >> 0_M$ and the strictly positive vector of input prices $w \equiv [w_1,...,w_N] >> 0_N$. The value of primary inputs used by the sector during period t is then $w \cdot x \equiv \Sigma_{n=1}^{N} w_n x_n$. Denote the *period t production possibilities set* for the sector by S^t.[2] Define the sector's *period t cost constrained value added function,* $R^t(p,w,x)$ as follows:[3]

[1] Let $(y,x) \in S^t$ where $y = [y_1,...,y_M]$ and $x \equiv [x_1,...,x_N] \geq 0_N$. If $y_m > 0$, then the sector produces the mth net output during period t while if $y_m < 0$, then the sector uses the mth net output as an intermediate input.

[2] We assume that S^t satisfies the following regularity conditions: (i) S^t is a closed set; (ii) for every $x \geq 0_N$, $(0_M,x) \in S^t$; (iii) if $(y,x) \in S^t$ and $y^* \leq y$, then $(y^*,x) \in S^t$ (free disposability of net outputs); (iv) if $(y,x) \in S^t$ and $x^* \geq x$, then $(y,x^*) \in S^t$ (free disposability of primary inputs); (v) if $x \geq 0_N$ and $(y,x) \in S^t$, then $y \leq b(x)$ where the upper bounding vector b can depend on x (bounded primary inputs implies bounded from above net outputs). When applying our methodology, we will need somewhat stronger conditions that will imply that that the cost constrained value added function is *positive* when evaluated at observed data points.

[3] Note that $R^t(p,w,x)$ is well defined even if there are increasing returns to scale in production; i.e., the constraint $w \cdot z \leq w \cdot x$ leads to a finite value for $R^t(p,w,x)$. The cost constrained value added function is analogous to Diewert's (1983; 1086) *balance of trade restricted value added function* and Diewert and Morrison's (1986; 669) *domestic sales function*. However, the basic idea can be traced back to Shephard's (1974) *maximal return function*, Fisher and Shell's (1998; 48) *cost restricted sales function* and Balk's (2003; 34) *indirect revenue function*. See also Färe, Grosskopf and Lovell (1992; 286) and Färe and Primont (1994; 203) on Shephard's formulation. Shephard, Fisher and Shell and Balk defined their functions as $IR^t(p,w,c) \equiv \max_{y,z} \{p \cdot y : w \cdot z \leq c ; (y,z) \in S^t\}$ where $c > 0$ is a scalar cost constraint. It can be seen that our cost constrained value added function replaces c in the above definition by $w \cdot x$, a difference which will be important in forming our input indexes and hence our value added decompositions. Another difference is that our y vector is a net output vector; i.e., some components of y can be negative. Excluding Diewert and Morrison (1986) and Diewert (1983), the other authors required that y be nonnegative. This makes a difference to our analysis. Also, our regularity conditions are weaker than the ones that are usually used.

(1) $R^t(p,w,x) \equiv \max_{y,z} \{p\cdot y : (y,z)\in S^t;\ w\cdot z \le w\cdot x\}$.

If (y^*,z^*) solves the constrained maximization problem defined by (1), then sectoral value added $p\cdot y$ is maximized subject to the constraints that (y,z) is a feasible production vector and primary input expenditure $w\cdot z$ is equal to or less than "observed" primary input expenditure $w\cdot x$. Thus if the sector faces the prices $p^t >> 0_M$ and $w^t >> 0_N$ during period t and (y^t, x^t) is the sector's observed production vector, then production will be *value added efficient* if the observed value added, $p^t\cdot y^t$, is equal to the optimal value added, $R^t(p^t,w^t,x^t)$. However, production may not be efficient and so the following inequality will hold:

(2) $p^t\cdot y^t \le R^t(p^t,w^t,x^t)$.

Following the example of Balk (1998; 143), we define the *value added* or *net revenue efficiency* of the sector during period t, e^t, as follows:

(3) $e^t \equiv p^t\cdot y^t/R^t(p^t,w^t,x^t) \le 1$

where the inequality in (3) follows from (2). Thus if $e^t = 1$, then production is allocatively efficient in period t and if $e^t < 1$, then production for the sector during period t is allocatively inefficient. Note that the above definition of value added efficiency is a net revenue counterpart to Farrell's (1957; 255) cost based measure of *overall efficiency* in the DEA context, which combined his measures of technical and (cost) allocative efficiency. DEA or *Data Envelopment Analysis* is the term used by Charnes and Cooper (1985) and their co-workers to denote an area of analysis which is called the nonparametric approach to production theory[4] or the measurement of the efficiency of production[5] by economists.

The cost constrained value added function has some interesting mathematical properties. For fixed w and x, $R^t(p,w,x)$ is a convex and linearly homogeneous function of p.[6] For fixed p and w, $R^t(p,w,x)$ is nondecreasing in x. If S^t is a convex set, then $R^t(p,w,x)$ is also concave in x. For fixed p and x, $R^t(p,w,x)$ is homogeneous of degree 0 in w.

It is possible to get more insight into the properties of R^t if we introduce the sector's *period t value added function* $\Pi^t(p,x)$. Thus for $p >> 0_M$ and $x \ge 0_N$, define $\Pi^t(p,x)$ as follows:[7]

[4] See Hanoch and Rothschild (1972), Diewert and Parkan (1983), Varian (1984) and Diewert and Mendoza (2007).
[5] See Farrell (1957), Afriat (1972), Färe and Lovell (1978), Färe, Grosskopf and Lovell (1985), Coelli, Rao and Battese (1997) and Balk (1998) (2003).
[6] A version of Hotelling's Lemma also holds for $R^t(p,w,x)$. Suppose y^*,x^* is a solution to the constrained maximization problem that defines $R^t(p^*,w^*,x^*)$ and $\nabla_p R^t(p^*,w^*,x^*)$ exists. Then $y^* = \nabla_p R^t(p^*,w^*,x^*)$. See Diewert and Morrison (1986; 670) for the analogous properties for their sales function.
[7] This function is known as the GDP function or the national product function in the international trade literature; see Kohli (1978) (1990), Woodland (1982) and Feenstra (2004; 76). It is known as the gross, restricted or variable profit function in the duality literature; see Gorman (1968), McFadden (1978) and Diewert (1973) (1974). Sato (1976) called it a value added function. It was introduced into the economics

(4) $\Pi^t(p,x) \equiv \max_y \{p\cdot y: (y,x)\in S^t\}$.

Using definitions (1) and (4), it can be seen that the cost constrained value added function $R^t(p,w,x)$ has the following representation:

$$
\begin{aligned}
(5)\ R^t(p,w,x) &\equiv \max_{y,z} \{p\cdot y : (y,z)\in S^t;\ w\cdot z \le w\cdot x;\} \\
&= \max_z \{\Pi^t(p,z) : w\cdot z \le w\cdot x;\ z \ge 0_N\}.
\end{aligned}
$$

Holding p constant, we can define the period t *"utility" function* $f^t(z) \equiv \Pi^t(p,z)$ and the second maximization problem in (5) becomes the following "utility" maximization problem:

(6) $\max_z \{f^t(z) : w\cdot z \le w\cdot x;\ z \ge 0_N\}$

where $w\cdot x$ is the consumer's "income". For u in the range of $\Pi^t(p,z)$ over the set of nonnegative z vectors and for $w >> 0_N$, we can define the *cost function* $C^t(u,w)$ that corresponds to $f^t(z)$ as follows:[8]

(7) $C^t(u,w) \equiv \min_z \{w\cdot z : f^t(z) \ge u;\ z \ge 0_N\} = \min_z \{w\cdot z : \Pi^t(p,z) \ge u;\ z \ge 0_N\}$.

If $\Pi^t(p,z)$ increases as all components of z increase, then $C^t(u,w)$ will be increasing in u and we can solve the following maximization problem for a unique u^*:

(8) $\max_u \{u: C^t(u,w) \le w\cdot x\}$.

Using the solution to (8), we will have the following solution for the maximization problem that defines $R^t(p,w,x)$:

(9) $R^t(p,w,x) = u^*$

with $C^t(u^*,w) = w\cdot x$.

The above formulae simplify considerably if S^t is a cone, so that production is subject to constant returns to scale. If S^t is a cone, then $\Pi^t(p,z)$ is linearly homogeneous in z and hence, so is $f^t(z) \equiv \Pi^t(p,z)$. Define the unit cost function c^t that corresponds to f^t as follows:[9]

(10) $c^t(w,p) \equiv \min_z \{w\cdot z : \Pi^t(p,z) \ge 1;\ z \ge 0_N\}$.

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literature by Samuelson (1953). We use the cost constrained value added function as our basic building block in this chapter rather than the conceptually simpler GDP function because the cost constrained value added function allows us to deal with technologies which exhibit global increasing returns to scale.

[8] Of course, $f^t(z)$ should be denoted as $f^t(z,p)$ and $C^t(u,w)$ should be denoted as $C^t(u,w,p)$.

[9] $c^t(w,p)$ will be linearly homogeneous and concave in w for fixed p and it will be homogeneous of degree minus one in p for fixed w. If $\Pi^t(p,z)$ is increasing in p_m, then $c^t(w,p)$ will be decreasing in p_m.

The total cost function, $C^t(u,w) = C^t(u,w,p)$ is now equal to $uc^t(w,p)$ and the solution to (8) is the following u^*:

$$(11)\ u^* = R^t(p,w,x) \equiv w\cdot x/c^t(w,p).$$

3. Decomposing Value Added Growth for a Sector into Explanatory Factors

We assume that we can observe the net outputs and inputs used by the sector or production unit for two consecutive periods, say period t−1 and t. The observed net output and input vectors for the two periods are denoted by y^{t-1}, y^t, $x^{t-1} >> 0_N$ and $x^t >> 0_N$. The observed output and input price vectors are the strictly positive vectors p^{t-1}, p^t, w^{t-1} and w^t. We also assume that $p^i\cdot y^j > 0$ and $w^i\cdot x^j > 0$ for $i = t-1,t$ and $j = t-1,t$. Our task in this section is to decompose (one plus) the growth in observed nominal value added over the two periods, $p^t\cdot y^t/p^{t-1}\cdot y^{t-1}$, into explanatory growth factors.

One of the explanatory factors will be the *growth in the value added efficiency* of the sector or production unit. In the previous section, we defined the period t value added efficiency as $e^t \equiv p^t\cdot y^t/R^t(p^t,w^t,x^t)$. Define the corresponding period t−1 efficiency as $e^{t-1} \equiv p^{t-1}\cdot y^{t-1}/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})$. Given the above definitions of revenue efficiency in periods t−1 and t, we can define an index of the *change in value added efficiency* ε^t for the sector over the two periods as follows:

$$(12)\ \varepsilon^t \equiv e^t/e^{t-1} = [p^t\cdot y^t/R^t(p^t,w^t,x^t)]/[p^{t-1}\cdot y^{t-1}/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})].$$

Thus if $\varepsilon^t > 1$, then value added efficiency has *improved* going from period t−1 to t whereas it has *fallen* if $\varepsilon^t < 1$.

Notice that the cost constrained value added function for the production unit in period t, $R^t(p,w,x)$, depends on four sets of variables:

- The time period t and this index t serves to indicate that the period t technology set S^t is used to define the period t value added function;
- The vector of net output prices p that the production unit faces;
- The vector of primary input prices w that the production unit faces and
- The vector of primary inputs x which is available for use by the production unit during period t.

At this point, we will follow the methodology that is used in the economic approach to index number theory that originated with Konüs (1939) and Allen (1949) and we will use the value added function to define various *families of indexes* that vary only *one* of the four sets of variables, t, p, w and x, between the two periods under consideration and hold constant the other sets of variables.[10]

[10] The theory which follows is largely adapted from Diewert (1980a; 455-461) (1983, 1054-1076) (2014), Diewert and Morrison (1986), Kohli (1990), Fox and Kohli (1998) and the IMF, ILO, OECD, UN and the

Our first family of factors that explain sectoral value added growth is a family of *net output price indexes*, $\alpha(p^{t-1},p^t,x,t)$:

(13) $\alpha(p^{t-1},p^t,w,x,s) \equiv R^s(p^t,w,x)/R^s(p^{t-1},w,x)$.

Thus the net output price index $\alpha(p^{t-1},p^t,w,x,s)$ defined by (13) is equal to the (hypothetical) cost constrained value added $R^s(p^t,w,x)$ generated by the best practice technology of period s while facing the period t net output prices p^t and the reference primary input prices w and using the reference primary input vector x, divided by the cost constrained value added $R^s(p^{t-1},w,x)$ generated by the best practice technology of period s while facing the period t−1 net output prices p^{t-1} and the reference primary input prices w and using the same reference primary input vector x. Thus for each choice of technology (i.e., s could equal t−1 or t) and for each choice of reference vectors of input prices w and quantities x, we obtain a possibly different net output price index.

