Income Taxation in a Life Cycle Model with Human Capital

By

Michael P. Keane

University of Technology Sydney
Arizona State University

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Abstract: I examine the effect of labor income taxation in a very simple life-cycle model where work experience builds human capital. There are four key findings: First, contrary to conventional wisdom, in such a model permanent tax changes can have larger effects on labor supply than temporary tax changes. Second, even with small returns to work experience, conventional methods of estimating the inter-temporal elasticity of substitution will be very seriously biased towards zero. (This includes methods that rely on exogenous changes in tax regimes). Third, for plausible parameter values, both compensated and uncompensated labor supply elasticities are likely to be quite a bit larger than (conventional) estimates of the inter-temporal elasticity of substitution (despite the fact that the latter is typically viewed as an upper bound on the former). Fourth, for plausible parameter values, large welfare losses from proportional income taxation are quite consistent with existing (small) estimates of labor supply elasticities.

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I. Introduction

This paper examines the effect of income taxation in a very simple life-cycle model where work experience builds human capital. In such a model the wage rate is no longer equivalent to the opportunity cost of time. This has important implications for how workers respond to tax changes, and for the estimation and interpretation of wage elasticities of labor supply. In particular, I will show that, in this simple context, permanent tax changes can have larger effects on current labor supply than transitory tax changes. This result, which holds at quite reasonable parameter values, contradicts the widespread presumption that transitory tax (or wage) changes should have larger effects.

The introduction of human capital into the life-cycle model also has important implications for the intertemporal elasticity of substitution in labor supply (Frisch elasticity), as shown by Imai and Keane (2004). For instance, say we calibrate our simple life-cycle model so the true intertemporal elasticity is large (e.g., 2 or 4). Then, using data generated from the model, we can calculate the intertemporal elasticity, *using conventional empirical methods*. These methods involve regressing hours changes on wage changes and ignoring human capital. Consistent with the existing labor supply literature, this procedure gives small values for the intertemporal elasticity (much smaller than the true value). And I show this is true even if returns to experience are very "small" (in a sense made precise below).

I then go further and show how failure to account for human capital may also lead to misleading conclusions regarding Marshallian and Hicks elasticities. The Frisch elasticity is an upper bound on these elasticities.[1] Thus, the low estimates of the Frisch elasticity typically obtained in the literature have contributed to the broad consensus that Marshallian and Hicks elasticities are also small. However, in the model presented here, I show that both permanent and transitory tax changes can have much larger effects on labor supply than the (incorrectly estimated) Frisch elasticity would suggest. This contradicts the notion that the Frisch elasticity – as conventionally calculated – gives an upper bound on tax effects.

Of course, we are also interested in how labor supply effects of wages and/or taxes are decomposed into income and substitution effects. This affects the welfare loss from the tax. The calculations here suggest that the compensated substitution effect of a permanent tax change may be much greater than the conventionally measured intertemporal substitution effect. Hence, the small Frisch elasticities obtained in prior work (ignoring human capital) should not be viewed as an upper bound on plausible compensated substitution effects.

[1] In a model with assets but no human capital it is well known that the intertemporal elasticity of substitution (Frisch) is an upper bound on the compensated elasticity (Hicks) which in turn is an upper bound on the total (Marshallian) elasticity. See, e.g., Blundell and MaCurdy (1999).

These findings about labor supply behavior in models that include human capital are in sharp contrast to the consensus of the existing literature, which is based almost entirely on either static models or dynamic models that include savings but not human capital.[2] The consensus is summed up nicely in a recent survey by Saez, Slemrod and Giertz (2009), who state: "… optimal progressivity of the tax-transfer system, as well as the optimal size of the public sector, depend (inversely) on the compensated elasticity of labor supply …. With some exceptions, the profession has settled on a value for this elasticity close to zero… In models with only a labor-leisure choice, this implies that the efficiency cost of taxing labor income … is bound to be low as well."[3, 4] The results presented here challenge this consensus view, by showing that, in a model with human capital, conventional econometric methods (designed for models without human capital) will tend to seriously understate labor supply elasticities, and hence the welfare costs of income taxation.

Section II presents a very simple two period version of the basic life-cycle labor supply model that has played a major role in empirical work over the past 30 years. Section III discusses the extension of this model to include human capital. Section IV presents a series of simulations that show how the introduction of human capital radically alters the behavior of the model, such that a very small Frisch elasticity (as conventionally measured) is consistent with large responses to tax changes, and large welfare losses from labor income taxation. Section V concludes.

II. A Simple Life-Cycle Model without Human Capital

I start by presenting a simple model of life-cycle labor supply of the type that has strongly influenced economists' thinking on the subject since the pioneering work by MaCurdy (1981). In order to make the points I wish to make, I do not need all the features of MaCurdy's model. In particular, it will be sufficient to have two periods, and I abstract from uncertainty about future wages. The period utility function is given by:

$$(1) \qquad U_t = \frac{C_t^{1+\eta}}{1+\eta} - \beta \frac{h_t^{1+\gamma}}{1+\gamma} \qquad t = 1,2 \qquad \eta \le 0,\ \gamma \ge 0$$

Here C_t is consumption in period t and h_t is hours of labor supplied in period t. The present

[2] Two notable exceptions are Imai and Keane (2004) and the pioneering early work by Heckman (1973).
[3] Inclusion of this quote is not meant a criticism of Saez, Slemrod and Giertz (2009). They are simply making a statement of fact. I quote them only because they state the consensus and its implications so succinctly.
[4] As Ballard and Fullerton (1992) note, if a wage tax is used to finance compensating lump sum transfers (as in the Harberger approach), the welfare cost depends only on the compensated elasticity. But if it is used to finance a public good (that has no impact on labor supply) it is the uncompensated elasticity that matters. Saez (2001) presents optimal tax rate formulas for a Mirrlees (1971) model (with both transfers and government spending on a public good) and shows that, in general, both elasticities matter for optimal tax rates (see, e.g., his equation 9).

value of lifetime utility is given by:

(2) $$V = \frac{[w_1 h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta} - \beta\frac{h_1^{1+\gamma}}{1+\gamma} + \rho\left\{\frac{[w_2 h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta} - \beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

Here w_1 and w_2 are wage rates in periods 1 and 2, while τ_1 and τ_2 are tax rates on labor earnings in periods 1 and 2, respectively. People are free to borrow or lend across periods at the interest rate r. The quantity b is net borrowing in period 1, while $b(1+r)$ is the net repayment in period 2. Parameter ρ is the discount factor. (I assume there is no non-labor income. This simplifies the subsequent analysis while not changing any of the results).

