Web Appendices to Accompany: "A Dynamic Equilibrium Model of the U.S. Wage Structure, 1968-1996"

by

Matthew Johnson
CRA International, Oakland California

and

Michael P. Keane
ARC Federation Fellow, University of Technology Sydney
Research Professor, Arizona State University

March, 2007
Revised March 2009

Appendix A: Solution and Estimation Methods

Estimation of the model requires nesting a fixed-point algorithm for finding the equilibrium wage vector (at any given parameter vector) within a parameter search algorithm. So, at the highest level of abstraction, we simply have:

Algorithm A. Parameter Search Algorithm

Step 1: Choose an initial guess for the parameter vector θ_0.

Step 2: Solve for the equilibrium wage vector $\tilde{w}(\theta_0)$. [See *Algorithm B*].

Step 3: Calculate the statistical objective function $q(\theta_0)$ that evaluates the fit of the model to the observe wage, occupational and educational choice data at θ_0.

Step 4: Using a search algorithm that seeks to improve $q(\theta)$, update the trial parameter vector to θ_1.

Repeat Steps 2 through 4 until the algorithm converges to an estimate $\hat{\theta}$ that gives a maximum of $q(\theta)$. Let r denote iterations, and θ_r the current trial parameter.

In Step 2, at each trial parameter vector θ_r,we must solve 29 fixed point problems to obtain the 160-vector of equilibrium wages in each of the 29 years of the model economy. This done using the simple algorithm:

Algorithm B. Equilibrium Wage Vector Computation

Step 1: Choose an initial guess for the equilibrium wage vector $\tilde{w}_0(\theta_r)$.

Step 2: Using equations (10)-(12), determine the allocation of labor across occupations and the home sector, as well as college attendance, in each period of the model, conditional on wage vector $\tilde{w}_0(\theta_r)$.

Step 3: Given the supply of each type of labor to each occupation determined in Step 2, use equation (2) to determine the marginal product of each type of labor in each occupation in each period.

Step 4: Update the wage vector to $\tilde{w}_1(\theta_r)$ by setting $w_{g,e,a,k,t} = \partial Y/\partial L_{g,e,a,k,t}$ for each g, e, a, k and t (i.e., set wage equal to marginal product for each type of labor in each period).

Repeat steps 2 through 4 until convergence is achieved to a wage vector $\tilde{w}(\theta_r)$ that satisfies the equilibrium condition in equation (2). That is, wages equal marginal products when workers allocate based on those wages.

In the vast majority of cases, algorithm B converges quickly to the equilibrium wage vector. However, in the course of searching for the parameter vector $\hat{\theta}$ that maximizes the objective function $q(\theta)$, we must examine many trial parameter vectors θ_t – on the order of a thousand. Furthermore, Step 4 of *Algorithm A* requires us to obtain derivatives of $q(\theta_r)$ with respect to each element of θ_r (as these are needed to update the parameter vector). Derivatives are obtained numerically, by "bumping" the trial parameter vector one element at a time and calculating the change in the objective function with each bump. As the model contains 311 parameters, this means solving for 312 equilibria given each trial parameter vector. Thus, *in toto*, we must solve for the equilibrium wage vector about 1000・312・29 ≈ 9 million times.

The fact that in estimation we most solve a 160-dimensional fixed point problem roughly 9 million times creates two problems:

First, it means estimation of the model is computationally burdensome, analogous to the problem that arises in estimating discrete dynamic programming (DP) models (where solution of a DP problem is nested within a parameter search algorithm so the DP problem must be solved at many trial parameter vectors).

Second, if the fixed point solution algorithm fails to find an equilibrium wage vector even in very rare circumstances (say once in every 10^5 or even 10^6 attempts), this is sufficient to cause the parameter search algorithm to break down. In our early work we found that this is exactly what happened, for the following reason:

On rare occasions, for a certain a trial wage vector, the occupational choice decision rule in (10)-(12) may imply almost no workers of a certain type choose a certain occupation. In that case, their marginal product in that occupation is driven to a very high value. Thus, in Step 4, their wage in that occupation is set very high, and almost all such workers choose the occupation in Step 2. This drives their marginal product to near zero, again causing very few to choose the occupation, etc.. On rare occasions, such oscillatory behaviour causes the search algorithm to break down.

We found a very effective way to eliminate this (very rare) non-convergence problem is to assume a small fraction of workers of each gender/education/age type chooses occupations randomly and with equal probability. Thus, we replace (12) with:

$$\mathrm{P}\left(d_{kt}=1 \mid g,e,a\right)=\pi\frac{1}{11}+(1-\pi)\exp\left(U(g,e,a)_{k,t}\right)\Big/\sum_{k=0}^{10}\exp\left(U(g,e,a)_{k,t}\right) \qquad (12')$$

where π is the proportion of workers that chooses randomly and uniformly over the eleven options. In our empirical work we set π = 0.05 for each type, which places a lower bound on the choice frequency for each occupation of 0.05/11 = 0.45%.

On very rare occasions it also occurs during the course of the search process that, at a certain trial wage vector $\tilde{w}_i(\theta_r)$, the return to college is so large (small) that nearly everyone (no one) chooses to attend college,[1] also creating non-convergence problems. To deal with this problem, we also assume a small fraction π_e of each gender/parental background type chooses randomly and with equal probability whether to attend college. Thus we replace equation (15) with:

$$\mathrm{P}_t\left(e = COL \middle| g,b\right) = \pi_e \frac{1}{11} + (1-\pi_e)\frac{\exp(V_t^C(g,b))}{\exp(V_t^{HS}(g,b)) + \exp(V_t^C(g,b))} \qquad (15')$$

where π_e is the fraction of workers that chooses randomly whether to attend college. In our empirical work we set $\pi_e = 0.05$ for each gender/parental background type, placing a lower bound of 0.05/2 = 2.5% on the college attendance rate for each type.

Notably, our choices of π and π_e have little influence on the behaviour of the model at the optimized parameter vector $\hat{\theta}$, because the fraction who choose each occupation and who choose college as a result of the random choice process is small relative to the fractions who choose each option as a result of an optimization process.

We next describe the estimation algorithm in detail. As noted in Section II.5, the exogenous (endogenous) capital version of the model contains K = 306 (311) parameters and fits 9,464 (9,493) data elements. These consist of observed occupation specific full-time equivalent earnings and employment shares for each of 16 education -gender-age cells, as well as age nineteen college attendance probabilities for each of 8 gender-parental education types, all over a period of 29 years. We use the method of moments (MOM) to estimate the parameter vector $\hat{\theta}$ that minimizes the distance from zero of the empirical moments – which consist of differences between PSID observations and model predictions for each of the data elements noted above.

Given the large dimension of the parameter vector and the large number of moments being fit, we eschew the traditional MOM approach of attempting to estimate an optimal weighting matrix, out of concerns about numerical stability. Instead, we assume a simplified error structure and estimate the variance of the cell errors using their sample variance across periods. This both increases speed to convergence and reduces sensitivity of the search algorithm given the large dimension of the parameter space.

[1] Note that this can occur exactly as in our earlier discussion of non-convergence, in a case where, during the search process, the wage is set very high for college labor in one of the occupations.