Following the example of Konüs (1939) in his analysis of the true cost of living index, it is natural to single out two special cases of the family of net output price indexes defined by (13): one choice where we use the period t−1 technology and set the reference input prices and quantities equal to the period t−1 input prices and quantities w^{t-1} and x^{t-1} (which gives rise to a *Laspeyres type net output price index*) and another choice where we use the period t technology and set the reference input prices and quantities equal to the period t prices and quantities w^t and x^t (which gives rise to a *Paasche type net output price index*). We define these special cases $\alpha_L{}^t$ and $\alpha_P{}^t$ as follows:

(14) $\alpha_L{}^t \equiv \alpha(p^{t-1},p^t,w^{t-1},x^{t-1},t-1) \equiv R^{t-1}(p^t,w^{t-1},x^{t-1})/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})$;
(15) $\alpha_P{}^t \equiv \alpha(p^{t-1},p^t,w^t,x^t,t) \equiv R^t(p^t,w^t,x^t)/R^t(p^{t-1},w^t,x^t)$.

Since both output price indexes, $\alpha_L{}^t$ and $\alpha_P{}^t$, are equally representative, a single estimate of net output price change should be set equal to a symmetric average of these two estimates. We choose the geometric mean as our preferred symmetric average and thus our preferred overall measure of net output price growth is the following overall *net output price index*, α^t:[11]

(16) $\alpha^t \equiv [\alpha_L{}^t\,\alpha_P{}^t]^{1/2}$.

Our second family of factors that explain value added growth is a family of *input quantity indexes*, $\beta(x^{t-1},x^t,w)$:

World Bank (2004; 455-456). This approach to the net output quantity and input price indexes is an adaptation of the earlier work on theoretical price and quantity indexes by Konüs (1939), Allen (1949), Fisher and Shell (1972) (1998), Samuelson and Swamy (1974), Archibald (1977) and Balk (1998).

[11] Choosing the geometric mean leads to a measure of net output price inflation that satisfies the time reversal test; i.e., the resulting index has the property that it is equal to the reciprocal of the corresponding index that measures price change going backwards in time rather than forward in time; see Diewert (1997) and Diewert and Fox (2017) on this point.

(17) $\beta(x^{t-1},x^t,w) \equiv w\cdot x^t/w\cdot x^{t-1}$.

The input quantity index $\beta(x^{t-1},x^t,w)$ defined by (17) is equal to a ratio of simple linear aggregates of the observed input vectors for periods t−1 and t, x^{t-1} and x^t, where we use the vector of strictly positive input prices $w >> 0_N$ as weights. We note that this family of input quantity index does not use the cost constrained value added function. An alternative definition for a family of input quantity indexes that uses the cost restricted value added function for period s and reference vectors p and w is $\beta^*(x^{t-1},x^t,p,w,s) \equiv R^s(p^s,w^s,x^t)/R^s(p^s,w^s,x^{t-1})$.[12] If the period s technology set is a cone, then using (11), it can be seen that $\beta^*(x^{t-1},x^t,p,w,s) = w\cdot x^t/w\cdot x^{t-1} = \beta(x^{t-1},x^t,w)$. In the general case where the period s technology is not a cone, the input growth measure $\beta^*(x^{t-1},x^t,p,w,s)$ will also incorporate the effects of nonconstant returns to scale. In this general case, it seems preferable to isolate the effects of nonconstant returns to scale and the use of the simple input quantity indexes defined by (17) will allow us to do this as will be seen below.

It is natural to single out two special cases of the family of input quantity indexes defined by (17): one choice where we use the period t−1 input prices w^t which gives rise to the *Laspeyres input quantity index* $\beta_L{}^t$ and another choice where we set the reference input prices equal to w^t (which gives rise to the *Paasche input quantity index* $\beta_P{}^t$. Thus define these special cases $\beta_L{}^t$ and $\beta_P{}^t$ as follows:

(18) $\beta_L{}^t \equiv w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1}$;
(19) $\beta_P{}^t \equiv w^t\cdot x^t/w^t\cdot x^{t-1}$.

Since both input quantity indexes, $\beta_L{}^t$ and $\beta_P{}^t$, are equally representative, a single estimate of input quantity change should be set equal to a symmetric average of these two estimates. We choose the geometric mean as our preferred symmetric average and thus our preferred overall measure of input quantity growth is the following overall *input quantity index*, β^t:[13]

(20) $\beta^t \equiv [\beta_L{}^t\,\beta_P{}^t]^{1/2}$.

Our next family of indexes will measure the effects on cost constrained value added of a change in input prices going from period t−1 to t. We consider a family of measures of the relative change in cost constrained value added of the form $R^s(p,w^t,x)/R^s(p,w^{t-1},x)$. Since $R^s(p,w,x)$ is homogeneous of degree 0 in the components of w, it can be seen that we cannot interpret $R^s(p,w^t,x)/R^s(p,w^{t-1},x)$ as an input price index. If there is only one primary input, $R^s(p,w^t,x)/R^s(p,w^{t-1},x)$ is identically equal to unity and this measure of input price change will be independent of changes in the price of the single input. It is best to interpret $R^s(p,w^t,x)/R^s(p,w^{t-1},x)$ as measuring the effects on cost constrained value added of a change in the relative proportions of primary inputs used in production or in

[12] The counterpart to this family of input quantity indexes was defined by Sato (1976; 438) and Diewert (1980a; 456) using value added functions (i.e., the functions $\Pi^s(p,x)$) with the assumption that there was no technical progress between the two periods being compared.
[13] This index is Fisher's (1922) ideal input quantity index.

the *mix* of inputs used in production that is induced by a change in relative input prices when there is more than one primary input. Thus define the family of *input mix indexes* $\gamma(w^{t-1},w^t,p,x,s)$ as follows:[14]

(21) $\gamma(w^{t-1},w^t,p,x,s) \equiv R^s(p,w^t,x)/R^s(p,w^{t-1},x)$.

As usual, we will consider two special cases of the above family of input mix indexes, a Laspeyres case and a Paasche case. However, the Laspeyres case $\gamma_{LPP}{}^t$ will use the period t cost constrained value added function and the period t−1 reference vectors p^{t-1} and x^{t-1} while the Paasche case $\gamma_{PLL}{}^t$ will use the use the period t−1 cost constrained value added function and the period t reference vectors p^t and x^t:

(22) $\gamma_{LPP}{}^t \equiv \gamma(w^{t-1},w^t,p^{t-1},x^t,t) \quad \equiv R^t(p^{t-1},w^t,x^t)/R^t(p^{t-1},w^{t-1},x^t)$;
(23) $\gamma_{PLL}{}^t \equiv \gamma(w^{t-1},w^t,p^t,x^{t-1},t-1) \equiv R^{t-1}(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^{t-1},x^{t-1})$.

The reason for these rather odd looking choices for reference vectors will be justified below in more detail but, basically, we make these choices in order to have value added growth decompositions into explanatory factors that are exact without making restrictive assumptions on the technology sets.

As usual, the above two indexes are equally representative and so it is natural to take an average of these two measures. We choose the geometric mean as our preferred symmetric average and thus our preferred overall measure of input mix change is the following overall *input mix index*, γ^t:

(24) $\gamma^t \equiv [\gamma_{LPP}{}^t\gamma_{PLL}{}^t]^{1/2}$.

We turn now to the effects on cost constrained value added due to the effects of technical progress; i.e., as time marches on, new techniques are developed that allow increased net outputs using the same inputs or that allow the same net outputs to be produced by fewer inputs. Thus we use the cost constrained value added function in order to define a family of *technical progress indexes* going from period t−1 to t, $\tau(p,w,x)$, for reference vectors of output and input prices, p and w, and a reference vector of input quantities x as follows:[15]

[14] It would be more accurate to say that $\gamma(w^{t-1},w^t,p,x,s)$ represents the hypothetical proportional change in cost constrained value added for the period s reference technology due to the effects of a change in the input price vector from w^{t-1} to w^t when facing the reference net output prices p and the reference vector of inputs x. Thus we shorten this description to say that γ is an "input mix index". If there is only one primary input, then since $R^s(p,w,x)$ is homogeneous of degree 0 in w, it can be seen that $\gamma(w^{t-1},w^t,p,x,s) \equiv R^s(p,w^t,x)/R^s(p,w^{t-1},x) = [(w_1{}^t)^0\, R^s(p,1,x)]/[(w_1{}^{t-1})^0\, R^s(p,1,x)] = 1$; i.e., if there is only one primary input, then the input mix index is identically equal to 1. For alternative mix definitions, see Balk (2001) and Diewert (2014; 62).
[15] This family of technical progress measures was defined by Diewert and Morrison (1986; 662) using the value added function $\Pi^t(p,x)$. A special case of the family was defined earlier by Diewert (1983; 1063). Balk (1998; 99) also used this definition and Balk (1998; 58), following the example of Salter (1960), also used the joint cost function to define a similar family of technical progress indexes.

(25) $\tau(t-1,t,p,w,x) \equiv R^t(p,w,x)/R^{t-1}(p,w,x)$.

Technical progress measures are usually defined in terms of upward shifts in production functions or outward shifts of production possibilities sets due to the discovery of new techniques or managerial innovations over time. If there is positive technical progress going from period t−1 to t, then $R^t(p,w,x)$ will be greater than $R^{t-1}(p,w,x)$ and hence $\tau(p,w,x)$ will be greater than one and this measure of technical progress is equal to the proportional increase in value added that results from the expansion of the underlying best practice technology sets due to the passage of time. For each choice of reference vectors of output and input prices, p and w, and reference vector of input quantities x, we obtain a possibly different measure of technical progress.

Again, we will consider two special cases of the above family of technical progress indexes, a Laspeyres case and a Paasche case. However, the Laspeyres case $\tau_L{}^t$ will use the period t input vector x^t as the reference input vector and the period t−1 reference output and input price vectors p^{t-1} and w^{t-1} while the Paasche case $\tau_P{}^t$ will use the use the period t−1 input vector x^{t-1} as the reference input and the period t reference output and input price vectors p^t and w^t:

(26) $\tau_L{}^t \equiv \tau(t-1,t,p^{t-1},w^{t-1},x^t) \equiv R^t(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^t)$.
(27) $\tau_P{}^t \equiv \tau(t-1,t,p^t,w^t,x^{t-1}) \quad \equiv R^t(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^t,x^{t-1})$.

Using (11), recall that if the reference technologies in periods t and t−1 are cones, then we have $R^t(p,w,x) = w\cdot x/c^t(w,p)$ and $R^{t-1}(p,w,x) = w\cdot x/c^{t-1}(w,p)$. Thus in the case where the reference technology is subject to constant returns to scale, $\tau_L{}^t \equiv \tau(t-1,t,p^{t-1},w^{t-1},x^t)$ turns out to be independent of x^t and $\tau_P{}^t \equiv \tau(t-1,t,p^t,w^t,x^{t-1})$ turns out to be independent of x^{t-1}. These “mixed” indexes of technical progress are then true Laspeyres and Paasche type indexes.

We have one more family of indexes to define and that is a family of returns to scale measures. Our measures are analogous to the global measures of returns to scale that were introduced by Diewert (2014; 62) using cost functions. Here we will use the cost restricted value added function in place of the cost function. Our returns to scale measure will be a measure of output growth divided by input growth from period t−1 to t but the technology is held constant when we compute the output growth measure. Our measure of input growth will be $w\cdot x^t/w\cdot x^{t-1}$ where w is a positive vector of reference input prices. Now pick positive reference price vector p that will value our M net outputs. If we hold the technology constant at period t−1 levels, our measure of output growth will be $R^{t-1}(p,w,x^t)/R^{t-1}(p,w,x^{t-1})$. If we hold the technology constant at period t levels, our measure of output growth will be $R^t(p,w,x^t)/R^t(p,w,x^{t-1})$. Thus for the reference technology set indexed by s (equal to t−1 or t) and reference price vectors p and w, define the family of *returns to scale measures* $\delta(x^{t-1},x^t,p,w,s)$ as follows:

(28) $\delta(x^{t-1},x^t,p,w,s) \equiv [R^s(p,w,x^t)/R^s(p,w,x^{t-1})]/[w\cdot x^t/w\cdot x^{t-1}]$.

We define the Laspeyres and Paasche special cases of (28):

(29) $\delta_L{}^t \equiv \delta(x^{t-1},x^t,p^{t-1},w^{t-1},t-1) \equiv [R^{t-1}(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})]/[w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1}]$;
(30) $\delta_P{}^t \equiv \delta(x^{t-1},x^t,p^t,w^t,t) \equiv [R^t(p^t,w^t,x^t)/R^t(p^t,w^t,x^{t-1})]/[w^t\cdot x^t/w^t\cdot x^{t-1}]$.