In the standard life cycle model, there is no human capital accumulation via returns to work experience. That is, hours of work in period 1 do not affect the wage rate in period 2. Thus, the consumer treats the wage path $\{w_1, w_2\}$ as exogenously given, and the first order conditions for his/her optimization problem are simply:

(3) $$\frac{\partial V}{\partial h_1} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} = 0$$

(4) $$\frac{\partial V}{\partial h_2} = \left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta} w_2(1-\tau_2) - \beta h_2^{\gamma} = 0$$

(5) $$\frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} - \rho\left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r) = 0$$

Equation (5) can be simplified to read $\left[C_1\right]^{\eta} \big/ [C_2]^{\eta} = \rho(1+r)$, which is the classic inter-temporal optimality condition that requires one to set the borrowing level b so as to equate the ratio of the marginal utilities of consumption in the two periods to $\rho(1+r)$. Utilizing this condition, we can divide (4) by (3) obtain:

(6) $$\left(\frac{h_2}{h_1}\right)^{\gamma} = \frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\frac{1}{\rho(1+r)}$$

Taking logs we obtain:

(7) $$\ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1} + \ln\frac{(1-\tau_2)}{(1-\tau_1)} - \ln\rho(1+r)\right\}$$

From (7) we obtain:

(8) $$\frac{\partial \ln(h_2/h_1)}{\partial \ln(w_2/w_1)} = \frac{1}{\gamma}$$

Thus, the intertemporal (or Frisch) elasticity of substitution, the rate at which a worker shifts hours of work from period 1 to period 2 as the relative wage increases in period 2, is simply $1/\gamma$. The elasticity with respect to a change in the tax ratio $(1-\tau_2)/(1-\tau_1)$ is identical.

Notice that we could rearrange (7) to obtain:

$$\ln(h_2) = \frac{1}{\gamma}\{\ln w_2(1-\tau_2) - \ln w_1(1-\tau_1) - \ln \rho(1+r)\} - \ln h_1$$

We would then obtain that $\dfrac{\partial \ln h_2}{\partial \ln w_2} = \dfrac{1}{\gamma} - \dfrac{\partial \ln h_1}{\partial \ln w_2}$. The second term is a negative income effect on period 1 labor supply that arises because an increase in w_2 increases lifetime wealth.

Before solving (4)-(5) to obtain the labor supply functions for h_1 and h_2, it is useful to first look at the static case, which would arise if (i) there is only one period, (ii) there is no borrowing and lending across periods, or (iii) people are myopic. Then the utility function in (1) would generate the labor supply function:

$$\text{(9)} \qquad \ln h = \frac{1+\eta}{\gamma-\eta}\ln w - \frac{1}{\gamma-\eta}\ln \beta$$

Thus, $\dfrac{1+\eta}{\gamma-\eta}$ is the Marshallian (or uncompensated or total) labor supply elasticity. As $\eta<0$, we see that the Frisch elasticity must exceed the Marshallian. The two approach each other as $\eta \to 0$ (the case of utility linear in consumption, so there are no income effects).

Next, we use the Slutsky equation to find the income and compensated substitution effects in the static model. Writing the Slutsky equation in elasticity form we have:

$$\text{(10)} \qquad \frac{w}{h}\frac{\partial h}{\partial w} = \frac{w}{h}\frac{\partial h}{\partial w}\bigg|_u + \frac{wh}{N}\frac{N}{h}\frac{\partial h}{\partial N}$$

where N represents non-labor income. The two terms on the right are the compensated substitution (Hicks) elasticity and the income effect. Using (9), we can easily verify that the income effect (evaluated at N=0) is equal to $\dfrac{\eta}{\gamma-\eta}$. Thus, we have that the compensated substitution (or Hicks) elasticity is simply $\dfrac{1}{\gamma-\eta}$. As $\eta < 0$, we see that this is smaller than the Frisch elasticity but larger than the Marshallian.

Now return to the dynamic model with saving. In what follows I will assume that $\rho(1+r)=1$, so that (5) requires the consumer to equate the marginal utility of consumption in both periods. Furthermore, as the simple model in (1) contains no changing preferences over time, this is equivalent to equalizing consumption in the two periods. None of the points I wish to make hinge on this assumption, and it simplifies the analysis considerably.

From (3) we have that:

$$(11) \qquad \frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1)$$

where $C_1=w_1h_1+b$ is consumption in period 1. This is the familiar within-period optimality condition which says to set the ratio of the marginal utility of leisure to the marginal utility of consumption equal to the opportunity cost of time, which in this case is just the after tax wage rate. Given our assumption that $\rho(1+r)=1$, we just have $C_1=C_2=C$, and C is just the present value of earnings times the factor $(1+r)/(2+r)$:

$$(12) \qquad C = \{w_1(1-\tau_1)h_1(1+r) + w_2(1-\tau_2)h_2\}/(2+r)$$

Now we use equation (6), with $\rho(1+r)=1$, to substitute out for h_2 in (12), obtaining:

$$(13) \qquad C = \{w_1(1-\tau_1)h_1(1+r) + w_2(1-\tau_2)\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right]^{1/\gamma} h_1\}/(2+r)$$

It is convenient to factor out h_1 and rewrite this as:

$$(13') \qquad C = h_1C^* = h_1\{w_1(1-\tau_1)(1+r) + w_2(1-\tau_2)\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right]^{1/\gamma}\}/(2+r)$$

Here C^* contains all the factors that govern lifetime wealth. We can now write (11) as:

$$(14) \qquad \ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau_1) - \ln\beta + \eta\ln C^*\right\}$$

Notice that $\partial\ln h_1/\partial\ln w_1$, holding C^* fixed, is $1/(\gamma-\eta)$, the compensated substitution effect, while $\partial\ln h_1/\partial\ln C^* = \eta/(\gamma-\eta)$ is the income effect.

We are now in a position to consider effects of permanent vs. temporary changes in tax rates. Via some tedious algebra we can obtain the effect of a tax reduction in period 1:

$$(15) \qquad \frac{\partial\ln h_1}{\partial\ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{1+x}\right] \quad where \quad x \equiv \left[\frac{w_1(1-\tau_1)}{w_2(1-\tau_2)}\right]^{(1+\gamma)/\gamma}(1+r)$$

Notice that the first term on the right is the Marshallian elasticity. The second term is positive because $\eta<0$, so the elasticity with respect to a temporary tax change exceeds the Marshallian.