Specifically, we assume that, at the "true" parameter vector θ^*, differences between observed and predicted cell frequencies are iid normally distributed, both across cells c for a given time t, and within cells over time. Thus, we assume the observed data elements $d_{c,t}$ are related to the model predictions $P_{c,t}(\theta^*)$ as follows:

$$d_{c,t} = P_{c,t}\left(\theta^*\right) + \varepsilon_{c,t} \qquad \text{where} \qquad \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right),$$

Hence:

$$f\left(\varepsilon_{c,t}\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{{\varepsilon_{c,t}}^2}{2{\sigma_c}^2}\right),$$

where θ is the 306×1 vector of model parameters, t=1,…29 indexes periods, and c=1,…328 indexes cells, ordered by wages (1-160), occupational choice probabilities (161-320) and college choice probabilities (321-328).[2] The log-likelihood is therefore

$$\ln L(\theta) = \sum\nolimits_{c,t} \ln f\left(\varepsilon_{c,t}\right) = \sum\nolimits_{c,t} \ln f\left(d_{c,t} - P_{c,t}(\theta)\right)$$

We then construct moments based on the score, which we denote by $s(\theta)$:

$$s(\theta) = \frac{\partial \ln L(\theta)}{\partial \theta} = \sum\nolimits_{c,t} \frac{\partial \ln f\left(d - P(\theta)\right)}{\partial \theta} = \sum\nolimits_{c,t} \frac{\partial f\left(d_{c,t} - P_{c,t}(\theta)\right) / \partial \theta}{f\left(d_{c,t} - P_{c,t}(\theta)\right)}$$

$$= \sum\nolimits_{c,t} \frac{\partial P_{c,t}(\theta)/\partial \theta}{{\sigma_c}^2}\left(d_{c,t} - P_{c,t}(\theta)\right)$$

Now, at $\hat{\theta}_{ML}$ we have:

$$s\left(\hat{\theta}_{ML}\right) = \sum\nolimits_{c,t} \frac{\partial P_{c,t}\left(\hat{\theta}_{ML}\right) / \partial \hat{\theta}_{ML}}{{\sigma_c}^2}\left(d_{c,t} - P_{c,t}\left(\hat{\theta}_{ML}\right)\right) = 0_{306}, \tag{A1}$$

as the (306 x 1) score (or derivative) vector $q(\theta)$ of the log-likelihood is set to a zero vector at the ML estimate. This expression can be written more compactly as:

$$s\left(\hat{\theta}_{ML}\right) = W'(\hat{\theta}_{ML})\left(d - P\left(\hat{\theta}_{ML}\right)\right) = 0$$

where (d-$P(\theta)$) is a vector of population moments, such that E[(d-$P(\theta^*)$)]=0 at the true parameter vector θ^*, and $W'(\theta) = P_\theta V^{-1}$ is a weighting matrix that depends on the derivatives of the model predictions with respect to θ, as well as the variances of the cell specific errors,

---

[2] As we discussed in Section IV.2, three occupational cells are dropped because they have extremely small shares: college and high school female transport operatives, and college female laborers.

as noted in equation (A1). Note that matrix $W'(\theta)$ has dimension cT x K = 9464 x 306, while V and P_θ denote, respectively, a cT x cT (9464 x 9464) diagonal variance matrix, and a cT x K (9464 x 306) matrix of first partials of each model prediction with respect to each parameter.

In the method of moments approach one obtains a consistent estimator of θ^* by using the expression:

$$q(\hat{\theta}_{MOM}) \equiv W'\left(d - P\left(\hat{\theta}_{MOM}\right)\right) = 0 \qquad \text{(A2)}$$

where W is now a fixed matrix of instruments, with the property that $\mathrm{E}W'P_\theta(\theta^*)$ converges in probability to a fixed positive definite matrix as sample size $cT \to \infty$. One obtains an efficient MOM estimator by choosing instruments that are asymptotically equivalent to the weights that appear in the score expression (A1).

Of course, one can only construct the optimal weights if one has an initial consistent estimate of θ in hand. In our application, we start with weights of the form $W'(\theta_0) = P_\theta(\theta_0)V(\theta_0)^{-1}$ evaluated at an initial guess for θ, denoted θ_0, and periodically update the value of θ at which the weights are evaluated as we iterate.

As it is often not possible to satisfy equation (A2) exactly, the MOM estimator is typically defined as:

$$\hat{\theta}_{MOM} = \arg\min_\theta \left(d - P(\theta)\right)' WW'\left(d - P(\theta)\right) \qquad \text{(A3)}$$

The parameter search algorithm relies on a Taylor series approximation around a trial parameter vector θ_r. For simplicity of exposition, further define the KxK (306 x 306) matrix of first partials of the empirical moments $Q_\theta(\theta) \equiv \partial q(\theta)/\partial\theta$, with elements of the form (for each parameter pair k and j):

$$\frac{\partial q_k(\theta_r)}{\partial \theta_j} = -\sum\nolimits_{c,t} \frac{\partial P_{c,t}(\theta_0)/\partial\theta_k}{\sigma_c^{\,2}(\theta_0)} \times \frac{\partial P_{c,t}(\theta_r)}{\partial\theta_j}$$

or, in matrix notation,

$$Q_\theta(\theta_r) = -P_\theta(\theta_0)'V(\theta_0)^{-1} \cdot P_\theta(\theta_r) = -W(\theta_0)'P_\theta(\theta_r)$$

where we emphasize that the instruments $\dfrac{\partial P_{c,t}(\theta_0)/\partial\theta_i}{\hat{\sigma}_c^{\,2}}$ are evaluated at the initial estimate θ_0 and are henceforth regarded as fixed as we iterate.

Our assumptions on the error structure lead to a diagonal covariance matrix *V* in which the 328 (329 when capital is endogenous) cell variances $\sigma_c^2(\theta)$ are repeated over *T*=29 diagonal blocks. Variances are estimated by the sample variances of errors across periods within each cell.

Given a trial parameter vector θ_r, we approximate the value θ_{r+1} that satisfies the moment conditions $q(\theta_{r+1})=0$ by a Taylor expansion around θ_r as follows:

$$q(\theta_{r+1}) = q(\theta_r) + \left.\frac{\partial q(\theta)}{\partial \theta}\right|_{\theta=\theta_r} \times (\theta_{r+1} - \theta_r) = 0_{306}$$

$$\theta_{r+1} = \theta_r - \left[Q_\theta(\theta)\big|_{\theta=\theta_r} \right]^{-1} q(\theta_r)$$

yielding the updated parameter estimate θ_{r+1}. In practice, the search algorithm based on this updating equation adds a step size λ to the direction vector:

$$\theta_{r+1} = \theta_r - \lambda \left[Q_\theta(\theta)\big|_{\theta=\theta_r} \right]^{-1} q(\theta_r) \tag{A4}$$

A line search on λ is then used to find the value of θ_{r+1} that minimizes (A3).

Standard errors of the estimates are obtained using the usual MOM formula:

$$\text{Est. } Var\left[\hat{\theta}_{MOM}\right] = \left[G\left(\hat{\theta}_{MOM}\right)' \left\{ \hat{V}ar\left[\bar{m}\left(\hat{\theta}_{MOM}\right)\right]\right\}^{-1} G\left(\hat{\theta}_{MOM}\right)\right]^{-1}$$

where the $\bar{m}\left(\hat{\theta}_{MOM}\right)$ are the sample moments, $G\left(\hat{\theta}_{MOM}\right)$ is the matrix of derivatives of the moments with respect to parameters, and $\hat{V}ar\left[\bar{m}\left(\hat{\theta}_{MOM}\right)\right]$ is the estimated variance-covariance matrix of the moments. In our notation, this reduces to:

$$\text{Est. } Var\left[\hat{\theta}_{MOM}\right] = \left[Q_\theta\left(\hat{\theta}_{MOM}\right)' \left\{ q(\hat{\theta}_{MOM}) q'(\hat{\theta}_{MOM}) \right\}^{-1} Q_\theta\left(\hat{\theta}_{MOM}\right)\right]^{-1}$$

Computational Search Algorithm

The updating equation (A4) is the basis of our parameter search algorithm. However, as we noted earlier, unchecked adherence to the updating rule can lead to trial parameter values that do not produce convergence to an equilibrium wage vector. In addition to the measures described earlier (i.e., placing lower bounds on occupational choice probabilities), the search algorithm was also modified to help avoid parameter values that lead to non-convergence. In addition, the algorithm considers only trial parameter vectors that produce non-zero gradients (to machine accuracy) for all parameters, and hence non-singular $Q_\theta(\theta_r)$.