In the case where the period t−1 reference production possibilities set S^{t-1} is a cone so that production is subject to constant returns to scale, then using (11), it can be seen that $\delta_L{}^t$ is equal to 1 and if S^t is a cone, then $\delta_P{}^t$ defined by (30) is also equal to 1.

Our preferred measure of returns to scale to be used in empirical applications is the geometric mean of the above special cases:

(31) $\delta^t \equiv [\delta_L{}^t\,\delta_P{}^t]^{1/2}$.

We are now in a position to decompose (one plus) the growth in value added for the production unit going from period t−1 to t as the product of six explanatory growth factors:

- The change in cost constrained value added efficiency over the two periods; i.e., $\varepsilon^t \equiv e^t/e^{t-1}$ defined by (12) above;
- Growth (or changes) in net output prices; i.e., a factor of the form $\alpha(p^{t-1},p^t,w,x,s)$ defined above by (13);
- Growth (or changes) in input quantities; i.e., a factor of the form $\beta(x^{t-1},x^t,w)$ defined by (17);
- Growth (or changes) in input prices; i.e., an input mix index of the form $\gamma(w^{t-1},w^t,p,x,s)$ defined by (21);
- Changes due to technical progress; i.e., a factor of the form $\tau(t-1,t,p,w,x)$ defined by (25) and
- A returns to scale measure $\delta(x^{t-1},x^t,p,w,s)$ of the type defined by (28).

Straightforward algebra using the above definitions shows that we have the following exact decompositions of the observed value added ratio going from period t−1 to t into explanatory factors of the above type:[16]

(32) $p^t\cdot y^t/p^{t-1}\cdot y^{t-1} = \varepsilon^t\,\alpha_P{}^t\,\beta_L{}^t\,\gamma_{LPP}{}^t\,\delta_L{}^t\,\tau_L{}^t$;
(33) $p^t\cdot y^t/p^{t-1}\cdot y^{t-1} = \varepsilon^t\,\alpha_L{}^t\,\beta_P{}^t\,\gamma_{PLL}{}^t\,\delta_P{}^t\,\tau_P{}^t$.

Now multiply the above decompositions together and take the geometric mean of both sides of the resulting equation. Using the above definitions, it can be seen that we obtain

[16] Diewert (2011) obtained decompositions of cost growth analogous to (32) and (33) under the assumption that the production unit was cost efficient in each period.

the following *exact decomposition of value added growth into the product of six explanatory growth factors*:[17]

(34) $p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \alpha^t \beta^t \gamma^t \delta^t \tau^t$.

If the reference technology exhibits constant returns to scale in periods t−1 and t, then $\delta_L{}^t = \delta_P{}^t = \delta^t = 1$ and the returns to scale factors drop out of the decompositions on the right hand sides of (32)-(34).

Total Factor Productivity growth for the production unit under consideration going from period t−1 to t can be defined (following Jorgenson and Griliches (1967)) as an index of output growth divided by an index of input growth. An appropriate index of output growth is the value added ratio divided by the value added price index α^t. An appropriate index of input growth is β^t. Thus define the *period t TFP growth rate*, $TFPG^t$, for the production unit as follows:[18]

(35) $TFPG^t \equiv \{[p^t \cdot y^t / p^{t-1} \cdot y^{t-1}]/\alpha^t\}/\beta^t = \varepsilon^t \gamma^t \delta^t \tau^t$

where the last equality in (35) follows from (34). Thus in general, period t TFP growth is equal to the product of period t value added efficiency change ε^t, a period t input mix index γ^t (which typically will be small in magnitude), period t technical progress τ^t and period t returns to scale for the best practice technology δ^t. If the reference best practice technologies are subject to constant returns to scale, then the returns to scale term is identically equal to 1 and drops out of the decomposition given by (35).

We follow the example of Kohli (1990) and obtain a levels decomposition for the observed level of nominal value added in period t, $p^t \cdot y^t$, relative to its observed value in period 1, $p^1 \cdot y^1$. We assume that we have price and quantity data for the primary inputs used and net outputs produced by the production unit (p^t,w^t,y^t,x^t) for periods t = 1, 2,...,T. We also assume that we have estimates for the cost constrained value added functions, $R^t(p,w,x)$, that correspond to the best practice technology sets S^t for t = 1,2,...,T. Thus for

[17] Balk (2003; 9-10) introduced the term "profitability" to describe the period t ratio of revenue to cost π^t but he considered this concept earlier; see Balk (1998; 66) for historical references. Diewert and Nakamura (2003; 129) described the same concept by the term "margin". If we divide both sides of (34) through by (one plus) the rate of cost growth, $w^t \cdot x^t / w^{t-1} \cdot x^{t-1}$, we obtain an expression for (one plus) the rate of growth of profitability, π^t/π^{t-1}, which will equal $\varepsilon^t \alpha^t \gamma^t \delta^t \tau^t / \beta^{t**}$ where β^{t**} is the Fisher ideal input price index that matches up with the Fisher ideal input quantity index $\beta^{t;}$ i.e., $\beta^t \beta^{t**} = w^t \cdot x^t / w^{t-1} \cdot x^{t-1}$. This decomposition of profitability growth can be compared to the alternative profitability growth decompositions obtained by Balk (2003; 22), Diewert and Nakamura (2003), O'Donnell (2010; 531) and Diewert (2014; 63). Our present decomposition of profitability is closest to that derived by Diewert. The problem with Diewert's decomposition is that his measure of returns to scale combined returns to scale with mix effects.

[18] There are similar decompositions for TFP growth using just quantity data and Malmquist gross output and input indexes; see Diewert and Fox (2014) (2017). For additional decompositions of TFP growth using both price and quantity data, see Balk (1998), (2001), (2003), Caves, Christensen and Diewert (1982), Diewert and Morrison (1986), Kohli (1990) and Diewert and Fox (2008) (2010). However, we believe that our present decomposition is the most comprehensive decomposition of TFP growth into explanatory factors that makes use of observable price and quantity data for both outputs and inputs.

$t = 2,3,...,T$, we can calculate the period to period growth factors ε^t, α^t, β^t, γ^t, τ^t and δ^t. Define the cumulated explanatory variables as follows:

$$(36)\ E^1 \equiv 1;\ A^1 \equiv 1;\ B^1 \equiv 1;\ C^1 \equiv 1;\ D^1 \equiv 1;\ T^1 \equiv 1.$$

For $t = 2,3,...,T$, define the above variables recursively as follows:

$$(37)\ E^t \equiv \varepsilon^t E^{t-1};\ A^t \equiv \alpha^t A^{t-1};\ B^t \equiv \beta^t B^{t-1};\ C^t \equiv \gamma^t C^{t-1};\ D^t \equiv \delta^t D^{t-1};\ T^t \equiv \tau^t T^{t-1}.$$

Using the above definitions and (34), it can be seen that we have the following *levels decomposition* for the level of period t observed value added relative to its period 1 level:

$$(38)\ p^t \cdot y^t / p^1 \cdot y^1 = A^t\, B^t\, C^t\, D^t\, E^t\, T^t\ ; \qquad t = 2,...,T.$$

Define the period t level of Total Factor Productivity, TFP^t, as follows:

$$(39)\ TFP^1 \equiv 1; \text{ for } t = 2,...,T, \text{ define } TFP^t \equiv (TFPG^t)(TFP^{t-1})$$

where $TFPG^t$ is defined by (35) for $t = 2,...,T$. Using (35)-(39), it can be seen that we have the following *levels decomposition for TFP* using the cumulated explanatory factors defined by (36) and (37):

$$(40)\ TFP^t = [p^t \cdot y^t / p^1 \cdot y^1]/[A^t\, B^t] = C^t\, D^t\, E^t\, T^t\ ; \qquad t = 2,...,T.$$

In the following section, we explain a practical method for obtaining estimates for the cost constrained value added function for a sector.

4. A Nonparametric Approximation to the Cost Constrained Value Added Function

We assume that (y^t,x^t) is the production unit's observed net output and primary input vector respectively, where $x^t > 0_N$ and the observed vector of net output and primary input prices is (p^t,w^t), with $p^t >> 0_M$ and $w^t >> 0_N$ for $t = 1,2,...,T$.[19] We assume that the production unit's period t production possibilities set S^t is the conical free disposal hull of the period t actual production vector and past production vectors that are in our sample of time series observations for the unit.[20] Using this assumption, for strictly positive price vectors p and w and nonnegative input quantity vector x, we define the *period t cost constrained value added function* $R^t(p,w,x)$ for the production unit as follows:

[19] We also assume that $p^s \cdot y^t > 0$ for $s = 1,...,T$ and $t = 1,...,T$. This will ensure that all of our explanatory factors are strictly positive.

[20] Diewert (1980b; 264) suggested that the convex, conical, free disposal hull of past and current production vectors be used as an approximation to the period t technology set S^t when measuring TFP growth. Tulkens (1993; 201-206), Tulkens and Vanden Eeckaut (1995a) (1995b) and Diewert and Fox (2014)(2017) dropped the convexity and constant returns to scale assumptions and used free disposal hulls of past and current production vectors to represent the period t technology sets. In this chapter, we also drop the convexity assumption but maintain the free disposal and constant returns to scale assumptions. We also follow Diewert and Parkan (1983; 153-157) and Balk (2003; 37) in introducing price data into the computations.

$$\text{(41) } R^t(p,w,x) \equiv \max_{y,z} \{p\cdot y : w\cdot z \le w\cdot x \,;\, (y,z)\in S^t\}$$
$$\ge \max_\lambda \{p\cdot\lambda y^s : w\cdot\lambda x^s \le w\cdot x;\ \lambda \ge 0\} \qquad \text{since } (\lambda y^s,\lambda x^s)\in S^t \text{ for all } \lambda \ge 0$$
$$= \max_\lambda \{\lambda p\cdot y^s : \lambda w\cdot x^s \le w\cdot x;\ \lambda \ge 0\}$$
$$= (w\cdot x/w\cdot x^s)p\cdot y^s.$$

The inequality in (41) will hold for all s = 1,2,...,t. Thus we have:

$$\text{(42) } R^t(p,w,x) \ge \max_s \{p\cdot y^s\, w\cdot x/w\cdot x^s : s = 1,2,...,t\}.$$

The rays $(\lambda y^s,\lambda x^s)\in S^t$ for $\lambda \ge 0$ generate the efficient points in the set S^t so the strict inequality in (42) cannot hold and so we have:

$$\text{(43) } R^t(p,w,x) \equiv \max_{y,z} \{p\cdot y : w\cdot z \le w\cdot x \,;\, (y,z)\in S^t\}$$
$$= \max_s \{p\cdot y^s\, w\cdot x/w\cdot x^s : s = 1,2,...,t\}$$
$$= \max_{\lambda_1,\ldots,\lambda_t} \{p\cdot(\Sigma_{s=1}^t\, y^s\lambda_s) \,;\, w\cdot(\Sigma_{s=1}^t\, x^s\lambda_s) \le w\cdot x \,;\, \lambda_1 \ge 0 ,..., \lambda_t \ge 0\}$$

where the last line in (43) follows from the fact that the solution to the linear programming problem is an extreme point and thus its solution is equal to the second line in (43). Thus all three equalities in (43) can serve to define $R^t(p,w,x)$. Our assumption that all inner products of the form $p\cdot y^s$ and $w\cdot x^s$ are positive rules out the possibility of a $\lambda_s = 0$ solution to the third line in (43). The last expression in (43) can be use to show that when we assume constant returns to scale for our nonparametric representation for S^t, the resulting $R^t(p,w,x)$ is linear and nondecreasing in x, is convex and linearly homogeneous in p and is homogeneous of degree 0 in w.

If t numbers, $\mu_1,...,\mu_t$ are all positive, then it can be seen that $\max_s \{\mu_s: s = 1,...,t\} = 1/\min_s \{1/\mu_s: s = 1,...,t\}$. Using this equality and (43), it can be seen that we can rewrite $R^t(p,w,x)$ as follows:

$$\text{(44) } R^t(p,w,x) = w\cdot x \max_s \{p\cdot y^s/w\cdot x^s : s = 1,2,...,t\}$$
$$= w\cdot x/\min_s \{w\cdot x^s/p\cdot y^s : s = 1,2,...,t\}$$
$$= w\cdot x/c^t(w,p)$$

where we define the *period t nonparametric unit cost function* $c^t(w,p)$ as follows:

$$\text{(45) } c^t(w,p) \equiv \min_s \{w\cdot x^s/p\cdot y^s : s = 1,2,...,t\}.$$

Thus we have an explicit functional form for the unit cost function $c^t(w,p)$ that was defined earlier by (10) above. It can be seen that $c^t(w,p)$ defined by (45) is a linear nondecreasing function of w (and hence is linearly homogeneous and concave in w which is a necessary property for unit cost functions) and is convex and homogeneous of degree minus one in p.