If $w_1=w_2$ and $\tau_1=\tau_2$ then the second term in (15) takes on a simple form. We just get:

$$\text{(16)} \qquad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{2+r}\right]$$

Notice that if the term $(1+\gamma)/\gamma(2+r)$ exceeds one then the elasticity in (16) will even exceed the Hicks elasticity. This will be true if $0 < \gamma < (1+r)^{-1}$. In a 2 period model where each period corresponds to roughly 20 years of a working life, a plausible value for $1+r$ is about $(1+.03)^{20} \approx 1.806$, or $(1+r)^{-1} \approx 0.554$. So (16) will exceed the Hicks elasticity if the Frisch elasticity $(1/\gamma)$ is at least $(.554)^{-1}=1.8$.

Now consider a permanent tax change. We assume that $\tau_1 = \tau_2 = \tau$, and look at the effect of a change in $(1- \tau)$. With $\tau_1 = \tau_2 = \tau$ equation (13') becomes:

$$\text{(13'')} \qquad C = h_1(1-\tau)C^{**} = h_1(1-\tau)\left\{ w_1(1+r) + w_2\left[\frac{w_2}{w_1}\right]^{1/\gamma}\right\}/(2+r)$$

And we can rewrite (14) as:

$$\text{(17)} \qquad \ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau) - \ln\beta + \eta\ln(1-\tau)C^{**}\right\}$$

It is then clear that:

$$\text{(18)} \qquad \frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{1+\eta}{\gamma-\eta}$$

which is just the Marshallian elasticity. So, comparing (16) and (18), we have the well known result that the labor supply elasticity with respect to a temporary tax change is greater than that with respect to a permanent change in the standard life-cycle model. In (18) the extra term $-\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{2+r}$ is the inter-temporal substitution effect (i.e., the extra effect of a wage or tax change that is only temporary). As noted above, it is positive (as $\eta<0$) and increasing in the parameter $(1/\gamma) > 0$, which governs people's willingness to substitute labor inter-temporally. Also note that as η becomes a larger negative number (making income effects grow larger) the inter-temporal substitution effect grows stronger.

The result that transitory changes in taxes (or after tax wages) should have a greater effect on labor supply than permanent changes is firmly entrenched as the conventional wisdom in the profession. For example, Saez, Slemrod and Giertz (2009) state: "The labor supply literature … developed a dynamic framework to distinguish between responses to temporary changes vs. permanent changes in wage rates.… Because of inter-temporal substitution, and barring adjustment costs, responses to temporary changes will be larger than responses to permanent changes." The interesting thing about this statement is its generality. The only qualification is that "adjustment costs" (e.g., restrictions on hours) might make it difficult for workers to react to temporary wage/tax changes as much as they would like.

In the next two sections I will show how introduction of human capital into the standard labor supply model undermines this conventional wisdom, such that permanent tax changes can have larger effects than temporary changes (for a wide range of reasonable parameter values). I begin in Section III.A by introducing human capital into a simple model with no borrowing or lending. This makes the impact of human capital clear. Then in Section III.B I present a model that includes both human capital and borrowing/lending.

III. Incorporating Human Capital in the Life-Cycle Model

III.A. A Life-Cycle Model with Human Capital and Borrowing Constraints

Next I will assume that the wage in period 2, rather than being exogenously fixed, is an increasing function of hours of work in period 1. Specifically, I assume that:

$$(19) \qquad w_2 = w_1(1+\alpha h_1)$$

where α is the percentage growth in the wage per unit of work. Given a two period model with each period corresponding to 20 years, it is plausible in light of existing estimates that αh_1, the percentage growth in the wage rate over 20 years, is on the order of 1/3 to 1/2. For instance, using the PSID, Geweke and Keane (2000) estimate that for men with a high school degree, average earnings growth from age 25 to 45 is 33%. For men with a college degree they estimate a rate of 52%. They also estimate that earnings growth essentially ceases after about age 45. At least for figures on the low end of the growth range, the approximation $\ln W_2 \approx \ln W_1 + \alpha h_1$ would not be bad. Thus, (19) is similar to a conventional earnings function, but without the usual quadratic in hours. I will introduce that in the simulation section, but for purposes of obtaining analytical results (19) is much more convenient.

In a model with human capital but no borrowing or lending, equation (2) is replaced by:

$$(20) \qquad V = \frac{[w_1 h_1 (1-\tau_1)]^{1+\eta}}{1+\eta} - \beta \frac{h_1^{1+\gamma}}{1+\gamma} + \rho \left\{ \frac{[w_1(1+\alpha h_1) h_2 (1-\tau_2)]^{1+\eta}}{1+\eta} - \beta \frac{h_2^{1+\gamma}}{1+\gamma} \right\}$$

and the first order conditions (3)-(5) are replaced by:

$$(21) \qquad \frac{\partial V}{\partial h_1} = \left[w_1 h_1 (1-\tau_1)\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} + \rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta} w_1 \alpha h_2 (1-\tau_2) = 0$$

$$(22) \qquad \frac{\partial V}{\partial h_2} = \left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta} w_1(1+\alpha h_1)(1-\tau_2) - \beta h_2^{\gamma} = 0$$

It is useful to rewrite (21) in the form:

$$(23) \qquad \frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1) + \rho\left[\frac{C_2^{\eta}}{C_1^{\eta}}\right]\left\{w_1 \alpha h_2 (1-\tau_2)\right\}$$

where $C_1=w_1h_1(1-\tau_1)$ and $C_2=w_1(1+\alpha h_1)h_2(1-\tau_2)$ are consumption in periods 1 and 2. The main point of this paper can be seen simply by comparing equations (11) and (23). Each equates the marginal rate of substitution between consumption and leisure to the opportunity cost of time. But in the standard life-cycle model (11) this is simply the after tax wage rate $w_1(1-\tau_1)$. The human capital model adds the additional term $\rho\left[C_2^{\eta}/C_1^{\eta}\right]\left\{w_1\alpha h_2(1-\tau_2)\right\}$, which is the human capital investment component of the opportunity cost of time.