The algorithm proceeds as follows:

1. Choose a starting parameter vector $\theta = \theta_0$.
2. Iterate the equilibrium model to convergence, and
   a. Compute the errors $\varepsilon_{c,t} = d_{c,t} - P_{c,t}(\theta)$.
   b. Use the $\varepsilon_{c,t}$ to calculate diagonal elements σ_c^2 of covariance matrix V.
3. Compute numerical derivatives of each $P_{c,t}(\theta)$ with respect to each of the K parameters, and:
   a. Populate the cT x K matrix P_θ.
   b. Build the fixed (over this series of iterations) instrument array $W(\theta_0) = V(\theta_0)^{-1}P_\theta(\theta_0)$.
   c. Compute the empirical moments $q = W'\varepsilon$ and the matrix $Q_\theta = -W'P_\theta$.
   d. Compute initial criterion function value $\Gamma_0 = q'q$.

Start of Main Iteration Loop:

4. Set the stepsize λ (starting at λ=1) and update the estimate of θ according to

   $$\theta_{r+1} = \theta_r - \lambda \left[\hat{Q}_\theta(\theta)\Big|_{\theta=\theta_r} \right]^{-1} q(\theta_r)$$ where r denotes the iteration number.

   Attempt to compute a new equilibrium at θ_{r+1} and:
   a. If the model converges to equilibrium, continue to step 5.
   b. If the model does not converge, reduce step size to $\lambda = \lambda/2$ and return to beginning of step 4.[3]
5. Evaluate the objective Γ at θ_{r+1}, and denote it Γ_{start}. Denote current step size λ_0.
6. Rescale step size to $\lambda = 2\lambda_0$, update the parameter vector as in Step 4, and attempt to compute an equilibrium at the updated parameter value. Then,
   a. If the model converges to equilibrium, compute a new Γ, and set s, the direction of the line search for λ (i.e., $\lambda = 2^s\lambda_0$)according to:
      i. If $\Gamma \geq \Gamma_{start}$ set s = -1.
      ii. If $\Gamma < \Gamma_{start}$ set s = +1.
   b. If the model does not converge, reduce the step size to $\lambda = \lambda_0/2$, update parameters, and attempt to compute a new equilibrium. Then:

[3] We also place reasonable upper bounds on the substitution parameters (ρ). When a parameter update leads to values above these bounds it is treated as equivalent to non-convergence.

i. If the model converges to equilibrium, compute a new Γ and

1. If $\Gamma \geq \Gamma_{start}$, stop λ line search at λ_0. Skip to Step 8.
2. If $\Gamma < \Gamma_{start}$ set s = -1.

ii. If the model does not converge to an equilibrium, stop the λ line search at λ_0, and skip to Step 8.

7. Set the step size to $2^s\lambda$, where s = -1 or 1, update parameter vector, and attempt to compute a new equilibrium:
   a. If the model converges to equilibrium, compute a new Γ and
      i. If $\Gamma \geq \Gamma_{start}$ reset λ to its value at update just prior.
      ii. If $\Gamma < \Gamma_{start}$ continue line search by repeating Step 7.
   b. If the model does not converge to an equilibrium, reset λ to its value at update just prior.
8. Update parameters to θ_{r+1} using the current λ. Compute Γ_{r+1} and compare it to the initial criterion value under Γ_r. Then,
   a. If $\Gamma_{r+1} \geq \Gamma_r$ then reduce step size $\lambda = \lambda/2$ and repeat Step 8. If Step 8 repetitions reach a predetermined limit, exit algorithm.
   b. If $\Gamma_{r+1} < \Gamma_r$ we have improved objective, so continue to Step 9.
9. Compute numerical derivatives of each prediction cell evaluated at θ_{iter} by each of the K parameters, and
   a. Populate the cT x K matrix P_θ, and
      i. If P_θ does not have full column rank (resulting from one or more zeroed columns), then reduce step size $\lambda = \lambda/2$. Update parameter vector (using prior iteration's P_θ) using new step size and repeat step 9. If step 9 repetitions reach a predetermined limit, exit algorithm.
      ii. If P_θ has full column rank, continue to step 9b.
   b. Compute errors $\varepsilon_{c,t}$ using equilibrium model predictions $P_{c,t}(\theta)$.
   c. Compute the empirical moments $q = \hat{W}'\varepsilon$ and the matrix $\hat{Q}_\theta = -\hat{W}'P_\theta$.
10. Compute iteration final criterion function value $\Gamma_{FINAL} = q'q$. Then,
    a. If $\left|(\Gamma_0 - \Gamma_{FINAL})/\Gamma_0\right| \leq m$, or maximum iterations are reached, exit algorithm.
    b. Otherwise, reset $\Gamma_0 = \Gamma_{FINAL}$ and return to Step 4.

Appendix B: Technician Occupation Codes

(1970 Census Occupation Classification)

Code	Description
24	Farm management advisors
25	Foresters and conservationists
26	Home management advisors
32	Librarians
73	Health practitioners, n.e.c.
74	Dieticians
75	Registered nurses
76	Therapists
80	Clinical laboratory technologists and technicians
81	Dental hygienists
82	Health record technologists and technicians
83	Radiologic technologists and technicians
84	Therapy assistants
85	Health technologists and technicians, n.e.c.
86	Clergymen
90	Religious workers, n.e.c.
100	Social workers
101	Recreation workers
141	Adult education teachers
142	Elementary school teachers
143	Pre-kindergarten and kindergarten teachers
144	Secondary school teachers
145	Teachers, except college and university, subject not specified
150	Agriculture and biological technicians, except health
151	Chemical technicians
152	Draftsmen
153	Electrical and electronic engineering technicians
154	Industrial engineering technicians
155	Mechanical engineering technicians
156	Mathematical technicians
161	Surveyors
162	Engineering and science technicians, n.e.c.
165	Embalmers
171	Radio operators
172	Tool programmers, numerical control
173	Technicians, n.e.c.
174	Vocational and educational counsellors
175	Actors

Code	Description
180	Athletes and kindred workers
181	Authors
182	Dancers
183	Designers
184	Editors and reporters
185	Musicians and composers
190	Painters and sculptors
191	Photographers
192	Public relations men and publicity writers
193	Radio and television announcers
194	Writers, artists, and entertainers, n.e.c.

Note: We classify the other 3-digit occupations in the 001-195 range as "Professionals."

Appendix C: Details of Data Construction

Construction of Occupational Employment Shares and Wage rates

We arrive at a measure of full-time hours for each occupation as follows: Figure A1 reports the average hours of "full-time" male workers in each occupation in each year (where full-time is defined as working at least 35 weeks and 30 hours per week). Clearly, average full-time hours differ significantly across occupations, with managers and administrators the highest and laborers the lowest. Hours levels also vary within occupations over time. For each occupation, we use the maximum of the full-time hours measure across all 29 years in Figure A1 as the occupation specific full-time hours level. Denote this by FTE_k for k=1,10.