From (43) we can see that our cost constrained value added function defined by (41) (which did not involve the unit cost function) does in fact conform to equation (11), which we used to simplify our explanatory factors when we had technology sets which were cones.

Now we are in a position to apply the decompositions of value added growth (34), of TFP growth (35) and for the level of TFP (40), using the specific functional form for a sector's cost constrained value added function defined by (43). However, with the assumption of constant returns to scale in production, the returns to scale growth factor δ^t is identically equal to one and so this factor vanishes from the decompositions of value added and TFP growth defined by (34) and (35) above. The levels return to scale growth factor D^t in (40) is also identically equal to one and hence vanishes from the decomposition (40).

In the following two sections, we apply our decomposition to two major sectors of the U.S. economy, the Corporate Nonfinancial Sector and the Noncorporate Nonfinancial Sector, respectively.

5. The U.S. Corporate Nonfinancial Sector, 1960-2014

The US Bureau of Economic Analysis (BEA), in conjunction with the Bureau of Labor Statistics (BLS) and the Board of Governors of the Federal Reserve, have developed a new set of production accounts (the Integrated Macroeconomic Accounts or IMA) for two major private sectors of the US economy: the Corporate Nonfinancial Sector and the Noncorporate Nonfinancial Sector. The Balance Sheet Accounts in the IMA cover the years 1960-2014 but do not provide a decomposition of output, input and asset values into price and quantity components. Diewert and Fox (2016a) provided such a decomposition and we will use their data in this study.

In this section, we will use their output and input data for the U.S. Corporate Nonfinancial Sector (which we denote as Sector 1) for the 55 years 1960-2014. The year t output y^{1t} is real value added[21] and the corresponding year t value added deflator is denoted as p^{1t}. The ten inputs used by this sector are labour and the services of nine types of asset.[22] The output and input data are listed in Appendix A of Diewert and Fox (2016b). The year t input vector for this sector is $x^{1t} \equiv [x_1^{1t},x_2^{1t},...,x_{10}^{1t}]$ where x_1^{1t} is year t labour input measured in billions of 1960 dollars and $x_2^{1t},...,x_{10}^{1t}$ are capital service inputs measured in billions of 1960 capital stock dollars. The corresponding year t input price vector for Sector 1 is $w^{1t} \equiv [w_1^{1t},w_2^{1t},...,w_{10}^{1t}]$ for t = 1960,...,2014.

[21] There is only a single value added output for this sector. The published data for this sector did not allow Diewert and Fox (2016a) to decompose real value added into gross output and intermediate input components.

[22] The nine types of asset used in this sector and the corresponding input numbers are as follows: 2 = Equipment; 3 = Intellectual property products; 4 = Nonresidential structures; 5 = Residential structures; 6 = Residential land; 7 = Farm land; 8 = Commercial land; 9 = Beginning of year inventory stocks and 10 = Beginning of the year real holdings of currency and deposits. The prices are user costs that use predicted asset inflation rates rather than ex post inflation rates but balancing rates of return were used that make the value of input in each year equal to the corresponding value of output.

Our year t technology set for Sector 1, S^{1t}, is defined as the free disposal cone spanned by the observed output and input vectors for the sector up to and including the year t observation. However, as was shown in the previous section, the free disposal cone can be replaced by the convex free disposal cone spanned by previous observations. For convenience, we label the years 1960-2014 as years 1-55 in definitions (46)-(50) below. Thus S^{1t} is defined as follows:

$$(46)\ S^{1t} \equiv \{(y,x): y \le \Sigma_{s=1}^{t} y^{1s} \lambda_s;\ x \ge \Sigma_{s=1}^{t} x^{1s} \lambda_s;\ \lambda_1 \ge 0, ..., \lambda_s \ge 0\}; \qquad t = 1,...,55.$$

We adapt definition (43) of section 4 to the present situation and define the *Sector 1 year t cost constrained value added function* $R^{1t}(p,w,x)$ for $p > 0$, $w >> 0_{10}$ and $x >> 0_{10}$ as follows:

$$\begin{aligned}(47)\ R^{1t}(p,w,x) &\equiv \max_{y,z} \{py: (y,z)\in S^{1t};\ w\cdot z \le w\cdot x\} \qquad t = 1,...,55 \\ &= \max_{\lambda_1,...,\lambda_t} \{p(\Sigma_{s=1}^{t} y^{1s}\lambda_s)\ ;\ w\cdot(\Sigma_{s=1}^{t} x^{1s}\lambda_s) \le w\cdot x\ ;\ \lambda_1 \ge 0\ ,...,\ \lambda_t \ge 0\} \\ &= \max_s \{py^{1s}\ w\cdot x/w\cdot x^{1s} : s = 1,2,...,t\} \\ &= w\cdot x \max_s \{py^{1s}/w\cdot x^{1s} : s = 1,2,...,t\}.\end{aligned}$$

Using the cost constrained value added functions defined by (47), we can readily calculate the Sector 1 counterparts to the year t generic value added growth decompositions (32)-(33) that we derived in section 3 above. Using our present notation for the Sector 1 prices and quantities, these decompositions can be written as follows for t = 2,...,55:[23]

$$(48)\ v^{1t}/v^{1,t-1} = p^{1t}y^{1t}/p^{1,t-1}y^{1,t-1} = \varepsilon^{1t}\,\alpha_P^{\ 1t}\,\beta_L^{\ 1t}\,\gamma_{LPP}^{\ 1t}\,\tau_L^{\ 1t}\ ;$$
$$(49)\ v^{1t}/v^{1,t-1} = \varepsilon^{1t}\,\alpha_L^{\ 1t}\,\beta_P^{\ 1t}\,\gamma_{PLL}^{\ 1t}\,\tau_P^{\ 1t}\ ;$$
$$(50)\ v^{1t}/v^{1,t-1} = \varepsilon^{1t}\,\alpha^{1t}\,\beta^{1t}\,\gamma^{1t}\,\tau^{1t}\ .$$

As in section 3, we define year t Total Factor Productivity Growth for Sector 1 as value added growth divided by output price growth α^{1t} times input quantity growth β^{1t}:

$$(51)\ TFPG^{1t} \equiv [v^{1t}/v^{1,t-1}]/[\alpha^{1t}\beta^{1t}] = \varepsilon^{1t}\,\gamma^{1t}\,\tau^{1t}\ ; \qquad t = 1961,...,2014.$$

Since we have only a single value added output, $\alpha^{1t} \equiv p^{1t}/p^{1,t-1}$ can be interpreted as a Fisher output price index and $[v^{1t}/v^{1.t-1}]/\alpha^{1t}$ can be interpreted as a Fisher output quantity index going from year t−1 to year t. β^{1t} is the Fisher input quantity index going from year t−1 to year t. Thus $TFPG^{1t}$ is equal to a conventional Fisher productivity growth index in this one output case.

[23] Since our Sector 1 technology sets are cones, our returns to scale explanatory factors are all equal to unity; i.e., $\delta_L^{\ 1t} = \delta_P^{\ 1t} = \delta^{1t} = 1$ for t =2,...,55. Thus these explanatory factors do not appear in the decompositions (48)-(50). Since there is only one output for Sector 1, we have $\alpha_L^{\ 1t} = \alpha_P^{\ 1t} = \alpha^{1t} = p^{1t}/p^{1,t-1}$.

The (one plus) growth factors for our Sector 1 that appear in the decomposition given by (51) are listed in Table 1. In addition, we list the cost constrained value added efficiency levels e^{1t} that are the Sector 1 counterparts to the e^t defined by (3).

It can be verified that the TFP growth decomposition defined by (51) holds; i.e., for each year t, nonparametric TFP growth $TFPG^t$ equals the product of value added efficiency growth ε^{1t} times the year t input mix growth factor γ^{1t} times the year t technical progress measure τ^{1t}. It can be seen that the input mix factors are all very close to one. It can also be seen when value added efficiency in year t, e^{1t}, is less than one, then the year t technical progress measure τ^{1t} always equals one so that there is no technical progress in years where the value added efficiency is less than one. Our nonparametric measure of technical progress τ^{1t} is always equal to or greater than one; i.e., our measure never indicates technological regress. Another important empirical regularity emerges from Table 1: since the input mix growth factors γ^{1t} are always very close to one, then when the year t value added efficiency growth factor ε^t is equal to one, our nonparametric measure of TFP growth, $TFPG^t$, is virtually equal to our year t measure of technical progress τ^{1t}. Finally, the last row of Table 1 lists the arithmetic averages of the various growth factors. It can be seen that the arithmetic average rate of TFP growth (and of technical progress) for Sector 1 is 1.70% per year which is a very high average rate of TFP growth over 55 years.

To conclude this section, apply the definitions (37)-(40) to Sector 1 in order to obtain the following levels decomposition for Total Factor Productivity in year t relative to the year 1960, TFP^{1t}:

(52) $TFP^{1t} = [v^{1t}/v^{1,1960}]/[A^{1t}\,B^{1t}] = C^{1t}\,E^{1t}\,T^{1t}$; $\quad$ t = 1960,...,2014.

Table 2 lists the various levels that appear in (52). We note that the returns to scale level for Sector 1 in year t relative to 1960, D^{1t}, is identically equal to one and so it does not appear in the decomposition defined by (52).

Table 1: U.S. Corporate Nonfinancial Value Added Growth $v^{1t}/v^{1,t-1}$, Output Price Growth α^{1t}, Input Quantity Growth β^{1t}, TFP Growth $TFPG^{1t}$, Value Added Efficiency Growth ε^{1t}, Input Mix Growth Factors γ^{1t} and Technical Progress Growth Factors τ^{1t} and Value Added Efficiency Factors e^{1t}