To understand this extra term, notice that $dw_2/dw_1 = w_1\alpha$ is the increment to the period 2 wage for each additional unit of work hours in period 1. This is multiplied by h_2 to obtain the corresponding increment in earnings, and further multiplied by $(1-\tau_2)$ to obtain after tax earnings. Of course, it is also discounted back to period 1, and multiplied by the ratio of marginal utilities of consumption in each period, to accommodate that an extra unit of consumption at $t=2$ may be valued differently from that at $t=1$ (we have not yet introduced borrowing into the model).

Now, a key point is that a temporary tax change in period 1 affects only $(1-\tau_1)$, and hence it only affects the first component of the opportunity cost of time (the current wage rate). In contrast, a permanent tax change also affects both $(1-\tau_1)$ and $(1-\tau_2)$, shifting both components of the opportunity cost of time. As we will see, this means that in the model with human capital and no borrowing/lending a permanent tax change will have a larger impact on time t labor supply than would a temporary tax change that is in effect only at time t.

To solve the model for h_1 we use (22) to solve for h_2 and substitute this into (21). This gives the following implicit function for h_1:

$$(24) \quad \beta h_1^{\gamma} = \left[w_1(1-\tau_1)\right]^{1+\eta} h_1^{\eta} + \rho\alpha\beta^{-(1+\eta)/(\gamma-\eta)}\left[w_1(1-\tau_2)\right]^{1+\eta+(1+\eta)^2/(\gamma-\eta)}(1+\alpha h_1)^{(1+2\eta+\gamma\eta)/(\gamma-\eta)}$$

As it is not possible to isolate h_1, we totally differentiate and obtain the elasticity of hours in period 1 with respect to $(1-\tau_1)$:

$$(25)\qquad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \frac{(1+\eta)[w_1(1-\tau_1)]^{1+\eta} h_1^{\eta}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau_1)]^{1+\eta} h_1^{\eta} - \rho\alpha^2[w_1(1-\tau_2)]^{\Gamma_0} \beta^{-\Gamma_3}\Gamma_1(1+\alpha h_1)^{\Gamma_2}}$$

where $\Gamma_0 \equiv (1+\eta)(1+\gamma)/(\gamma-\eta)$, $\Gamma_1 \equiv (1+2\eta+\gamma\eta)/(\gamma-\eta)$, $\Gamma_2 \equiv (1+3\eta+\gamma\eta-\gamma)/(\gamma-\eta)$, $\Gamma_3 \equiv (1+\eta)/(\gamma-\eta)$. Obviously this expression simplifies to the Marshallian elasticity $(1+\eta)/(\gamma-\eta)$ if $\alpha=0$ (i.e., the case of no human capital accumulation), because the third term in the denominator vanishes.

This third term captures substitution and income effects of the wage change at $t=2$ induced by changes in the tax rate at time $t=1$. To the extent this $t=1$ tax change raises hours of work at $t=1$, it will raise the wage rate at $t=2$ (substitution effect). But it also increases income at $t=2$ (income effect). Thus, the sign of the third term in the denominator of (25) is ambiguous. It is determined by the sign of $\Gamma_1 = (1+2\eta+\gamma\eta)$.

Note that if $\eta = -1$ we have log (C) utility and income effects are so strong that they completely counteract substitution effects, rendering the Marshallian elasticity zero. In this case $\Gamma_1 = -1-\gamma < 0$, so the third term *increases* the denominator. Of course this is irrelevant because the numerator is zero, but for somewhat larger values of η we see that the human capital effect will render the elasticity in (25) – i.e., that with respect to temporary tax/wage changes – *smaller* than the Marshallian.

At the other extreme is the case where $\eta = 0$, so utility is linear in C and there are no income effects. This case is adopted in almost all of the structural literature on dynamic models of human capital formation (see, e.g., Eckstein and Wolpin (1989), Keane and Wolpin (1997)) in order to avoid having to also model saving (as, with $\eta = 0$, the human capital investment and consumption/savings decisions separate). In this case $\Gamma_1 = 1$, and the third term must reduce the denominator. Thus, the elasticity with respect to temporary wage/tax changes given by (25) must *exceed* the Marshallian.

Indeed, for any value of η in the -1 to -.5 range the elasticity in (25) must be less than the Marshallian. The critical value of η is -.5. For values closer to zero it is possible to find values of γ small enough that the substitution effect dominates and (25) is larger than the Marshallian elasticity. Strikingly, the change occurs radically. For of η slightly larger than -.5 a nearly infinite Frisch elasticity of substitution $(1/\gamma)$ is necessary for the substitution effect to dominate. But for of $\eta = -0.40$ all we need is $\gamma < .50$, or $1/\gamma > 2$. These are the sort of values typically used in calibrating real business cycle models (see Prescott (1986, 2006)).

Now consider the effect of a permanent tax increase. To simplify the analysis I will assume that $\tau_1 = \tau_2 = \tau$. This modifies (20)-(24) so that τ replaces that τ_1 and τ_2. As a result, when we totally differentiate (24), we get the new term:

$$(26) \qquad \rho\alpha\beta^{-\Gamma_3}\left\{\frac{(1+\eta)(1+\gamma)}{\gamma-\eta}[w_1(1-\tau)]^{\frac{(1+2\eta-\gamma\eta)}{\gamma-\eta}} w_1 d(1-\tau)\right\}(1+\alpha h_1)^{\Gamma_1}$$

This term captures the fact that the tax cut at t=2, by increasing the fraction of earnings that a worker gets to keep at t=2, increases the return to human capital investment (and hence the opportunity cost of time) at t=1.

As a result of the new term in (26), equation (25) is replaced by:

$$(27) \quad \frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{(1+\eta)[w_1(1-\tau)]^{1+\eta}h_1^{\eta} + \rho\alpha\dfrac{(1+\eta)(1+\gamma)}{\gamma-\eta}[w_1(1-\tau)]^{\frac{(1+\eta)(1+\gamma)}{\gamma-\eta}}(1+\alpha h_1)^{\Gamma_1}\beta^{-\Gamma_3}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau)]^{1+\eta}h_1^{\eta} - \rho\alpha^2[w_1(1-\tau)]^{\Gamma_0}\Gamma_1(1+\alpha h_1)^{\Gamma_2}\beta^{-\Gamma_3}}$$

Note that the denominators of (25) and (27) are identical. The only difference is the additional human capital term in the numerator. The sign of this second term depends on the term $(1+\eta)(1+\gamma)/(\gamma-\eta)$. Notice that $(1+\gamma)$ must be positive, as $\gamma>0$. Thus, the sign of the second term depends on that of $(1+\eta)/(\gamma-\eta)$, the Marshallian elasticity itself. Thus, as long as the Marshallian elasticity is positive (i.e., the income effect does not dominate the substitution effect), the labor supply elasticity with respect to a permanent tax change (27) will exceed that with respect to a temporary tax change (25).