Next, occupational choice frequencies for each gender/education/age group are calculated using reported hours worked relative to our measures of full-time hours. Letting i=1,…, $N(t)$ index PSID core sample members in year t, for each group (g, e, a) we have:

$$\mathrm{P}\left(d_{kt}=1 \mid g,e,a\right)=\sum_{i=1}^{N(t)}\left(\alpha_{it}n_i \times I(d_{ikt}=1)\right)\Big/\sum_{i=1}^{N(t)}n_i \qquad for \quad k=1,10 \qquad \text{(C1)}$$

where $N(t)$ is the number of 25-64 year olds in the sample in year t, d_{ikt} is an indicator for whether person i chose occupation k in year t (equal to one for at most one occupation) and:

$$n_i = \text{ PSID core sample weight,} \qquad \alpha_{it}=hours_{it}/FTE_k\,, \quad 0<\alpha_i\le 1$$

where α_{it} is the fraction of a full-time unit of labor that person i supplies to occupation k.

As we assign each person to one main occupation, α_{it} may be non-zero for only one occupation per person. Any residual time (1- α_{it})n_i is assigned to the home sector. Thus, the number of FTEs assigned to home consists not only of workers who are fully unemployed or out of the labor force, but also includes some fraction of the time of part-time workers.

Analogous to (C1), the "wage rate" in occupation k – i.e., the annual earnings of a full-time equivalent worker who chooses that occupation – is given by:

$$w_{g,e,a,k,t}=\sum_{i=1}^{N(t)}\left(wage_{it}\cdot FTE_k\cdot\alpha_{it}n_i\times I(d_{ikt}=1)\right)\Big/\sum_{i=1}^{N(t)}\alpha_{it}n_i\times I(d_{ikt}=1)\;\; for\;\; k=1,10 \qquad \text{(C2)}$$

where $wage_{it} = earnings_{it} / hours_{it}$. The earnings data are put in 1999 US dollars using the CPI-U. Before applying (C2), we screen outliers by dropping the top and bottom 0.5% of full-time wage estimates in each year by gender.[4]

[4] The hours data in the 1969, 1970 and 1974 PSID are often missing or unreliable. In such cases, we are unable to construct α_{it}, the fraction of an FTE that i supplies to an occupation. Rather, we impute hours using group averages from the surrounding years. Specifically, we compute average hours worked in surrounding years for each gender, education, occupation and ten-year age group (conditional on positive hours). Mean hours are then computed for each group using core weights. These imputations of average hours are then used to stand in for cases where individual observations are missing/unreliable in the suspect years ('69, '70 and '74).

Construction of Occupational Employment Shares and Wage Rates

As our capital stock measure we use Bureau of Economic Analysis (BEA) estimates of nonresidential fixed investment, reported in June 2003.[5] We add equipment and software (E&S) and non-residential structures (omitting government and residential fixed assets). Several prior studies, including Krusell et al (2000), attempt to improve upon standard BEA measures of capital by incorporating quality-adjusted measures of equipment prices. But, comparing the BEA measures to those used in Krusell et al (2000), it appears the gap between the two has narrowed considerably, as a result of a 2003 revision of the methods used to construct the national income and product accounts. Included in this revision were new methodologies for computing the real stock of fixed assets.[6] Thus we decided to use the standard BEA measures. Consistent with the rest of our data, the capital stock is expressed in 1999 dollars (see Figure A2).[7]

[5] Specifically, BEA Table 1.1 "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods", and BEA Table 1.2 "Chain-Type Quantity Indexes for Net Stock of Fixed Assets and Consumer Durable Goods".
[6]In describing the overhaul, Moulton , "Note on the Upcoming Comprehensive Revision of the National Income and Product Accounts," *Survey of Current Business*, Nov. 2002, p. 6-7, states: "BEA is pursuing research on developing quality-adjusted prices indexes for software, photocopy equipment, and nonresidential structures. The BEA is also evaluating the use of quality-adjusted price indexes for communications equipment developed by the Federal Reserve Board."
[7] We use separate BEA deflators for equipment and software (E&S) and Structures. This is important as the two can behave rather differently (e.g., from 1996 to 1999 the price level for E&S fell by roughly 3%, while that for structures grew 15%). Note also that the BEA readily provides quantity indexes rather than value deflators. We used the quantity indexes to get the implied deflators and use these to convert current values of capital to 1999 dollars.