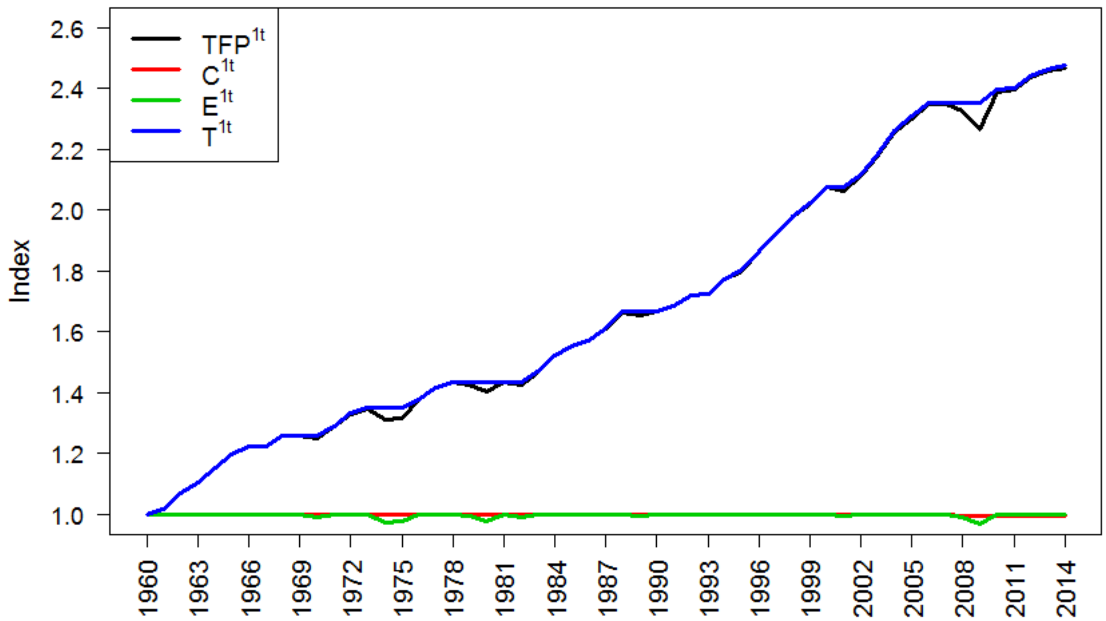
Year t	$v^{1t}/v^{1,t-1}$	α^{1t}	β^{1t}	$TFPG^{1t}$	ε^{1t}	γ^{1t}	τ^{1t}	e^{1t}
1961	1.02696	1.00305	1.00469	1.01906	1.00000	1.00000	1.01906	1.00000
1962	1.09170	1.00647	1.03460	1.04841	1.00000	1.00000	1.04842	1.00000
1963	1.06692	1.00495	1.02435	1.03642	1.00000	1.00000	1.03641	1.00000
1964	1.08005	1.00921	1.02752	1.04153	1.00000	1.00000	1.04154	1.00000
1965	1.10313	1.01758	1.04519	1.03721	1.00000	1.00000	1.03722	1.00000
1966	1.10528	1.02920	1.05110	1.02171	1.00000	1.00000	1.02172	1.00000
1967	1.05161	1.02232	1.02739	1.00123	1.00000	1.00000	1.00123	1.00000
1968	1.09790	1.03102	1.03470	1.02914	1.00000	1.00000	1.02914	1.00000
1969	1.08381	1.04212	1.04058	0.99944	0.99950	0.99994	1.00000	0.99950
1970	1.02816	1.03715	0.99899	0.99233	0.99337	0.99894	1.00000	0.99287
1971	1.07692	1.03612	1.00738	1.03176	1.00718	1.00006	1.02436	1.00000
1972	1.11407	1.03557	1.04114	1.03330	1.00000	1.00000	1.03330	1.00000
1973	1.12293	1.05864	1.04779	1.01236	1.00000	1.00000	1.01235	1.00000
1974	1.08158	1.09825	1.01129	0.97383	0.97416	0.99966	1.00000	0.97416
1975	1.08256	1.09815	0.98241	1.00345	1.00312	1.00035	1.00000	0.97720
1976	1.13447	1.04863	1.03550	1.04476	1.02333	1.00061	1.02032	1.00000
1977	1.13447	1.05665	1.04253	1.02985	1.00000	1.00000	1.02985	1.00000
1978	1.14104	1.07144	1.05167	1.01263	1.00000	1.00000	1.01262	1.00000
1979	1.11671	1.08202	1.03863	0.99368	0.99367	1.00001	1.00000	0.99367
1980	1.08280	1.09350	1.00423	0.98604	0.98644	0.99960	1.00000	0.98019
1981	1.13011	1.08602	1.01929	1.02090	1.02021	1.00019	1.00048	1.00000
1982	1.03636	1.05950	0.98579	0.99226	0.99289	0.99937	1.00000	0.99289
1983	1.06824	1.01840	1.01903	1.02936	1.00716	1.00019	1.02185	1.00000
1984	1.12268	1.03086	1.04890	1.03830	1.00000	1.00000	1.03830	1.00000
1985	1.06489	1.01773	1.02518	1.02064	1.00000	1.00000	1.02064	1.00000
1986	1.04040	1.01396	1.01493	1.01098	1.00000	1.00000	1.01099	1.00000
1987	1.07299	1.01895	1.02831	1.02405	1.00000	1.00000	1.02404	1.00000
1988	1.08863	1.02562	1.02632	1.03421	1.00000	1.00000	1.03422	1.00000
1989	1.04995	1.03037	1.02506	0.99409	0.99408	1.00002	1.00000	0.99408
1990	1.04513	1.03022	1.00733	1.00709	1.00596	1.00003	1.00108	1.00000
1991	1.01671	1.02198	0.98427	1.01074	1.00000	1.00000	1.01073	1.00000
1992	1.04361	1.01273	1.00954	1.02075	1.00000	1.00000	1.02077	1.00000
1993	1.04598	1.02083	1.02097	1.00359	1.00000	1.00000	1.00360	1.00000
1994	1.07768	1.01521	1.03256	1.02806	1.00000	1.00000	1.02807	1.00000
1995	1.06258	1.01365	1.03237	1.01541	1.00000	1.00000	1.01541	1.00000
1996	1.06562	1.00663	1.02243	1.03537	1.00000	1.00000	1.03538	1.00000
1997	1.07519	1.00793	1.03774	1.02794	1.00000	1.00000	1.02795	1.00000
1998	1.05955	1.00264	1.02397	1.03203	1.00000	1.00000	1.03204	1.00000
1999	1.06140	1.00662	1.03224	1.02149	1.00000	1.00000	1.02148	1.00000
2000	1.06697	1.01154	1.02730	1.02676	1.00000	1.00000	1.02677	1.00000
2001	0.99271	1.01425	0.98433	0.99434	0.99529	0.99905	1.00000	0.99529
2002	1.00792	0.99938	0.98456	1.02437	1.00473	0.99999	1.01955	1.00000
2003	1.03213	1.01016	0.99016	1.03190	1.00000	1.00000	1.03189	1.00000
2004	1.06661	1.02069	1.00978	1.03488	1.00000	1.00000	1.03488	1.00000
2005	1.06847	1.03438	1.01215	1.02056	1.00000	1.00000	1.02057	1.00000
2006	1.07032	1.03063	1.01851	1.01964	1.00000	1.00000	1.01964	1.00000
2007	1.03033	1.02012	1.01042	0.99959	0.99969	0.99991	1.00000	0.99969
2008	1.00803	1.02121	0.99622	0.99084	0.99113	0.99970	1.00000	0.99082
2009	0.94412	1.01625	0.95309	0.97475	0.97620	0.99851	1.00000	0.96724
2010	1.05625	1.00079	1.00150	1.05384	1.03387	1.00112	1.01818	1.00000
2011	1.04785	1.02221	1.02168	1.00332	1.00000	1.00000	1.00332	1.00000
2012	1.05777	1.01674	1.02335	1.01661	1.00000	1.00000	1.01660	1.00000
2013	1.03693	1.00644	1.02162	1.00849	1.00000	1.00000	1.00849	1.00000
2014	1.04003	1.00803	1.02686	1.00476	1.00000	1.00000	1.00476	1.00000
Mean	1.06620	1.02880	1.01910	1.01700	1.00000	0.99995	1.01700	0.99736

Table 2: U.S. Corporate Nonfinancial Value Added Year t Levels $v^{1t}/v^{1,1960}$, Output Price Levels A^{1t}, Input Quantity Levels B^{1t}, TFP Levels TFP^{1t}, Input Mix Levels C^{1t}, Value Added Efficiency Levels E^{1t} and Technical Progress Levels T^{1t} where all Levels are Relative to 1960

Year t	$v^{1t}/v^{1,1960}$	A^{1t}	B^{1t}	TFP^{1t}	C^{1t}	E^{1t}	T^{1t}
1960	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1961	1.02696	1.00305	1.00469	1.01906	1.00000	1.00000	1.01906
1962	1.12113	1.00954	1.03945	1.06839	1.00000	1.00000	1.06840
1963	1.19615	1.01454	1.06476	1.10730	1.00000	1.00000	1.10730
1964	1.29190	1.02388	1.09407	1.15329	1.00000	1.00000	1.15330
1965	1.42514	1.04188	1.14350	1.19620	1.00000	1.00000	1.19622
1966	1.57518	1.07230	1.20194	1.22217	1.00000	1.00000	1.22220
1967	1.65648	1.09623	1.23486	1.22368	1.00000	1.00000	1.22370
1968	1.81864	1.13024	1.27771	1.25934	1.00000	1.00000	1.25937
1969	1.97106	1.17785	1.32956	1.25864	0.99994	0.99950	1.25937
1970	2.02656	1.22161	1.32822	1.24899	0.99888	0.99287	1.25937
1971	2.18245	1.26574	1.33802	1.28866	0.99894	1.00000	1.29004
1972	2.43141	1.31076	1.39306	1.33157	0.99894	1.00000	1.33300
1973	2.73032	1.38762	1.45963	1.34803	0.99894	1.00000	1.34947
1974	2.95307	1.52395	1.47611	1.31275	0.99860	0.97416	1.34947
1975	3.19687	1.67352	1.45016	1.31728	0.99894	0.97720	1.34947
1976	3.62675	1.75491	1.50164	1.37625	0.99955	1.00000	1.37689
1977	4.11443	1.85432	1.56550	1.41733	0.99955	1.00000	1.41799
1978	4.69473	1.98679	1.64640	1.43524	0.99955	1.00000	1.43590
1979	5.24266	2.14974	1.70999	1.42617	0.99956	0.99367	1.43590
1980	5.67676	2.35075	1.71722	1.40627	0.99916	0.98019	1.43590
1981	6.41535	2.55297	1.75035	1.43566	0.99935	1.00000	1.43659
1982	6.64861	2.70487	1.72548	1.42454	0.99872	0.99289	1.43659
1983	7.10233	2.75464	1.75831	1.46636	0.99891	1.00000	1.46798
1984	7.97366	2.83966	1.84429	1.52252	0.99891	1.00000	1.52421
1985	8.49111	2.89001	1.89073	1.55395	0.99891	1.00000	1.55567
1986	8.83418	2.93035	1.91896	1.57102	0.99891	1.00000	1.57276
1987	9.47898	2.98587	1.97328	1.60880	0.99891	1.00000	1.61057
1988	10.31908	3.06238	2.02521	1.66384	0.99891	1.00000	1.66569
1989	10.83452	3.15539	2.07596	1.65401	0.99893	0.99408	1.66569
1990	11.32345	3.25074	2.09118	1.66573	0.99896	1.00000	1.66748
1991	11.51263	3.32219	2.05828	1.68363	0.99896	1.00000	1.68537
1992	12.01468	3.36449	2.07791	1.71857	0.99896	1.00000	1.72038
1993	12.56706	3.43456	2.12149	1.72473	0.99896	1.00000	1.72656
1994	13.54323	3.48679	2.19055	1.77314	0.99896	1.00000	1.77502
1995	14.39079	3.53437	2.26146	1.80046	0.99896	1.00000	1.80238
1996	15.33507	3.55779	2.31219	1.86415	0.99896	1.00000	1.86615
1997	16.48815	3.58599	2.39946	1.91624	0.99896	1.00000	1.91830
1998	17.47003	3.59545	2.45697	1.97761	0.99896	1.00000	1.97976
1999	18.54264	3.61926	2.53617	2.02010	0.99896	1.00000	2.02229
2000	19.78448	3.66103	2.60542	2.07417	0.99896	1.00000	2.07642
2001	19.64021	3.71320	2.56459	2.06243	0.99802	0.99529	2.07642
2002	19.79580	3.71089	2.52498	2.11269	0.99801	1.00000	2.11702
2003	20.43174	3.74861	2.50013	2.18008	0.99801	1.00000	2.18453
2004	21.79279	3.82615	2.52457	2.25612	0.99801	1.00000	2.26072
2005	23.28502	3.95768	2.55525	2.30252	0.99801	1.00000	2.30721
2006	24.92242	4.07891	2.60254	2.34773	0.99801	1.00000	2.35252
2007	25.67843	4.16098	2.62966	2.34678	0.99791	0.99969	2.35252
2008	25.88458	4.24922	2.61973	2.32528	0.99762	0.99082	2.35252
2009	24.43813	4.31825	2.49684	2.26658	0.99613	0.96724	2.35252
2010	25.81284	4.32164	2.50058	2.38861	0.99725	1.00000	2.39529
2011	27.04786	4.41763	2.55480	2.39655	0.99725	1.00000	2.40323
2012	28.61031	4.49159	2.61446	2.43635	0.99725	1.00000	2.44312
2013	29.66699	4.52053	2.67099	2.45703	0.99725	1.00000	2.46386
2014	30.85463	4.55684	2.74273	2.46873	0.99725	1.00000	2.47559

Note that the final level of TFP in 2014, 2.46873, is slightly less than the level of technology in 2014, which is 2.47559. This small difference is explained by the fact that the cumulative input mix level, 0.99725, is slightly less than 1 in 2014. We plot TFP^{1t}, C^{1t}, E^{1t} and T^{1t} in Figure 1.

Figure 1: Sector 1 Level of TFP, Input Mix, Value Added Efficiency and Technology



It can be seen that there was a substantial decline in value added efficiency over the years 2006-2009 and in fact, TFP has grown at a slower than average rate since 2006. The level of TFP also fell in the 1974, 1979, 1982, 1989 and 2001 recessions when efficiency growth dipped below one. However, on the whole, TFP growth in the U.S. Corporate Nonfinancial Sector has been satisfactory.

We turn now to an analysis of the performance of the U.S. Noncorporate Nonfinancial Sector.

6. The U.S. Noncorporate Nonfinancial Sector, 1960-2014

In this section, we use the Diewert and Fox (2016b) output and input data for the U.S. Noncorporate Nonfinancial Sector (which we denote as Sector 2) for the 55 years 1960-2014. The year t output y^{2t} is real value added for this sector and the corresponding year t value added deflator is denoted as p^{2t}. The 15 inputs used by this sector are labour and the services of 14 types of asset.[24] The output and input data are listed in Appendix A of

[24] The 14 types of asset used in this sector and the corresponding input numbers are as follows: 2 = Equipment held by sole proprietors; 3 = Equipment held by partners; 4 = Equipment held by cooperatives; 5 = Intellectual property products held by sole proprietors; 6 = Intellectual property products held by partners; 7 = Nonresidential structures held by sole proprietors; 8 = Nonresidential structures held by partners; 9 = Nonresidential structures held by cooperatives; ; 10 = Residential structures held by the noncorporate nonfinancial sector; 11 = Residential land held by the noncorporate nonfinancial sector; 12 =

Diewert and Fox (2016b). The year t input vector for this sector is $x^{2t} \equiv [x_1^{2t},x_2^{2t},...,x_{15}^{2t}]$ where x_1^{2t} is year t labour input measured in billions of 1960 dollars and $x_2^{2t},...,x_{15}^{2t}$ are capital service inputs measured in billions of 1960 capital stock dollars. The corresponding year t input price vector for sector 2 is $w^{2t} \equiv [w_1^{2t},w_2^{2t},...,w_{15}^{2t}]$ for t = 1960,...,2014.