In summary, we have now seen that in the model with borrowing but no human capital, there is an intertemporal substitution effect that tends to make the response to a temporary tax change greater than that to a permanent tax change. In the model with human capital and no borrowing, the human capital effect leads to the opposite outcome. In the next Section we present a model with both human capital and borrowing/saving. Not surprisingly, we will find that whether permanent or temporary tax cuts have a larger effect will depend on the relative strength of these human capital and intertemporal substitution effects.

III.B. A Life-Cycle Model with both Human Capital and Saving/Borrowing

In the model with both human capital and saving/borrowing equation (2) is replaced by:

$$(28) \qquad V = \frac{[w_1 h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta} - \beta\frac{h_1^{1+\gamma}}{1+\gamma} + \rho\left\{\frac{[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta} - \beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

and the first order conditions for the problem are:

$$(29)\quad \frac{\partial V}{\partial h_1} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} + \rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta} w_1 \alpha h_2(1-\tau_2) = 0$$

$$(30)\quad \frac{\partial V}{\partial h_2} = \left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta} w_1(1+\alpha h_1)(1-\tau_2) - \beta h_2^{\gamma} = 0$$

$$(31)\quad \frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} - \rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r) = 0$$

Equation (31) can be simplified to read $\left[C_1\right]^{\eta} \big/ [C_2]^{\eta} = \rho(1+r)$. As before, we will assume $\rho(1+r)=1$, to simplify the analysis. In that case $C_1=C_2=C$, and (29) can be rewritten:

$$(32)\quad \frac{\beta h_1^{\gamma}}{C^{\eta}} = w_1(1-\tau_1) + \rho\alpha w_1 h_2(1-\tau_2)$$

It is useful to compare this to (11), which is the MRS condition for the model without human capital. Here the opportunity cost of time is augmented by the term $\rho\alpha w_1 h_2(1-\tau_2)$, which captures the effect of an hour of work at t=1 on the present value of earnings at t=2.

Now, continuing to assume $\rho(1+r)=1$, we can divide (30) by (29) to obtain:

$$(33)\quad \left(\frac{h_2}{h_1}\right)^{\gamma} = \frac{w_1(1+\alpha h_1)(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)} = \frac{w_2(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)}$$

Taking logs we obtain:

$$(34)\quad \ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)}\right]$$

This equation illustrates clearly why the conventional procedure of regressing hours growth on wage growth leads to underestimates of the Frisch elasticity $1/\gamma$, and overestimates of the key utility function parameter γ. The effective wage rate at t=1 is understated by failure to account for the term $\rho\alpha w_1 h_2(1-\tau_2)$ that appears in the denominator.

We can get a better sense of the magnitude of the problem if we simplify by assuming $\tau_1 = \tau_2 = \tau$. Then we can rewrite (34) as:

$$(35)\quad \ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1}\frac{1}{(1+\rho\alpha h_2)}\right\} = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1} - \ln(1+\rho\alpha h_2)\right\} \approx \frac{1}{\gamma}\ln\frac{w_2}{w_1} - \frac{1}{\gamma}\rho\alpha h_2$$

If we solve this for $1/\gamma$ we obtain:[5]

$$\text{(36)} \qquad \frac{1}{\gamma} = \ln\left(\frac{h_2}{h_1}\right) \div \ln\left(\frac{w_2}{w_1(1+\rho\alpha h_2)}\right) = \ln\left(\frac{h_2}{h_1}\right) \div \left[\ln\left(\frac{w_2}{w_1}\right) - \ln(1+\rho\alpha h_2)\right]$$

Thus, wage growth from t=1 to t=2 would have to be adjusted downward by a factor of roughly $\rho\alpha h_2$ percent in order to correct for the missing human capital term (obtaining a valid estimate of the growth of the opportunity cost of time).

As we noted earlier, a reasonable estimate of αh_1 is about 33%. For illustration, let's suppose that h_2 is 20% greater than h_1, so that αh_2 is roughly 40%. As we also noted earlier, a reasonable value for ρ is 0.554, giving $\rho\alpha h_2$ = 22%. Hence, for these values, the growth in the opportunity cost of time is only 1 – 22/33 or 1/3 of the observed growth in wages. If we had used observed wage growth to calculate $1/\gamma$ we would obtain 20/33 ≈ .60 for the Frisch elasticity. But the correct value is $ln(1.20)/ln[1.33/1.22] \approx 2.1$. Thus, for reasonable parameter values, the downward bias in estimates of the Frisch elasticity due to ignoring human capital will tend to be substantial.[6]

Now consider the impact of permanent vs. temporary wage/tax changes in this model. First, solve (30) for h_2 to obtain:

$$\text{(37)} \qquad h_2 = \beta^{-1/\gamma}\left[w_1(1+\alpha h_1)(1-\tau_2)\right]^{1/\gamma} C^{\eta/\gamma}$$

Substituting this into (29) we obtain:

$$\text{(38)} \qquad \beta h_1^{\gamma} = w_1(1-\tau_1)C^{\eta} + \rho\alpha\beta^{-1/\gamma} w_1^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma}(1-\tau_2)^{(1+\gamma)/\gamma} C^{\eta(1+\gamma)/\gamma}$$

Next we must substitute out for C. Given our assumption that $\rho(1+r)=1$, we just have $C_1=C_2=C$, and C is just the present value of earnings times the factor $(1+r)/(2+r)$:

$$\text{(39)} \qquad C = \{w_1(1-\tau_1)h_1(1+r) + w_2(1-\tau_2)h_2\}/(2+r)$$

In the model without human capital we were able to substitute for h_2 in this equation using the intertemporal optimization condition (equation (6)), obtaining an equation for C only in

[5] The 3rd and 4th terms on the right hand side of (35) play no role in the subsequent exposition. I include them only because they suggest a possible approach to estimating $(1/\gamma)$, i.e., including h_2 on the right hand side of a conventional hours growth specification and then finding appropriate instruments for both (w_2/w_1) and h_2. I believe this would be difficult, but further examination of this issue is tangential to the purpose of this paper.