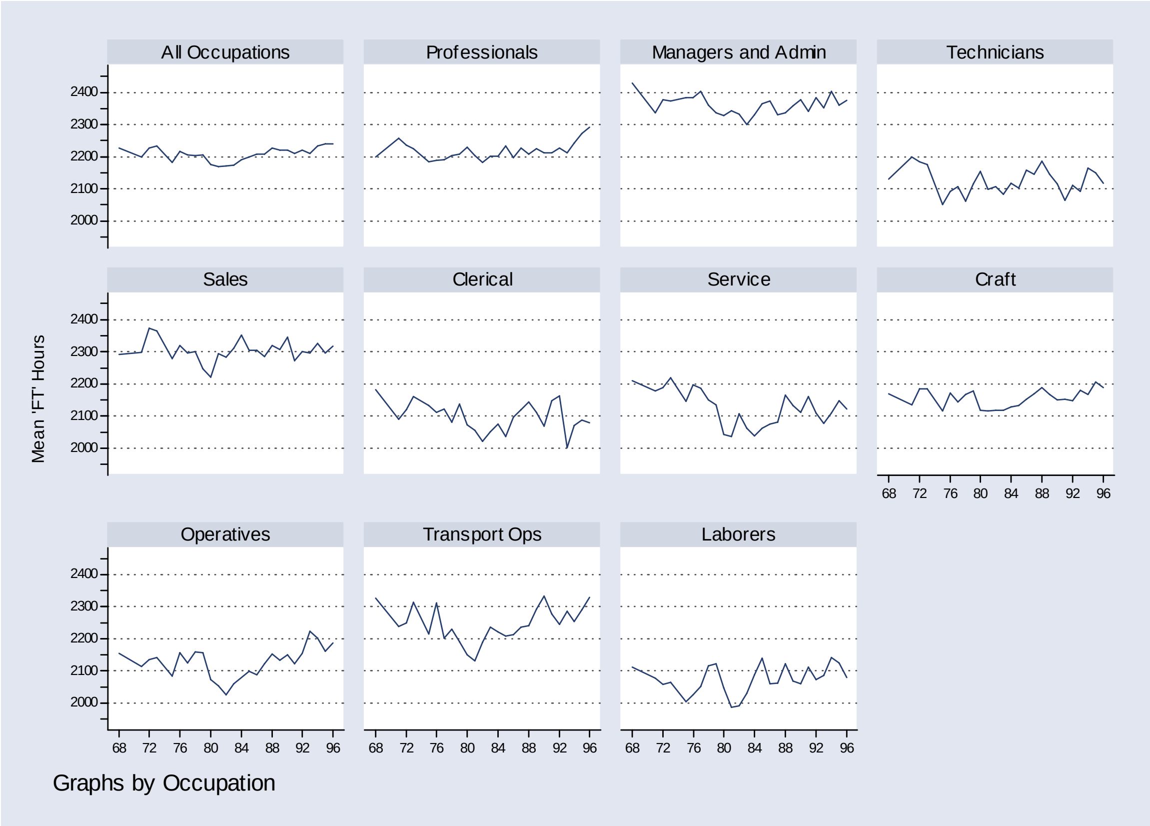

Figure A1 – Full Time Hours by Occupation

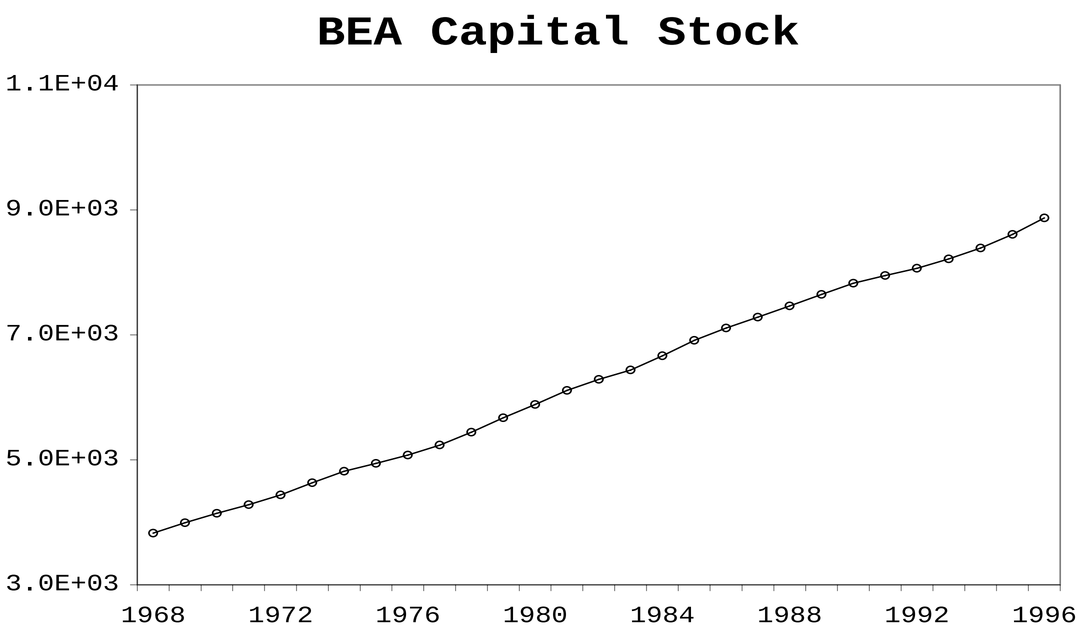

Figure A2

Appendix D: CES Share Parameter Estimates

Table D1 – CES Skill and Occupational Share Parameters

	Estimate	**Standard Error**		
Share of (Capital + Skilled Labor) Aggregate - λ_s				
Constant	-9.73E-01	(1.31E-01)	**	
Trend	-9.14E-02	(6.77E-03)	**	
Trend^2	2.28E-02	(4.42E-04)	**	
Trend^3	-1.29E-03	(2.16E-05)	**	
Trend^4	2.12E-05	(8.83E-07)	**	
λ_s in Selected	*1968*	*1982*		*1996*
Years	0.261	0.374		0.262
Share of Capital in (capital, Skilled Labor) Aggregate - λ_A				
Constant	1.77E+00	(7.04E-01)	**	
Trend	-1.14E-01	(1.14E-01)		
Trend^2	4.59E-02	(8.26E-03)	**	
Trend^3	-2.72E-03	(3.13E-04)	**	
Trend^4	4.47E-05	(4.51E-06)	**	
λ_A in Selected	*1968*	*1982*		*1996*
years	0.846	0.969		0.902
Share of Services in Unskilled Labor Aggregate - λ_u				
Constant	-8.55E-01	(2.80E-01)	*	
Trend	-1.01E-02	(1.73E-02)		
Trend^2	4.79E-03	(1.03E-04)	*	
Trend^3	-9.90E-05	(1.62E-06)	*	
λ_u in Selected	*1968*	*1982*		*1996*
years	0.297	0.434		0.614
Share of Professionals in Skilled Labor Aggregate - λ_1				
Constant	-1.21E-01	(1.64E-01)		
Trend	-3.50E-02	(1.38E-01)		
Trend^2	2.99E-03	(1.67E-02)		
Trend^3	-1.11E-04	(6.63E-04)		
Trend^4	1.69E-06	(8.65E-06)		
λ_1 in Selected	*1968*	*1982*		*1996*
years	0.462	0.435		0.466
Share of Technicians in Services Aggregate - λ_3				
Constant	1.48E-01	(4.54E-01)		
Trend	5.41E-02	(9.17E-02)		
Trend^2	-1.94E-03	(3.99E-03)		
Trend^3	2.73E-05	(6.55E-05)		
λ_3 in Selected	*1968*	*1982*		*1996*
years	0.248	0.324		0.363

Share of Sales in Services Aggregate - λ_4

Constant	-3.75E-02	(4.99E-01)		
Trend	-4.87E-03	(4.43E-03)		
Trend^2	8.74E-04	(7.47E-04)		
Trend^3	-1.77E-05	(2.11E-05)		
λ_4 in Selected	*1968*	*1982*		*1996*
years	0.195	0.180		0.195

Share of Clerical in Services Aggregate - λ_5

Constant	5.46E-01	(4.19E-02)	*	
Trend	1.24E-02	(3.81E-02)		
Trend^2	-6.42E-04	(1.54E-03)		
Trend^3	3.63E-06	(1.94E-05)		
λ_5 in Selected	*1968*	*1982*		*1996*
years	0.355	0.320		0.271

Share of Craft Workers in Blue Collar Aggregate - λ_7

Constant	1.88E+00	(7.50E-01)	*	
Trend	2.06E-02	(5.05E-02)		
Trend^2	-1.90E-03	(3.88E-03)		
Trend^3	4.57E-05	(8.92E-05)		
λ_7 in Selected	*1968*	*1982*		*1996*
years	0.493	0.538		0.534

Share of Operatives in Blue Collar Aggregate - λ_8

Constant	1.50E+00	(6.66E-01)	*	
Trend	-4.10E-02	(1.02E-02)	*	
Trend^2	1.17E-03	(2.21E-03)		
Trend^3	-3.16E-06	(6.90E-05)		
λ_8 in Selected	*1968*	*1982*		*1996*
years	0.317	0.246		0.243

Share of Transport Operatives in Blue Collar Aggregate - λ_9

Constant	4.39E-01	(2.71E-02)	*	
Trend	3.12E-03	(6.10E-02)		
Trend^2	2.45E-04	(4.73E-03)		
Trend^3	-2.44E-07	(1.11E-04)		
λ_9 in Selected	*1968*	*1982*		*1996*
years	0.115	0.136		0.150