Our year t technology set for Sector 2, S^{2t}, is defined in an analogous manner as for Sector 1 in (46), as the free disposal cone spanned by the observed output and input vectors up to and including the year t observation. We adapt definition (46) to the present situation and the *Sector 2 year t cost constrained value added function* $R^{2t}(p,w,x)$ for $p > 0$, $w >> 0_{15}$ and $x >> 0_{15}$ can be written as follows:

$$(53)\ R^{2t}(p,w,x) = w\cdot x \max{}_s \{py^{2s}/w\cdot x^{2s} : s = 1,2,...,t\}. \qquad t = 1,...,55$$

Using the cost constrained value added functions defined by (53), we can readily calculate the Sector 2 counterparts to the year t generic value added growth decompositions (32)-(33) that we derived in section 3. Using our present notation for the Sector 2 prices and quantities, these decompositions can be written in the same manner as (48)-(50) except that the superscript 2 replaces the superscript 1. As in section 3, we define year t Total Factor Productivity Growth for Sector 2 as value added growth divided by output price growth α^{2t} times input quantity growth β^{2t}, which leads to the following year t decomposition of TFP growth for Sector 2:[25]

$$(54)\ TFPG^{2t} \equiv [v^{2t}/v^{2,t-1}]/[\alpha^{2t}\beta^{2t}] = \varepsilon^{2t}\,\gamma^{2t}\,\tau^{2t}\ ; \qquad t = 1961,...,2014.$$

Since we have only a single value added output, $\alpha^{2t} \equiv p^{2t}/p^{2,t-1}$ can be interpreted as a Fisher output price index and $[v^{2t}/v^{2.t-1}]/\alpha^{2t}$ can be interpreted as a Fisher output quantity index going from year t−1 to year t. β^{2t} is the Fisher input quantity index going from year t−1 to year t for Sector 2. Thus $TFPG^{2t}$ is equal to a conventional Fisher productivity growth index in this one output case.

The growth factors for our Sector 2 that appear in the decomposition given by (54) are listed in Table 3. In addition, we list the cost constrained value added efficiency levels e^{2t} that are the Sector 2 counterparts to the e^t defined by (3).

Farm land held by the noncorporate nonfinancial sector; 13 = Commercial land held by noncorporate nonfinancial sector; 14 = Beginning of the year inventories held by the noncorporate nonfinancial sector and 15 = Beginning of the year real holdings of currency and deposits by noncorporate nonfinancial sector..

[25] The returns to scale measures δ^{2t} for Sector 2 are all equal to one and thus these growth factors do not appear in (53).

Table 3: U.S. Noncorporate Nonfinancial Value Added Growth $v^{2t}/v^{2,t-1}$, Output Price Growth α^{2t}, Input Quantity Growth β^{2t}, TFP Growth $TFPG^{2t}$, Value Added Efficiency Growth Factors ε^{2t}, Input Mix Growth Factors γ^{2t} and Technical Progress Growth Factors τ^{2t} and Value Added Efficiency Factors e^{2t}

Year t	$v^{2t}/v^{2,t-1}$	α^{2t}	β^{2t}	$TFPG^{2t}$	ε^{2t}	γ^{2t}	τ^{2t}	e^{2t}
1961	1.02611	1.01608	0.98336	1.02696	1.00000	1.00000	1.02695	1.00000
1962	1.03628	1.01432	0.99047	1.03148	1.00000	1.00000	1.03149	1.00000
1963	1.02537	1.01138	0.98980	1.02428	1.00000	1.00000	1.02426	1.00000
1964	1.05036	1.01681	1.00129	1.03166	1.00000	1.00000	1.03169	1.00000
1965	1.05693	1.01991	0.99748	1.03891	1.00000	1.00000	1.03889	1.00000
1966	1.06541	1.03070	1.00046	1.03319	1.00000	1.00000	1.03320	1.00000
1967	1.02600	1.03158	0.99474	0.99984	1.00000	0.99976	1.00003	1.00000
1968	1.05414	1.04532	0.99242	1.01614	1.00000	1.00000	1.01619	1.00000
1969	1.05337	1.04377	1.00590	1.00327	1.00000	1.00000	1.00330	1.00000
1970	1.03555	1.03923	0.99661	0.99986	1.00000	0.99959	1.00023	1.00000
1971	1.05931	1.04708	0.99649	1.01525	1.00000	1.00000	1.01529	1.00000
1972	1.10456	1.04810	1.01373	1.03960	1.00000	1.00000	1.03965	1.00000
1973	1.16895	1.04374	1.03526	1.08181	1.00000	1.00000	1.08176	1.00000
1974	1.05229	1.09421	1.02312	0.93996	0.94041	0.99956	1.00000	0.94041
1975	1.07140	1.10309	0.99317	0.97796	0.97815	0.99985	1.00000	0.91986
1976	1.09366	1.08706	1.00684	0.99924	0.99895	1.00030	1.00000	0.91889
1977	1.09102	1.08397	1.01453	0.99208	0.99188	1.00024	1.00000	0.91143
1978	1.13298	1.07146	1.02691	1.02970	1.02912	1.00063	1.00000	0.93797
1979	1.11623	1.11798	1.02918	0.97012	0.97030	0.99984	1.00000	0.91011
1980	1.05125	1.06681	1.01524	0.97062	0.97138	0.99923	1.00000	0.88406
1981	1.08929	1.09691	1.00518	0.98793	0.98828	0.99960	1.00000	0.87370
1982	1.04364	1.06451	1.00908	0.97157	0.97353	0.99798	1.00000	0.85057
1983	1.05757	1.07541	1.01796	0.96606	0.96782	0.99826	1.00000	0.82319
1984	1.15730	1.01282	1.02387	1.11602	1.11439	1.00148	1.00000	0.91736
1985	1.07921	1.04855	1.01081	1.01823	1.01809	1.00017	1.00000	0.93396
1986	1.05984	1.01768	1.01298	1.02808	1.03079	0.99734	1.00000	0.96272
1987	1.04901	1.04904	1.01400	0.98616	0.98582	1.00038	1.00000	0.94906
1988	1.08921	1.04848	1.01531	1.02319	1.02319	1.00004	1.00000	0.97107
1989	1.06461	1.05355	1.02123	0.98950	0.99138	0.99813	1.00000	0.96269
1990	1.04318	1.04174	1.01110	0.99039	0.99286	0.99751	1.00000	0.95582
1991	1.00925	1.04153	1.00458	0.96459	0.96850	0.99596	1.00000	0.92571
1992	1.06726	1.01569	0.98668	1.06495	1.06571	0.99930	1.00000	0.98654
1993	1.03876	1.02356	1.02364	0.99141	0.99190	0.99947	1.00000	0.97855
1994	1.05278	1.01154	1.01235	1.02807	1.02192	0.99940	1.00665	1.00000
1995	1.04291	1.04735	1.00941	0.98649	0.98648	0.99999	1.00000	0.98648
1996	1.07813	1.05142	1.00882	1.01645	1.01370	0.99997	1.00272	1.00000
1997	1.06300	1.03341	1.01934	1.00912	1.00000	1.00000	1.00916	1.00000
1998	1.08134	1.02197	1.00929	1.04836	1.00000	1.00000	1.04834	1.00000
1999	1.06748	1.01844	1.00857	1.03925	1.00000	1.00000	1.03922	1.00000
2000	1.08263	1.05231	1.01848	1.01015	1.00000	1.00000	1.01012	1.00000
2001	1.15232	1.04426	1.08093	1.02086	1.00000	1.00000	1.02087	1.00000
2002	1.04271	1.00449	1.01851	1.01918	1.00000	1.00000	1.01918	1.00000
2003	1.05478	1.00840	1.02963	1.01589	1.00000	1.00000	1.01587	1.00000
2004	1.08508	1.01994	1.03304	1.02984	1.00000	1.00000	1.02985	1.00000
2005	1.06903	1.02104	1.03269	1.01386	1.00000	1.00000	1.01387	1.00000
2006	1.09790	1.01809	1.03878	1.03814	1.00000	1.00000	1.03815	1.00000
2007	1.02757	1.02451	1.02994	0.97382	0.97366	1.00015	1.00000	0.97366
2008	1.05018	1.00395	1.00167	1.04431	1.02705	0.99990	1.01689	1.00000
2009	0.93797	0.98063	0.98224	0.97379	0.97479	0.99898	1.00000	0.97479
2010	1.03208	1.03383	0.99483	1.00349	1.00318	1.00033	1.00000	0.97788
2011	1.08243	1.01786	1.00189	1.06143	1.02262	1.00079	1.03713	1.00000
2012	1.05758	1.02064	1.01541	1.02047	1.00000	1.00000	1.02049	1.00000
2013	1.03553	1.02191	1.01072	1.00258	1.00000	1.00000	1.00261	1.00000
2014	1.04452	1.02358	1.01703	1.00338	1.00000	1.00000	1.00340	1.00000
Mean	1.06400	1.03890	1.01180	1.01260	0.99971	1.00030	1.01250	0.97086

It can be verified that the TFP growth decomposition defined by (54) holds; i.e., for each year t, nonparametric TFP growth in Sector 2, $TFPG^{2t}$, equals the product of value added efficiency growth ε^{2t} times the year t input mix growth factor γ^{2t} times the year t technical progress measure τ^{2t}. The arithmetic average rate of TFP growth for Sector 2 was 1.26% per year, which is well below the 1.70% per year rate of TFP growth for Sector 1, but is still quite good. As was the case with Sector 1, the Sector 2 input mix growth factors are all close to one and hence are not a significant determinant of TFP growth for the Noncorporate Nonfinancial Sector of the U.S. economy. Again, we see that when the year t efficiency factor e^{2t} is below one, then the year t rate of technological change τ^{2t} is equal to one. Moreover, the rate of technological change τ^{2t} is always greater than or equal to one. What is very surprising is the very large number of years where value added efficiency e^{2t} is below unity, indicating that Sector 2 is operating well within the production frontier during those years.[26] The mean level of the value added efficiency factors is equal to 0.97086. Compare this very low average level of efficiency with the corresponding average level of efficiency for Sector 1, which was 0.99736.[27] Nevertheless, we see that the average rate of TFP growth for Sector 2 was 1.26% per year which is very close to the average rate of technical progress for Sector 2, which was 1.25% per year.

To conclude this section, we apply definitions (37)-(40) to our present Sector 2 in order to obtain the following levels decomposition for Total Factor Productivity in year t relative to the year 1960, $TFP^{2t} \equiv [v^{2t}/v^{2,1960}]/[A^{2t}\, B^{2t}] = C^{2t}\, E^{2t}\, T^{2t}$. Table 4 lists these cumulative explanatory factors.

Note that the final level of TFP for Sector 2 in 2014, 1.91416, is somewhat less than the level of technology in 2014, which is 1.94563. This small difference is explained by the fact that the cumulative input mix level, 0.98424, is 1.5% less than 1 in 2014.[28] Note also that the final level of TFP in Sector 2, 1.91416, is much lower than the final level of TFP for Sector 1, which is 2.46873. We plot TFP^{2t}, C^{2t}, E^{2t} and T^{2t} in Figure 2.

[26] Recall that as we our approach rules out technical regress, loss of efficiency is gross loss of efficiency less any technical progress that occurs during recession years. Hence estimates of efficiency loss are a bit too low in magnitude, and our estimates of technical progress are biased downward.