[6] If we assume that hours grow by 10% rather then 20%, the conventional approach to measuring the Frisch elasticity would give 10/33 ≈.30, while the correct calculation is $ln(1.10)/ln[1.33/(1+(.554)(.33)(1.10)] \approx ln(1.10)/ln[1.33/1.20] \approx 0.93$

terms of h_1 (equation (13)). We were then able to substitute this into the first order condition for h_1 to obtain an explicit function for h_1 (equation (14)) that was fairly easy to differentiate. Things are much more difficult here, because the intertemporal optimization condition (33) cannot be solved explicitly for h_2 in terms of h_1. Instead, we use (37) to substitute for h_2. However, this only delivers an implicit function for C:

$$(40)\qquad C=\{w_1(1-\tau_1)h_1(1+r)+\left[w_1(1-\tau_2)(1-\tau_2)\right]^{(1+\gamma)/\gamma}\beta^{-1/\gamma}C^{\eta/\gamma}\}/(2+r)$$

We are now in a position to calculate labor supply elasticities of h_1 with respect to temporary tax changes, using the two equation system (38) and (40). First, we implicitly differentiate (40) to obtain an expression for $dC/d(1-\tau_1)$ that will involve $dh_1/d(1-\tau_1)$. Then we implicitly differentiate (38) to obtain an expression for $dh_1/d(1-\tau_1)$ that involves $dC/d(1-\tau_1)$. Finally, we substitute the former expression into the latter, group terms, and convert to elasticity form to obtain:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)}=$$

$$(41)\qquad \frac{A\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]+\eta\left[A+B\frac{1+\gamma}{\gamma}\right]D}{\gamma\left[A+B\left(1+\frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]-\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right]\frac{\alpha h_1}{1+\alpha h_1}}$$

where:

$$A\equiv w_1(1-\tau_1)C^{\eta}$$

$$B\equiv\rho\alpha\beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma}C^{\eta(1+\gamma)/\gamma}$$

$$D\equiv w_1h_1(1-\tau_1)(1+r)$$

$$E\equiv\beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{(1+\gamma)/\gamma}$$

The term B is the human capital affect that arises because an increase in h_1 increases income at t+2 (holding h_2 fixed). It is exactly the second term on the right hand side of (38). The term $EC^{\eta/\gamma}(\gamma-\eta)/\gamma$ is the standard income effect of the higher after tax wage in period t=1. The term $EC^{\eta/\gamma}\left({(1+\gamma)}/{\gamma}\right)\left({\alpha h_1}/{(1+\alpha h_1)}\right)$ is a special income effect that arises because an increase in h_1 increases the wage rate at t=2.

It can be verified via cumbersome algebra that (41) reduces to (15) – the elasticity of hours with respect to a temporary tax cut in the standard life-cycle model without human capital – if we set $\alpha = 0$. The simulations in Section IV.C will reveal that (41) is strongly decreasing in α (for given η and γ). This is intuitive: as human capital becomes more important, a temporary tax hits a smaller and smaller part of the opportunity cost of time.

We can now look at the effect of a permanent tax increase by setting $\tau_1 = \tau_2 = \tau$ in (38) and (40), and following the same solution procedure as above. This leads to the result:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau)} =$$

$$(42) \qquad \frac{\left[A+\left\{B\frac{1+\gamma}{\gamma}\right\}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]+\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}\right]}{\gamma\left[A+B\left(1+\frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]-\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right]\frac{\alpha h_1}{1+\alpha h_1}}$$

This expression reduces to the Marshallian elasticity (18) if we set $\alpha = 0$. Compared to equation (41), equation (42) has two new terms, both of which appear in curly brackets in the numerator. The first is $\left\{B\frac{1+\gamma}{\gamma}\right\}$ which is an additional human capital effect. It captures that a lower tax rate in period t=2 provides an additional incentive to accumulate human capital at t=1. The second is $\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}$ which captures an additional income effect (i.e., the lower tax in period 2 leads to higher lifetime income holding labor supply fixed).

Whether a permanent or a temporary tax change has a larger effect on labor supply depends on which of these two effects dominates. A permanent tax change will have the larger effect if the following condition holds:

$$\left\{B\frac{1+\gamma}{\gamma}\right\}\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]>\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}$$

Some tedious algebra reveals that this condition is equivalent to a bound on the parameter α, which governs how work experience in period 1 affects the wage in period 2. The bound is:

$$(43) \qquad \alpha > \frac{-\eta\left(\beta h_1^{\gamma}\right)C^{-\eta}}{\rho(2+r)C+\eta h_1\left(\beta_1 h_1^{\gamma}\right)C^{-\eta}} > 0$$

Note that the numerator of (43) is obviously positive, as $\eta<0$, and the next two terms are the marginal utilities of leisure and consumption respectively, which are both positive. But the sign of the denominator appears ambiguous, as the first term is positive while the second is negative. However, we can show it is positive as follows:

Utilizing the fact that $\rho(1+r)=1$, so that $\rho(2+r)=(1+\rho)$, we can see that, in order for the denominator to be positive, we must have:

$$(44) \qquad C > \frac{-\eta}{1+\rho} h_1 \frac{\left(\beta h_1^{\gamma}\right)}{C^{\eta}}$$

Now recall from equation (32) that $\frac{\beta h_1^{\gamma}}{C^{\eta}} = w_1(1-\tau) + \rho\alpha w_1 h_2(1-\tau)$. Thus we have that:

$$(45) \qquad C > \frac{-\eta}{1+\rho} h_1 \left[w_1(1-\tau) + \rho\alpha w_1 h_2(1-\tau) \right] = \frac{-\eta}{1+\rho} \left[w_1 h_1(1-\tau) + \frac{1}{1+r}(\alpha h_1) w_1 h_2(1-\tau) \right]$$

where in the second term on the right we have substituted $\rho(1+r)=1$. Of course we have that the present value of lifetime consumption equals that of lifetime income:

$$(46) \qquad \frac{2+r}{1+r} C = \left\{ w_1(1-\tau)h_1 + \frac{1}{1+r} w_1(1+\alpha h_2)(1-\tau)h_2 \right\}$$

Thus, the term in the square brackets in (45) is $\frac{2+r}{1+r}C - \frac{1}{1+r} w_1 h_2(1-\tau)$, which is lifetime income minus a part of period 2 earnings. So we can rewrite (45) as:

$$(47) \qquad C > \frac{-\eta}{1+1/(1+r)} \left[\frac{2+r}{1+r} C - \frac{1}{1+r} w_1 h_2(1-\tau) \right] = -\eta \left[C - \frac{1}{2+r} w_1 h_2(1-\tau) \right]$$

As long as η > -1 (i.e., substitution effects dominate income effects) this inequality must hold. The right hand side takes on its greatest value when η = -1, and then (47) just says that C is greater than a fraction of C.