Table D2 – CES Education Shares by Occupation

	Estimate	Standard Error		
High School Share among Professionals - μ_1				
Constant	-1.52E+00	(8.29E-01)		
Trend	6.65E-02	(1.73E-02)	*	
Trend^2	-9.71E-03	(4.51E-04)	*	
Trend^3	3.79E-04	(3.95E-05)	*	
Trend^4	-4.39E-06	(4.08E-07)	*	
μ_1 in Selected years	*1968*	*1982*		*1996*
	0.188	0.161		0.165
High School Share among Managers - μ_2				
Constant	-4.85E-01	(2.92E-02)	*	
Trend	7.59E-02	(2.20E-02)	*	
Trend^2	-1.20E-02	(5.31E-03)	*	
Trend^3	5.70E-04	(2.60E-04)	*	
Trend^4	-9.10E-06	(4.14E-06)	*	
μ_2 in Selected years	*1968*	*1982*		*1996*
	0.396	0.360		0.294
High School Share among Technicians - μ_3				
Constant	-8.03E-01	(3.75E-01)	*	
Trend	-6.26E-02	(3.97E-02)		
Trend^2	4.06E-03	(1.62E-03)	*	
Trend^3	-8.16E-05	(2.43E-05)	*	
μ_3 in Selected years	*1968*	*1982*		*1996*
	0.297	0.249		0.233
High School Share among Sales Workers - μ_4				
Constant	-1.13E-01	(2.75E-02)	*	
Trend	-4.91E-02	(5.87E-03)	*	
Trend^2	1.67E-04	(1.16E-03)		
Trend^3	2.18E-05	(3.71E-05)		
μ_4 in Selected years	*1968*	*1982*		*1996*
	0.460	0.323		0.296
High School Share among Clerical Workers - μ_5				
Constant	2.61E-01	(4.10E-01)		
Trend	5.04E-02	(1.19E-02)	*	
Trend^2	-5.60E-03	(2.24E-03)	*	
Trend^3	1.35E-04	(5.69E-05)	*	
μ_5 in Selected years	*1968*	*1982*		*1996*
	0.576	0.553		0.576

High School Share among Service Workers - μ_6

Constant	1.05E+00	(9.64E-01)		
Trend	-6.65E-02	(4.06E-02)		
Trend^2	1.36E-03	(9.33E-04)		
Trend^3	-1.05E-05	(8.34E-06)		
μ_6 in Selected years	*1968*	*1982*		*1996*
	0.728	0.580		0.502

High School Share among Craft Workers - μ_7

Constant	1.25E+00	(9.58E-01)		
Trend	-5.57E-02	(2.26E-02)	*	
Trend^2	1.71E-03	(2.70E-04)	*	
Trend^3	-2.27E-05	(1.80E-06)	*	
μ_7 in Selected years	*1968*	*1982*		*1996*
	0.767	0.672		0.626

High School Share among Operatives - μ_8

Constant	2.43E+00	(1.77E+00)		
Trend	-8.85E-02	(6.18E-02)		
Trend^2	-2.07E-04	(1.00E-03)		
Trend^3	5.61E-05	(4.02E-06)	*	
μ_8 in Selected years	*1968*	*1982*		*1996*
	0.912	0.777		0.742

High School Share among Transport Operatives - μ_9

Constant	7.62E-01	(8.57E-01)		
Trend	1.80E-01	(7.46E-02)	*	
Trend^2	-1.52E-02	(7.36E-03)	*	
Trend^3	3.16E-04	(1.63E-04)		
μ_9 in Selected years	*1968*	*1982*		*1996*
	0.716	0.754		0.721

High School Share among Laborers - μ_9

Constant	1.61E+00	(1.20E+00)		
Trend	-5.17E-02	(1.65E-03)	*	
Trend^2	1.56E-03	(2.46E-03)		
Trend^3	-4.89E-05	(6.08E-05)		
μ_{10} in Selected years	*1968*	*1982*		*1996*
	0.826	0.735		0.558

Table D3 – CES Gender Shares by Education/Occupation

	Estimate	Standard Error		
A. Male+Female High School Labor Aggregates, by Occupation				
HS Male Share of HS Aggregate, Professionals - ξ_1				
Constant	9.13E-01	(1.93E-01)	*	
Trend	2.57E-02	(1.33E-04)	*	
Trend^2	-2.57E-03	(2.37E-04)	*	
Trend^3	4.30E-05	(8.22E-06)	*	
ξ_1 in Selected years	*1968*	*1982*		*1996*
	0.718	0.704		0.633
HS Male Share of HS Aggregate, Managers - ξ_2				
Constant	1.31E+00	(3.20E-01)	*	
Trend	2.33E-02	(2.05E-02)		
Trend^2	-4.06E-03	(2.11E-03)		
Trend^3	7.40E-05	(3.50E-05)	*	
ξ_2 in Selected years	*1968*	*1982*		*1996*
	0.791	0.730		0.593
HS Male Share of HS Aggregate, Technicians - ξ_3				
Constant	3.60E-01	(1.30E-01)	*	
Trend	1.20E-01	(3.79E-02)	*	
Trend^2	-8.54E-03	(2.39E-03)	*	
Trend^3	1.48E-04	(3.77E-05)	*	
ξ_3 in Selected years	*1968*	*1982*		*1996*
	0.616	0.677		0.568
HS Male Share of HS Aggregate, Sales Workers - ξ_4				
Constant	1.11E+00	(6.92E-02)	*	
Trend	6.25E-02	(1.07E-03)	*	
Trend^2	-5.13E-03	(3.44E-04)	*	
Trend^3	8.84E-05	(7.60E-06)	*	
ξ_4 in Selected years	*1968*	*1982*		*1996*
	0.763	0.767		0.683
HS Male Share of HS Aggregate, Clerical Workers - ξ_5				
Constant	3.58E-01	(3.34E-01)		
Trend	3.37E-02	(1.96E-03)	*	
Trend^2	-3.95E-03	(6.18E-04)	*	
Trend^3	8.89E-05	(2.04E-05)	*	
ξ_5 in Selected years	*1968*	*1982*		*1996*
	0.596	0.568		0.544

HS Male Share of HS Aggregate, Service Workers - ξ_6

Constant	9.50E-01	(4.98E-02)	*	
Trend	-4.36E-02	(3.70E-02)		
Trend^2	1.69E-03	(2.06E-03)		
Trend^3	-2.51E-05	(3.30E-05)		
ξ_6 in Selected years	*1968*	*1982*		*1996*
	0.713	0.644		0.622

HS Male Share of HS Aggregate, Craft Workers - ξ_7

Constant	1.22E+00	(8.45E-01)		
Trend	1.03E-01	(4.67E-04)	*	
Trend^2	-7.51E-03	(3.71E-04)	*	
Trend^3	1.32E-04	(9.83E-06)	*	
ξ_7 in Selected years	*1968*	*1982*		*1996*
	0.788	0.820		0.751

HS Male Share of HS Aggregate, Operatives - ξ_8

Constant	8.42E-01	(1.96E-01)	*	
Trend	-7.50E-04	(2.07E-02)		
Trend^2	-8.22E-04	(8.48E-04)		
Trend^3	1.77E-05	(1.15E-05)		
ξ_8 in Selected years	*1968*	*1982*		*1996*
	0.699	0.669		0.637

HS Male Share of HS Aggregate, Transport Operatives - ξ_9

Constant	2.20E+00	(6.98E-01)	*	
Trend	-2.43E-02	(1.42E-03)	*	
Trend^2	-3.27E-03	(3.38E-05)	*	
Trend^3	1.03E-04	(1.84E-06)	*	
ξ_9 in Selected years	*1968*	*1982*		*1996*
	0.897	0.809		0.778

HS Male Share of HS Aggregate, Laborers - ξ_{10}

Constant	1.30E+00	(5.81E-01)	*	
Trend	-2.35E-02	(9.15E-03)	*	
Trend^2	8.71E-04	(7.61E-04)		
Trend^3	-3.84E-05	(2.15E-05)		
ξ_{10} in Selected years	*1968*	*1982*		*1996*
	0.782	0.734		0.602

B. Male+Female College Labor Aggregates, by Occupation

COL Male Share of COL Aggregate, Professionals - γ_1

Constant	1.07E+00	(4.78E-01)	*	
Trend	5.31E-02	(4.03E-02)		
Trend^2	-5.70E-03	(3.80E-03)		
Trend^3	1.13E-04	(7.30E-05)		
	1968	*1982*		*1996*
γ_1 in Selected years	0.754	0.725		0.642