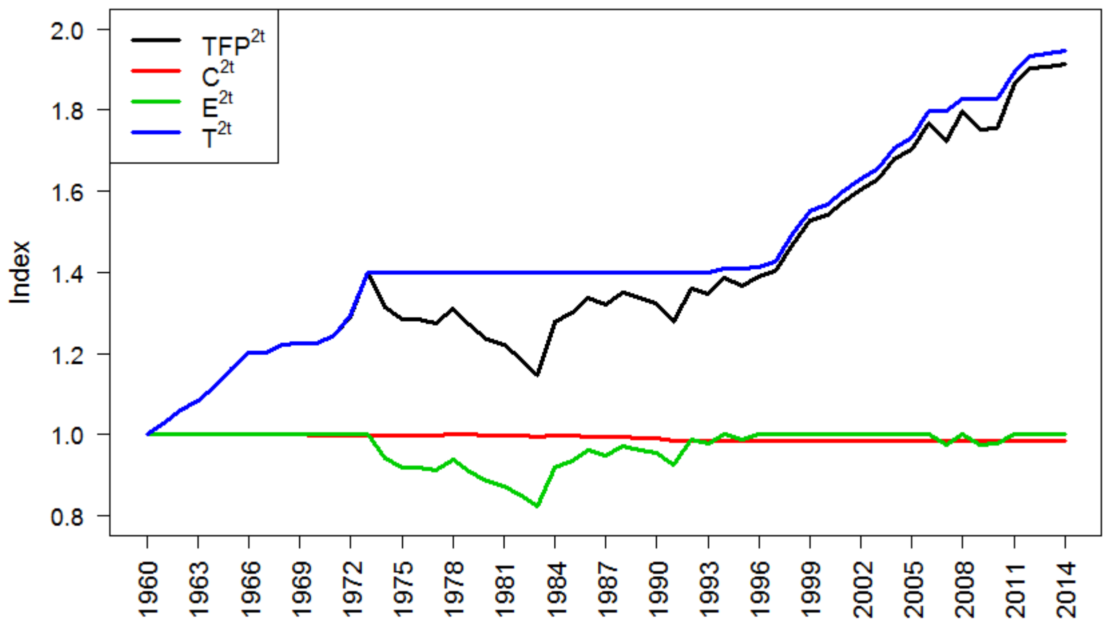
[27] The efficiency level e^{2t} was below unity for the years 1974-1993, 1995, 2007, 2009 and 2010, which is a total of 24 years. A possible explanation for the long stretch of inefficient years 1974-1993 is that Sector 2 uses a high proportion of structures and land to produce its net outputs and there may have been a boom in these investments prior to 1974. Once the recession of 1974 occurred, these relatively fixed inputs could not be contracted in line with the net outputs produced by this sector, leading to the long string of inefficient years. An alternative explanation is that there are measurement errors in our data for Sector 2.

[28] For most observations, γ^{2t} is only slightly less than one. But over time, the product of these γ^{2t} cumulate to 0.984 which is significantly below one.

Table 4: U.S. Noncorporate Nonfinancial Value Added Year t Levels $v^{2t}/v^{2,1960}$, Output Price Levels A^{2t}, Input Quantity Levels B^{2t}, TFP Levels TFP^{2t}, Input Mix Levels C^{2t}, Value Added Efficiency Levels E^{2t} and Technical Progress Levels T^{2t} where all Levels are Relative to 1960

Year t	$v^{2t}/v^{2,1960}$	A^{2t}	B^{2t}	TFP^{2t}	C^{2t}	E^{2t}	T^{2t}
1960	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1961	1.02611	1.01608	0.98336	1.02696	1.00000	1.00000	1.02695
1962	1.06334	1.03063	0.97399	1.05928	1.00000	1.00000	1.05930
1963	1.09031	1.04236	0.96406	1.08500	1.00000	1.00000	1.08499
1964	1.14522	1.05988	0.96531	1.11935	1.00000	1.00000	1.11938
1965	1.21041	1.08098	0.96288	1.16291	1.00000	1.00000	1.16291
1966	1.28958	1.11417	0.96332	1.20151	1.00000	1.00000	1.20152
1967	1.32310	1.14936	0.95825	1.20132	0.99976	1.00000	1.20156
1968	1.39474	1.20145	0.95099	1.22070	0.99976	1.00000	1.22102
1969	1.46917	1.25404	0.95660	1.22470	0.99976	1.00000	1.22505
1970	1.52140	1.30323	0.95336	1.22452	0.99935	1.00000	1.22532
1971	1.61164	1.36458	0.95001	1.24320	0.99935	1.00000	1.24406
1972	1.78014	1.43021	0.96305	1.29243	0.99935	1.00000	1.29339
1973	2.08089	1.49277	0.99701	1.39816	0.99935	1.00000	1.39913
1974	2.18970	1.63340	1.02006	1.31422	0.99891	0.94041	1.39913
1975	2.34606	1.80178	1.01309	1.28526	0.99877	0.91986	1.39913
1976	2.56580	1.95864	1.02002	1.28428	0.99906	0.91889	1.39913
1977	2.79932	2.12311	1.03484	1.27411	0.99930	0.91143	1.39913
1978	3.17157	2.27483	1.06269	1.31196	0.99993	0.93797	1.39913
1979	3.54018	2.54321	1.09370	1.27276	0.99977	0.91011	1.39913
1980	3.72163	2.71313	1.11037	1.23536	0.99900	0.88406	1.39913
1981	4.05393	2.97606	1.11613	1.22045	0.99861	0.87370	1.39913
1982	4.23086	3.16805	1.12627	1.18576	0.99659	0.85057	1.39913
1983	4.47442	3.40696	1.14649	1.14551	0.99485	0.82319	1.39913
1984	5.17824	3.45063	1.17385	1.27841	0.99632	0.91736	1.39913
1985	5.58841	3.61816	1.18655	1.30171	0.99649	0.93396	1.39913
1986	5.92280	3.68213	1.20195	1.33826	0.99385	0.96272	1.39913
1987	6.21307	3.86270	1.21878	1.31974	0.99422	0.94906	1.39913
1988	6.76733	4.04995	1.23744	1.35034	0.99426	0.97107	1.39913
1989	7.20459	4.26681	1.26371	1.33616	0.99240	0.96269	1.39913
1990	7.51567	4.44489	1.27773	1.32333	0.98993	0.95582	1.39913
1991	7.58523	4.62950	1.28359	1.27647	0.98593	0.92571	1.39913
1992	8.09540	4.70216	1.26649	1.35937	0.98524	0.98654	1.39913
1993	8.40916	4.81293	1.29644	1.34770	0.98472	0.97855	1.39913
1994	8.85297	4.86845	1.31245	1.38553	0.98413	1.00000	1.40844
1995	9.23288	5.09895	1.32480	1.36681	0.98412	0.98648	1.40844
1996	9.95424	5.36112	1.33647	1.38929	0.98410	1.00000	1.41227
1997	10.58131	5.54023	1.36232	1.40195	0.98410	1.00000	1.42521
1998	11.44201	5.66193	1.37498	1.46974	0.98410	1.00000	1.49411
1999	12.21415	5.76635	1.38676	1.52743	0.98410	1.00000	1.55271
2000	13.22343	6.06797	1.41238	1.54294	0.98410	1.00000	1.56842
2001	15.23758	6.33652	1.52669	1.57512	0.98410	1.00000	1.60115
2002	15.88833	6.36495	1.55495	1.60534	0.98410	1.00000	1.63186
2003	16.75867	6.41842	1.60103	1.63085	0.98410	1.00000	1.65776
2004	18.18448	6.54641	1.65393	1.67950	0.98410	1.00000	1.70725
2005	19.43982	6.68416	1.70799	1.70279	0.98410	1.00000	1.73094
2006	21.34295	6.80506	1.77422	1.76773	0.98410	1.00000	1.79697
2007	21.93133	6.97187	1.82734	1.72145	0.98425	0.97366	1.79697
2008	23.03184	6.99939	1.83040	1.79772	0.98415	1.00000	1.82732
2009	21.60307	6.86380	1.79789	1.75060	0.98315	0.97479	1.82732
2010	22.29602	7.09602	1.78859	1.75672	0.98347	0.97788	1.82732
2011	24.13380	7.22277	1.79196	1.86463	0.98424	1.00000	1.89516
2012	25.52348	7.37188	1.81957	1.90280	0.98424	1.00000	1.93399
2013	26.43038	7.53339	1.83908	1.90771	0.98424	1.00000	1.93904
2014	27.60715	7.71100	1.87039	1.91416	0.98424	1.00000	1.94563

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology



It can be seen that the loss of value added efficiency in Sector 2 was massive over the 20 years 1974-1993 and this loss of efficiency dragged down the level of Sector 2 TFP over these years. However, TFP growth resumed in 1994 and was excellent until 2006 when TFP growth again stalled with the exception of two good years of growth in 2011 and 2012.

It can be seen that our nonparametric methodology provides a useful supplement to traditional index number methods for calculating TFP growth. It illustrates the adverse influence of recessions when output falls but inputs cannot be adjusted optimally due to the fixity of many capital stock (and labour) components of aggregate input. Under these circumstances, production takes place in the interior of the production possibilities set and for Sector 2, the resulting waste of resources was substantial.[29]

We now consider the problem of how to decompose aggregate (across sectors) value added into explanatory factors.

7. Aggregation over Sectors: Weighted Average Approach

Diewert and Fox (2016c) considered different ways to go between sectoral and higher level of aggregation decompositions. In particular, a *sectoral weighted average approach* and an *aggregate cost constrained value added approach*. The first method is a "bottom

[29] We note that our empirical results in this section and the previous one, which use the cost restricted value added function, are very similar to our previous results for these sectors in Diewert and Fox (2016b), which used a cost function approach. However, our previous approach relied on the fact that we had only a single output in each sector. Our present approach is preferred if there are many sectoral net outputs.

up" approach, while the second method is a "top down" approach. Diewert and Fox found that both methods produced results that approximated each other very closely. Drawing on the material of the previous sections, we present here a summary of the results from the "bottom up" approach. This uses weighted averages of the sectoral decompositions to provide an approximate decomposition into explanatory components at the aggregate level.

Define *period t aggregate value added* v^t as the sum of the period t sectoral value added for each sector, $v^t \equiv v^{1t} + v^{2t}$. Then define the *period t share of aggregate value added for sector k* as $s^{kt} \equiv v^{kt}/v^t$, for k=1,2. Diewert and Fox (2016c) showed that, using the year t sector k explanatory growth factors, α_{kt}, β_{kt}, γ_{kt}, ε_{kt} and τ_{kt}, that are listed in Tables 1 and 3, we can write the following approximate decomposition of the (logarithm of the) aggregate value added ratio between periods t-1 and t:

$$
\begin{aligned}
(55)\ \ln v^t/v^{t-1} &\approx \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln(v^{kt}/v^{k,t-1}) \\
&= \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln(\alpha^{kt}\,\beta^{kt}\,\gamma^{kt}\,\varepsilon^{kt}\,\tau^{kt}) && \text{using (50)} \\
&= \ln \alpha^{t\bullet} + \ln \beta^{t\bullet} + \ln \gamma^{t\bullet} + \ln \varepsilon^{t\bullet} + \ln \tau^{t\bullet},
\end{aligned}
$$

where the terms in the last line of (55) are defined as follows:

$$(56)\ \ln \alpha^{t\bullet} \equiv \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln \alpha^{kt}\ ;$$
$$(57)\ \ln \beta^{t\bullet} \equiv \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln \beta^{kt}\ ;$$
$$(58)\ \ln \gamma^{t\bullet} \equiv \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln \gamma^{kt}\ ;$$
$$(59)\ \ln \varepsilon^{t\bullet} \equiv \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln \varepsilon^{kt}\ ;$$
$$(60)\ \ln \tau^{t\bullet} \equiv \Sigma_{k=1}{}^2\,(1/2)(s^{kt} + s^{k,t-1})\ln \tau^{kt}\ .$$

Period t aggregate Total Factor Productivity Growth, $TFPG^t$, can then be defined as aggregate real value added growth divided by aggregate primary input growth:

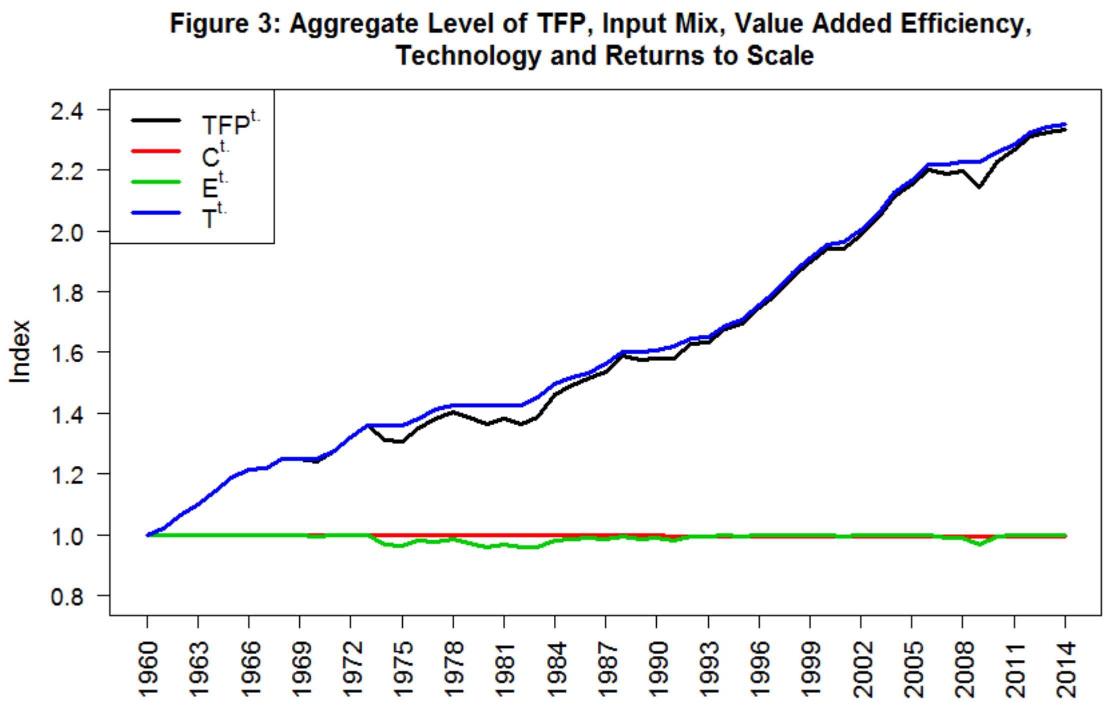
$$(61)\ TFPG^t \equiv [v^t/v^{t-1}]/\alpha^{t\bullet}\,\beta^{t\bullet} \approx \gamma^{t\bullet}\,\varepsilon^{t\bullet}\,\tau^{t\bullet}\ ; \qquad t = 2,\ldots,T$$

where the approximate equality in (61) follows from the approximate equality (55). Thus (61) provides an approximate decomposition of aggregate (one plus) TFP growth into the product of various aggregate explanatory growth factors (mix effects, returns to scale effects, cost constrained value added efficiency effects and technical progress effects). Using definitions (37)-(40) applied to aggregate value added, we obtain the following levels decomposition for approximate aggregate Total Factor Productivity in year t relative to the year 1960, $TFP^{t\bullet}$:

$$(62)\ TFP^{t\bullet} = [v^t/v^{1960}]/[A^{t\bullet}\,B^{t\bullet}] \approx C^{t\bullet}\,E^{t\bullet}\,T^{t\bullet}\ ; \qquad t = 1960,\ldots,2014.$$

Table 5: U.S. Aggregate Nonfinancial Value Added Growth v^t/v^{t-1}, Output Price Growth $\alpha^{t\bullet}$, Input Quantity Growth $\beta^{t\bullet}$, TFP Growth $TFPG^{t\bullet}$, Input Mix Growth Factors $\gamma^{t\bullet}$, Value Added Efficiency Growth Factors $\varepsilon^{t\bullet}$ and Technical Progress Growth Factors $\tau^{t\bullet}$ and Sector 1 Shares of Aggregate Value Added s^{1t}

Year t	v^t/v^{t-1}	$\alpha^{t\bullet}$	$\beta^{t\bullet}$	$TFPG^{t\bullet}$	$\gamma^{t\bullet}$	$\varepsilon^{t\bullet}$	$\tau^{t\bullet}$	s^{1t}
1960	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.70438
1961	1.02671	1.00688	0.99834	1.02139	1.00000	1.00000	1.02139	0.70455
1962	1.07533	1.00874	1.02160	1.04347	1.00000	1.00000	1.04348	0.71528
1963	1.05509	1.00675	1.01453	1.03300	1.00000	1.00000	1.03298	0.72330
1964	1.07183	1.01129	1.02027	1.03882	1.00000	1.00000	1.03883	0.72884
1965	1.09060	1.01820	1.03223	1.03766	1.00000	1.00000	1.03767	0.73721
1966	1.09480	1.02959	1.03773	1.02468	1.00000	1.00000	1.02468	0.74427
1967	1.04506	1.02466	1.01902	1.00088	0.99994	1.00000	1.00093	0.74893
1968	1.08691	1.03454	1.02408	1.02592	1.00000	1.00000	1.02592	0.75650
1969	1.07640	1.04252	1.03212	1.00037	0.99995	0.99962	1.00079	0.76171
1970	1.02992	1.03765	0.99842	0.99412	0.99910	0.99495	1.00005	0.76041
1971	1.07270	1.03872	1.00478	1.02781	1.00005	1.00547	1.02219	0.76340
1972	1.11182	1.03851	1.03461	1.03478	1.00000	1.00000	1.03479	0.76495
1973	1.13375	1.05506	1.04479	1.02851	1.00000	1.00000	1.02850	0.75765
1974	1.07448	1.09728	1.01411	0.96559	0.99964	0.96596	1.00000	0.76265
1975	1.07991	1.09932	0.98494	0.99737	1.00023	0.99716	1.00000	0.76452
1976	1.12486	1.05743	1.02877	1.03402	1.00054	1.01762	1.01556	0.77105
1977	1.12452	1.06275	1.03615	1.02121	1.00005	0.99816	1.02304	0.77788
1978	1.13925	1.07144	1.04613	1.01639	1.00014	1.00638	1.00981	0.77910
1979	1.11660	1.08986	1.03654	0.98843	0.99997	0.98846	1.00000	0.77917
1980	1.07583	1.08762	1.00662	0.98266	0.99952	0.98313	1.00000	0.78422
1981	1.12130	1.08833	1.01627	1.01380	1.00006	1.01333	1.00038	0.79038
1982	1.03789	1.06055	0.99064	0.98787	0.99908	0.98879	1.00000	0.78922
1983	1.06599	1.03011	1.01881	1.01573	0.99978	0.99877	1.01722	0.79088
1984	1.12992	1.02702	1.04355	1.05428	1.00031	1.02319	1.03007	0.78581
1985	1.06796	1.02429	1.02207	1.02012	1.00004	1.00387	1.01616	0.78356
1986	1.04461	1.01477	1.01450	1.01469	0.99942	1.00663	1.00858	0.78040
1987	1.06772	1.02543	1.02518	1.01568	1.00008	0.99690	1.01876	0.78425
1988	1.08876	1.03051	1.02393	1.03182	1.00001	1.00496	1.02674	0.78416
1989	1.05311	1.03536	1.02423	0.99309	0.99961	0.99349	1.00000	0.78180
1990	1.04470	1.03272	1.00815	1.00343	0.99948	1.00309	1.00084	0.78212
1991	1.01508	1.02620	0.98865	1.00053	0.99912	0.99307	1.00839	0.78337
1992	1.04873	1.01338	1.00450	1.03025	0.99985	1.01401	1.01619	0.77955
1993	1.04439	1.02143	1.02156	1.00090	0.99988	0.99821	1.00281	0.78074
1994	1.07222	1.01441	1.02813	1.02806	0.99987	1.00472	1.02338	0.78471
1995	1.05835	1.02076	1.02742	1.00915	1.00000	0.99710	1.01210	0.78785
1996	1.06827	1.01601	1.01951	1.03131	0.99999	1.00290	1.02833	0.78589
1997	1.07258	1.01331	1.03379	1.02390	1.00000	1.00000	1.02392	0.78781
1998	1.06417	1.00674	1.02081	1.03549	1.00000	1.00000	1.03551	0.78438
1999	1.06271	1.00916	1.02708	1.02530	1.00000	1.00000	1.02529	0.78342
2000	1.07036	1.02028	1.02537	1.02312	1.00000	1.00000	1.02312	0.78093
2001	1.02768	1.02115	1.00598	1.00042	0.99927	0.99638	1.00481	0.75436
2002	1.01647	1.00065	0.99290	1.02307	0.99999	1.00355	1.01946	0.74802
2003	1.03784	1.00971	1.00004	1.02781	1.00000	1.00000	1.02780	0.74391
2004	1.07134	1.02050	1.01572	1.03357	1.00000	1.00000	1.03358	0.74062
2005	1.06862	1.03090	1.01744	1.01881	1.00000	1.00000	1.01883	0.74052
2006	1.07748	1.02733	1.02378	1.02445	1.00000	1.00000	1.02446	0.73560
2007	1.02960	1.02128	1.01554	0.99272	0.99997	0.99275	1.00000	0.73612
2008	1.01915	1.01656	0.99768	1.00489	0.99975	1.00063	1.00450	0.72809
2009	0.94245	1.00646	0.96091	0.97449	0.99864	0.97582	1.00000	0.72938
2010	1.04971	1.00955	0.99971	1.04008	1.00091	1.02554	1.01327	0.73393
2011	1.05705	1.02104	1.01631	1.01865	1.00021	1.00604	1.01231	0.72754
2012	1.05772	1.01780	1.02118	1.01766	1.00000	1.00000	1.01766	0.72758
2013	1.03655	1.01063	1.01864	1.00688	1.00000	1.00000	1.00689	0.72784
2014	1.04125	1.01225	1.02417	1.00438	1.00000	1.00000	1.00439	0.72699
Mean	1.06550	1.03100	1.01720	1.01600	0.99990	1.00000	1.01600	0.76069



The growth decomposition components that appear in (61) are listed in Table 5, with the arithmetic means of the growth rates over the 54 years 1961-2014 listed in the last row. The average rate of aggregate TFP growth over these years was 1.60 percent per year, which is equal to the average rate of technical progress. There was no technical progress growth for eight of the years: 1974, 1975, 1979, 1980, 1982, 1989, 2007 and 2009. For these years, the rate of growth of value added efficiency was below unity and this translated into negative rates of TFP growth. The aggregate approximate input mix growth factors, the $\gamma^{t\bullet}$, are all very close to unity. The approximate equality in (61) was very close to being an equality, with the absolute value of the difference between $\mathrm{TFPG}^{t\bullet}$ and $\gamma^{t\bullet}\,\varepsilon^{t\bullet}\,\tau^{t\bullet}$ always less than 0.00003, and a mean difference of -0.0000034.

In Figure 3, we plot $\mathrm{TFP}^{t\bullet}$, and the explanatory factors $C^{t\bullet}$, $E^{t\bullet}$ and $T^{t\bullet}$ which appear in (62). Since Sector 1 is almost three times as big as Sector 2, it can be seen that the overall aggregate results are closer to the Sector 1 results. In particular, the huge value added inefficiency results that showed up in Sector 2 are no longer so huge in the aggregate results. However, inefficiency effects which are a result of recessions still show up as significant determinants of TFP at the aggregate level.

It can be seen that the input mix is not important in explaining U.S. Nonfinancial Private Sector TFP growth over the period 1960-2014. The most important explanatory factor is the level of technical progress but during recession years, the level of value added efficiency plays an important role. Also noteworthy is the very high rate of TFP growth for the Nonfinancial Sector over this long period: the geometric average rate of TFP growth was 1.583% per year

.

8. Conclusion

We have derived decompositions of nominal value added growth (and TFP growth) for individual sectors into explanatory factors. Starting with Denison (1962), various authors have presented decompositions of either aggregate labour productivity growth or TFP growth into sectoral explanatory factors by manipulating the index number formulae that are used to define the relevant aggregate.[30] The approach taken here relied instead on the economic approach to index number theory that started with Konüs (1939).

Rather than using the consumer's expenditure function in order to define various economic indexes, we used the sectoral cost constrained value added function, $R^t(p,w,x)$, as the basic building block in our approach. This function depends on four sets of variables: t (indicating which technology set is in scope), the output price vector p, the primary input price vector w and the primary input quantity vector x. Ratios of the cost constrained value added functions were used to define various explanatory "economic" indexes where three of the four sets of variables are held constant in the numerator and denominator and the remaining variable changes from a period $t-1$ level in the denominator to a period t level in the numerator.

With the goal of decomposing value added growth into a product of economic indexes, we operationalized our approach by assuming that an adequate approximation to a period t technology set can be obtained by taking the conical free disposal hull of past quantity observations for the sector under consideration. With a single output, we found that our approach generated estimates of TFP growth that are identical to standard index number estimates of TFP growth.

A main advantage of our approach is that our new nonparametric measure of technical progress never indicates technical regress. During recessions, value added efficiency drops below unity and depresses TFP growth. For our U.S. data set, TFP growth is well explained as the product of value added efficiency growth times the rate of technical progress. For the U.S. Noncorporate Nonfinancial Sector, we found that the cost of recessions was particularly high.

Implementation of the decompositions can provide key insights into the drivers of economic growth at a detailed sectoral level. Hence, we believe that they will provide new insights into the sources of economic growth. Our decompositions may also indicate data mismeasurement problems that can then be addressed by statistical agencies.

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[30] See for example, Tang and Wang (2004), Dumagan (2012), Balk (2014) (2015) (2016) and Diewert (2015) (2016).

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