Thus, equation (43) gives a positive lower bound that the human capital effect α must exceed in order for permanent tax changes to have a larger effect than temporary tax changes in the model with human capital and saving. Repeating (43) for convenience:

$$(43) \qquad \alpha > \frac{-\eta\left(\beta h_1^{\gamma}\right)C^{-\eta}}{\rho(2+r)C + \eta h_1\left(\beta_1 h_1^{\gamma}\right)C^{-\eta}} > 0$$

we see that, while this expression is difficult to further simplify, it is intuitive that the lower bound for α is increasing in ($-\eta$). As η approaches -1 (i.e., log(C) utility, stronger income effects) the numerator of (43) increases while the denominator decreases. It is also obvious that when utility is linear in consumption (no income effects) (43) reduces to $\alpha > 0$. In the simulations of Section IV.C we will see clearly how the lower bound for α increases in $(-\eta)$.

If we make the approximation that $\alpha^2 \approx 0$, which is reasonable given that, as noted earlier, a plausible value for αh_1 is about .33, we can obtain the more intuitive expression:

$$\text{(48)} \qquad \alpha > -\left[\frac{\eta}{1+\eta}\right]\frac{w_1(1-\tau)}{w_1(1-\tau)h_1 + \frac{1}{1+r}w_1(1-\tau)h_2}$$

which makes clear that the bound for α gets higher as income effects grow stronger.

IV. Simulations of the Model

IV.A. Model Calibration

Given that we have a two period model we can think of each period as 20 years of a 40 year working life (e.g., 25 to 44 and 45 to 64). I assume a real annual interest rate of 3%. Note that $1/(1+.03)^{20} = 0.554$. This implies a 20 year interest rate of $r = .806$. Thus, I will assume the discount factor $\rho = 1/(1+r) = 0.554$. I set the initial tax rates $\tau_1 = \tau_2 = .40$.

I will examine how the model behaves for a range of values of the key utility function parameters η and γ. I know of only two studies that estimate life-cycle models that include both savings and human capital investment, and that also assume CRRA utility. These are Keane and Wolpin (2001) and Imai and Keane (2004).[7] Keane-Wolpin estimate that $\eta \approx -.5$ while Imai-Keane estimate that $\eta \approx -.75$. Goeree, Holt and Palfrey (2003) present extensive experimental evidence, as well as evidence from field auction data, in favor of $\eta \approx -.4$ to $-.5$. Bajari and Hortacsu (2005) estimate $\eta \approx -.75$ from auction data. Thus, I will consider values of -.25, -.50 and -.75 for η, with most of the emphasis on the -.50 and -.75 cases. [8]

Of course, the value of γ has been the subject of great controversy in the literature. As discussed by Imai and Keane (2004), almost all the estimates of the intertemporal elasticity of substitution ($1/\gamma$) reported in the literature are quite small. Two rare exceptions are French (2005), who obtains a value of 1.33 for 60 year olds in the PSID, and Heckman and MaCurdy (1980), who obtain a value of 1.8 for married women in the PSID. Aside from this, estimates

[7] I believe that Shaw (1989) was the first to estimate a dynamic model that included both human capital and saving. But she assumed a translog utility function so the estimates are not very useful for calibrating (1).

[8] The value of $\eta \approx -.50$ obtained by Keane and Wolpin (2001) implies less curvature in consumption (i.e., higher willingness to substitute inter-temporally) than much of the prior literature. But their model includes liquidity constraints that limit the maximum amount of uncollateralized borrowing. Keane and Wolpin (2001, p. 1078) discuss how the failure of prior work to accommodate liquidity constraints will have led to downward bias in η. Specifically, in the absence of constraints on uncollateralized borrowing, one needs a large negative η to rationalize why youth with steep age-earnings profiles don't borrow heavily in anticipation of higher earnings in later life. Notably, their model fits the empirical distribution of assets for young men quite well.

of (1/γ) are generally in the 0 to .50 range. At the same time, many macro economists have argued that values of (1/γ) of 2 or greater are needed to explain business cycle fluctuations using standard models (see Prescott (1986, 2006)).

But Imai and Keane (2004) is a major exception to the prior literature, as they estimate that $\gamma \approx .25$. Theirs' is the only paper in this literature to include human capital, and they argue, for reasons similar to those discussed here, that failure to do so will have led prior work to severely underestimate (1/γ). It is notable that French (2005), who also obtained a reasonably high value of (1/γ), did so for 60 year olds. As both Shaw (1989) and Imai and Keane (2004) note, human capital investment is not so important for people late in the life-cycle. For them, the wage will be close to the opportunity cost of time, and the bias that results from ignoring human capital will be much less severe.

Given the controversy over γ, I will examine the behavior of the model for a wide range of values. Specifically, I look at $\gamma = \{0, 0.25, 0.50, 1, 2, 4\}$. But I will often focus on $\gamma = 0.50$. I consider this value plausible in light of Imai and Keane (2004) and results in Section III.B that prior estimates (ignoring human capital) are likely to be severely biased upward.

Next consider β. This is just a scaling parameter that depends on the units for hours and consumption, and has no bearing on the substantive behavior of the model. Thus, in each simulation, I set β so that optimal hours would be 100 in a static model. The initial wage w_1 is also set to 100. These values were chosen purely for ease of interpreting the results.

Finally, consider the wage function. In contrast to the simple function assumed for analytical convenience in Sections II-III, here I assume the more realistic function:

(49) $$w_2 = w_1 \exp(\alpha h_1 - \kappa\left(h_1^2/100\right) - \delta)$$

This corresponds more closely to a conventional Mincer log earnings specification:

(50) $$\ln w_2 = \ln w_1 + \alpha h_1 - \phi\left(h_1^2/100\right) - \delta$$

where w_1 plays the role of the initial skill endowment, and there is a quadratic in hours. However, I have also included the depreciation term δ which will cause earnings to fall if the person does not work sufficient hours in period one (see Keane and Wolpin (1997)).