COL Male Share of COL Aggregate, Professionals - γ_2

Constant	1.17E+00	(6.35E-01)		
Trend	1.26E-02	(1.37E-02)		
Trend^2	-3.04E-03	(4.79E-04)	*	
Trend^3	7.26E-05	(1.81E-05)	*	
	1968	*1982*		*1996*
γ_2 in Selected years	0.765	0.715		0.678

COL Male Share of COL Aggregate, Technicians - γ_3

Constant	5.93E-01	(1.58E-02)	*	
Trend	3.25E-02	(2.40E-02)		
Trend^2	-4.87E-03	(2.91E-03)		
Trend^3	1.17E-04	(6.59E-05)		
	1968	*1982*		*1996*
γ_3 in Selected years	0.650	0.594		0.574

COL Male Share of COL Aggregate, Sales Workers - γ_4

Constant	3.49E+00	(1.22E+00)	*	
Trend	-3.40E-01	(1.34E-01)	*	
Trend^2	1.62E-02	(7.20E-03)	*	
Trend^3	-2.73E-04	(1.36E-04)	*	
	1968	*1982*		*1996*
γ_4 in Selected years	0.960	0.754		0.651

COL Male Share of COL Aggregate, Clerical Workers - γ_5

Constant	5.28E-01	(1.87E-01)	*	
Trend	1.16E-02	(6.64E-03)		
Trend^2	-1.84E-03	(1.96E-04)	*	
Trend^3	3.82E-05	(8.04E-06)	*	
	1968	*1982*		*1996*
γ_5 in Selected years	0.631	0.603		0.563

COL Male Share of COL Aggregate, Service Workers - γ_6

Constant	7.15E-01	(8.77E-02)	*	
Trend	8.39E-02	(3.14E-02)	*	
Trend^2	-7.57E-03	(2.62E-03)	*	
Trend^3	1.71E-04	(5.89E-05)	*	
	1968	*1982*		*1996*
γ_6 in Selected years	0.688	0.700		0.719

COL Male Share of COL Aggregate, Craft Workers - γ_7

Constant	1.21E+00	(7.77E-01)		
Trend	4.82E-02	(5.05E-03)	*	
Trend^2	-5.86E-03	(8.85E-04)	*	
Trend^3	1.47E-04	(2.48E-05)	*	
	1968	*1982*		*1996*
γ_7 in Selected years	0.778	0.753		0.783

COL Male Share of COL Aggregate, Operatives - γ_8

Constant	-4.06E-01	(1.87E-01)	*	
Trend	2.53E-01	(6.49E-02)	*	
Trend^2	-1.73E-02	(3.76E-03)	*	
Trend^3	3.56E-04	(7.10E-05)	*	
	1968	*1982*		*1996*
γ_8 in Selected years	0.457	0.668		0.749

COL Male Share of COL Aggregate, Transport Operatives - γ_9

Constant	8.15E-01	(1.23E+00)		
Trend	1.19E-01	(1.79E-01)		
Trend^2	-7.25E-03	(1.13E-02)		
Trend^3	1.28E-04	(2.15E-04)		
	1968	*1982*		*1996*
γ_9 in Selected years	0.717	0.803		0.787

COL Male Share of COL Aggregate, Laborers - γ_{10}

Constant	-2.80E-02	(5.83E-01)		
Trend	5.22E-02	(6.84E-02)		
Trend^2	3.31E-03	(4.59E-03)		
Trend^3	-1.79E-04	(9.01E-05)		
	1968	*1982*		*1996*
γ_{10} in Selected years	0.507	0.710		0.473

Note: Three cells were dropped from the model due to extremely small cell counts: Female transport operatives and college female laborers.

Table D4 – CES Age Group Shares by Gender/Education/Occupation

	Estimate	Standard Error		
A. Combine HS Male Age Groups into HS Male Aggregate, by Occupation				
Age Group Shares of HS Male Aggregate, Skilled Labor Occupations (Professionals and Managers)- τ_{1-2}				
Constant: age 25-34	-1.69E-02	(1.14E-02)		
age 35-44	1.91E-01	(1.80E-02)	*	
age 45-54	1.37E-01	(1.22E-02)	*	
τ_{1-2} in selected years	*1968*	*1982*		*1996*
Age 25-34	0.225	0.215		0.221
Age 35-44	0.278	0.263		0.287
Age 45-54	0.267	0.279		0.277
Age Group Shares of HS Male Aggregate, Service Occupations: τ_{3-6}				
Constant: age 25-34	1.42E-01	(1.76E-02)	*	
age 35-44	2.08E-01	(1.63E-02)	*	
age 45-54	1.47E-01	(1.10E-02)	*	
τ_{3-6} in selected years	*1968*	*1982*		*1996*
Age 25-34	0.252	0.241		0.248
Age 35-44	0.270	0.256		0.279
Age 45-54	0.258	0.270		0.268
Age Group Shares of HS Male Aggregate, Blue Collar Occupations: τ_{7-10}				
Constant: age 25-34	1.35E-01	(1.24E-02)	*	
age 35-44	2.28E-01	(1.34E-02)	*	
age 45-54	1.59E-01	(6.43E-03)	*	
τ_{7-10} in selected years	*1968*	*1982*		*1996*
Age 25-34	0.249	0.238		0.245
Age 35-44	0.274	0.259		0.283
Age 45-54	0.260	0.271		0.269
Higher Order Polynomial Terms Common to HS Males, all 10 Occupations				
Age 25-34: (t)	-1.02E-03	(2.49E-03)		
Age 25-34: (t^2)	-9.82E-04	(7.77E-05)	*	
Age 25-34: (t^3)	3.71E-05	(2.67E-07)	*	
Age 35-44: (t)	2.51E-03	(3.57E-03)		
Age 35-44: (t^2)	-1.45E-03	(1.91E-04)	*	
Age 35-44: (t^3)	5.13E-05	(2.98E-06)	*	
Age 45-54: (t)	1.83E-02	(1.09E-03)	*	
Age 45-54: (t^2)	-2.03E-03	(3.92E-05)	*	
Age 45-54: (t^3)	5.33E-05	(2.72E-06)	*	

B. Combine HS Female Age Groups into HS Female Aggregate, by Occup.

Age Group Shares of HS Female Aggregate, Skilled Labor Occupations (Professionals and Managers)- σ_{1-2}

Constant: age 25-34	-2.15E-01	(8.49E-03)	*
age 35-44	-4.53E-02	(2.04E-02)	*
age 45-54	-6.56E-02	(1.32E-02)	*

σ_{1-2} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.221	0.244	0.235
Age 35-44	0.257	0.278	0.264
Age 45-54	0.254	0.245	0.282

Age Group Shares of HS Female Aggregate, Service Occupations: σ_{3-6}

Constant: age 25-34	-2.00E-01	(4.99E-03)	*
age 35-44	-1.17E-01	(6.03E-03)	*
age 45-54	-8.22E-02	(3.76E-03)	*

σ_{3-6} in selected years	*1968*	*1982*	*1996*
Age 30	0.228	0.252	0.244
Age 40	0.243	0.264	0.251
Age 50	0.254	0.245	0.283

Age Group Shares of HS Female Labor Aggregate, Blue Collar OCC: σ_{7-10}

Constant: age 25-34	-1.41E-01	(5.60E-03)	*
age 35-44	-2.93E-02	(8.42E-03)	*
age 45-54	-1.23E-02	(3.86E-03)	*

σ_{7-10} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.229	0.253	0.244
Age 35-44	0.252	0.273	0.259
Age 45-54	0.258	0.249	0.287

Higher Order Polynomial Terms Common to HS Females, all 10 Occupations

Age 25-34: (t)	1.52E-02	(1.25E-03)	*
Age 25-34: (t^2)	4.73E-04	(8.21E-05)	*
Age 25-34: (t^3)	-2.24E-05	(1.47E-06)	*
Age 35-44: (t)	-4.84E-03	(1.22E-03)	*
Age 35-44: (t^2)	2.24E-03	(5.23E-05)	*
Age 35-44: (t^3)	-6.17E-05	(6.73E-07)	*
Age 45-54: (t)	5.83E-03	(5.58E-04)	*
Age 45-54: (t^2)	4.13E-05	(1.61E-05)	*
Age 45-54: (t^3)	4.82E-06	(9.85E-08)	*

C. Combine COL Male Age Groups into COL Male Aggregate, by Occupation

Age Group Shares of COL Male Aggregate, Skilled Labor Occupations (Professionals and Managers)- v_{1-2}

Constant: age 25-34	1.25E-01	(2.26E-02)	*
age 35-44	4.06E-01	(1.94E-02)	*
age 45-54	3.50E-01	(1.83E-02)	*

v_{1-2} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.221	0.217	0.235
Age 35-44	0.293	0.264	0.264
Age 45-54	0.282	0.271	0.289

Age Group Shares of COL Male Aggregate, Service Occupations: v_{3-6}

Constant: age 25-34	1.51E-01	(4.34E-02)	*
age 35-44	3.92E-01	(2.08E-02)	*
age 45-55	3.53E-01	(1.84E-02)	*

v_{3-6} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.226	0.222	0.240
Age 35-44	0.288	0.260	0.260
Age 45-54	0.282	0.271	0.288

Age Group Shares of COL Male Aggregate, Blue Collar Occupations: v_{7-10}

Constant: age 25-34	3.53E-01	(7.06E-02)	*
age 35-44	6.27E-01	(4.62E-02)	*
age 45-54	4.90E-01	(2.50E-02)	*

v_{7-10} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.237	0.235	0.252
Age 35-44	0.312	0.283	0.282
Age 45-54	0.277	0.269	0.284

Higher Order Polynomial Terms Common to COL Males, all 10 Occupations

Age 25-34: (t)	-4.80E-02	(9.52E-03)	*
Age 25-34: (t^2)	2.52E-03	(5.86E-04)	*
Age 25-34: (t^3)	-3.09E-05	(1.01E-05)	*
Age 35-44: (t)	-4.69E-02	(3.74E-03)	*
Age 35-44: (t^2)	1.82E-03	(9.36E-05)	*
Age 35-44: (t^3)	-1.46E-05	(4.26E-07)	*
Age 45-54: (t)	-2.55E-02	(5.28E-03)	*
Age 45-54: (t^2)	2.44E-04	(2.50E-04)	
Age 45-54: (t^3)	2.02E-05	(3.03E-06)	*

D. Combine COL Female Age Groups into COL Female Aggregate, by Occup.

Age Group Shares of COL Female Aggregate, Skilled Labor Occupations (Professionals and Managers): ϖ_{1-2}

Constant: age 25-34	7.29E-02	(9.14E-03)	*
Age 35-44	-6.18E-02	(1.04E-02)	*
Age 45-54	3.30E-01	(1.12E-02)	*

ϖ_{1-2} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.247	0.262	0.266
Age 35-44	0.220	0.277	0.249
Age 45-54	0.298	0.233	0.270

Age Group Shares of COL Female Aggregate, Service Occupations: ϖ_{3-6}

Age 25-34	3.25E-02	(2.01E-03)	*
Age 35-44	-1.02E-01	(9.50E-04)	*
Age 45-54	3.82E-01	(2.06E-02)	*

ϖ_{3-6} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.238	0.254	0.257
Age 35-44	0.212	0.268	0.240
Age 45-54	0.315	0.248	0.286

Age Group Shares of HS Female Labor Aggregate, Blue Collar OCC: ϖ_{7-10}

Constant: age 25-34	-2.30E-01	(3.05E-03)	*
Age 35-44	-4.19E-01	(3.15E-02)	*
Age 45-54	7.30E-02	(4.53E-03)	*

ϖ_{7-10} in selected years	*1968*	*1982*	*1996*
Age 25-34	0.227	0.243	0.247
Age 35-44	0.192	0.243	0.219
Age 45-54	0.287	0.227	0.263

Higher Order Polynomial Terms Common to COL Females, all 10 Occup.

Age 25-34: (t)	-2.84E-02	(1.29E-03)	*
Age 25-34: (t^2)	3.31E-03	(9.38E-05)	*
Age 25-34: (t^3)	-7.47E-05	(1.44E-06)	*
Age 35-44: (t)	-9.05E-03	(3.75E-03)	*
Age 35-44: (t^2)	3.01E-03	(3.91E-04)	*
Age 35-44: (t^3)	-8.46E-05	(9.11E-06)	*
Age 45-54: (t)	-1.01E-01	(9.25E-03)	*
Age 45-54: (t^2)	7.50E-03	(7.56E-04)	*
Age 45-54: (t^3)	-1.43E-04	(1.55E-05)	*

Appendix E: Labor Supply Elasticity Calculations

Table 3 reports estimates of the occupational choice equation. The coefficient α_1 on the annual wage is .0000862. Thus, e.g., a ten thousand dollar increase in the annual wage in an occupation raises the latent index associated with that occupation (see equation (10)) by .862. To put this in perspective, suppose each occupation and the home sector originally had equal choice probabilities of 1/11 = .091. A ten thousand dollar increase in the annual wage of one occupation (relative to all others) would, other things equal, raise its choice frequency to .191, a substantial 109% increase. [This is obtained via simple MNL arithmetic: exp(.862)/(exp(.862)+10·exp(0)) = 2.368/12.368 = .191].

To put the ten thousand dollar earnings increase inducing this change in perspective, the average annual wage for 35-44 year old male college graduates was roughly $58,000 in 1996, while that for male high school graduates was roughly $36,000 (see Figure 15). Hence, we have 17% and 28% earnings increases for these groups, respectively. Thus, the elasticity of labor supply *to a single occupation* is roughly 4 to 6, depending on the group.

Consider next the elasticity of labor supply to the whole economy. To determine this, we calculate what happens to the home sector (and, conversely, total employment) if the wage in <u>every</u> occupation rises 1% - implying a $580 ($360) increase for college (high school) males. As an approximation, suppose the home shares are 15% and 24% for college and high school males, respectively, and that the employed are spread evenly across occupations. Given our estimate of α_1, the 1% wage increase raises the latent index associated with all occupations (equation (10)) by .05 for college males and .031 for high school males. Using the MNL formula, it is simple to calculate that this causes the home sector share to drop to .1437 and .2344 for these groups, respectively. Thus, employment increases by 0.74% for each (the equality is a coincidence), and we have an elasticity of labor supply to the economy as a whole of roughly .74 for males.[8]

[8] For women, we see in Figure 15 that the average annual wage rates in 1996 for 35-44 year old college and high school graduates were roughly $32,000 and $21,000, respectively. Thus, a 1% wage increase means $320 and $210 increases in income. This translates into .028 and .018 increases in the latent indices in equation (10). From Figures 11 and 13 we see that home sector shares in 1996 were about 38% and 47%, respectively. Thus, doing the same calculations as above, we obtain labor supply elasticities of 1.13 and 0.94 for college and high school educated women respectively. These figures are about 50% greater than the figure for males.