Given that β is chosen so hours will be close to 100 in period one,[9] let's think of 100 as corresponding roughly to full-time work and 50 as corresponding to part-time work. I decided to calibrate the model so that (i) the person must work at least part-time in order to

[9] Actually, agents will typically supply somewhat more than 100 units of labor when α>0, due to the incentives to acquire human capital in the dynamic model.

have the wage stay constant at 100 in period two, and (ii) that the return to additional work falls to zero at 200 units of work. Given these constraints, the wage function reduces to:

$$(51) \qquad w_2 = w_1 \exp\left(\alpha h_1 - \frac{\alpha}{4}\left(h_1^2/100\right) - \frac{175}{4}\alpha \right)$$

Thus, the single parameter α determines how work experience maps into human capital. I will calibrate α so that it is roughly consistent with the 33% to 50% wage growth for men from age 25 to 45 discussed earlier. As we'll see below, this requires α in the .008 to .010 range. However, I will also consider a range of other α values, to learn about how the behavior of the model changes when human capital is more or less important.

IV.B. Baseline Simulation

Table 1 reports baseline simulations of the model with η = -.75, η = -.50 and η = -.25. It reports units of work in periods 1 and 2 as well as the wage rate in period 2. Recall that the wage rate in period 1 is normalized to 100, so we can read off the amount of wage growth directly from the table. Results are reported for values of α ranging from 0 to .012. Recall also that β is normalized in all models so that hours = 100 in the static case. Thus, it is to be expected that the overall level of hours is rising as we move down the rows of the table and the return to human capital investment increases.

Consider first the models with η = -.50. Notice that with α = .007 the amount of wage growth from t=1 to t=2 ranges from 26% when γ = 4 to 37% when γ = 0, including a value of 32% for my preferred value of γ = .50. These are plausible values, but a bit low compared to the 33% to 52% values that Geweke and Keane (2000) estimated from the PSID. At α = .008 the amount of wage growth ranges from 31% when γ = 4 to 46% when γ = 0, including a value of 39% for my preferred value of γ = .50. These values are solidly in the range of the values that Geweke and Keane (2000) estimated from the PSID. At α = .010 the amount of wage growth ranges from 41% when γ = 4 to 66% when γ = 0, including a value of 54% for my preferred value of γ = .50. This brings us to the upper end of the range of values that Geweke and Keane (2000) estimated. Based on these simulation results, I would conclude that values of α in the .008 to .010 range are reasonable (when η = -.50).

A notable feature of the results in Table 1 is that the rate of wage growth is not very sensitive to the setting of η, although it gets slightly greater as η approaches zero (i.e., income effects become weaker). For instance, comparing η = -.75 vs. -.50 vs. -.25, and looking only at the γ = .50 column, for α = .008 we see wage growth of 35%, 39% and 44%, respectively. Thus, α = .008 appears to be a reasonable setting regardless of the value of η. However, if we look at α = .010, η = -.25, γ = .50 we get wage growth of 64%, which is a bit high. Thus, for η = -.25 the plausible range for α = .008 is more like .007 to .009.

The other thing we see in Table 1 is hours of work at t=1 and t=2. If we look at McGrattan and Rogerson (1998) we see that in 1990 the typical married male in the 25 to 44 age range worked 40 hours per week, while the typical married male in the 45 to 64 age range worked 34 hours per week. Thus, there was a 15% decline in hours between the two periods. None of the models in Table 1 matches this pattern, as all imply that hours increase, albeit modestly, from t=1 to t=2. For instance, the model with α = .008, η = -.50, and γ = .50 gives an increase in units of work from 121 to 133, or 10%.

There are two possible reactions to this. First, one could view this as a failure of the model. Second, one could accept that this is a very simple stylized model designed to clarify some issues about (i) how taxes affect labor supply in models with both human capital and saving and (ii) the misleading nature of conventional labor supply elasticity estimates in such models. In order to capture the decline in hours that occurs at later ages – ages 55 to 64 in particular – one would have to account for the factors that motivate retirement such as declining tastes for work with age, pensions, health, etc.. The simple model here abstracts from these issues entirely. (But see below for a qualification of this statement).

Perhaps more relevant for our purposes is that hours do follow a hump shape over the life cycle; as Imai and Keane (2004) note, for men in the PSID average annual hours rise from 2042 at age 25 to 2294 at age 35, a 12% increase. They then plateau before beginning to fall with retirement. Thus, our model with α = .008, η = -.50, and γ = .50, which generates 10% hours growth, can be charitably interpreted as successfully capturing the modest growth in hours that occurs over the life cycle prior to the onset of the forces that drive retirement.

Using this "modest" hours growth criterion (i.e., the model should generate hours growth in the 10%-15% range), we see that some specifications in Table 1 can be ruled out. In particular, if we look at α values in the plausible .007 to .010 range, we see that models with γ = 0 generate implausibly large increases in labor supply (e.g., 236/140 = 69% in the α = .008, η = -.50 case). If η = -.25 then the γ = .25 models can be ruled out as well.

Table 2 reports of the same set of baseline model simulations, except for the model of Section III.A, where no borrowing or lending is allowed. The first striking finding here is that levels of period 1 hours, and hence period 2 wages, are almost identical to those in the model with borrowing. For example, in the model with η = -.50 and α = .008, the amount of wage growth ranges from 31% when γ = 4 to 48% when γ = 0. Recall that in the model with borrowing the range was an essentially identical 31% to 46%.

The other striking finding is that hours growth is actually negative in the models with η = -.75 or -.50. For example, in the model with η = -.50, α = .008 and γ = .50, hours decline from 124 units in period 1 to 118 units in period 2, or 5%. If η = -.75 the decline is even greater (from 118 to 106, or 10%). In models with η = -.25 hours still increase, but more modestly than before. For example, with α = .007, hours increased from 130 to 148, or 14% in the model with borrowing, but only from 131 to 137, or 5%, in the model without.

There are two reasons why an hours decline occurs in the model with borrowing constraints. The first reason was also operative in the model with borrowing and lending. That is, the component of the opportunity cost of time that arises because of the return to human capital investment (i.e., the second term in equation (23) or (32)) vanishes in period 2, as there is no future. This force, which drives down the opportunity cost of time as people age, is in fact one factor that drives retirement behavior.

The second reason is the income effect that arises because wages are higher in period two than in period one. The inability to smooth consumption over time means this income effect is much stronger in the model with borrowing constraints. Clearly, the human capital effect alone is not sufficient to cause hours to fall in period two, but the human capital effect combined with the income effect is.

In summary, the results of this section suggest that human capital effects in the α = .008 to .010 range are plausible for the η = -.75 to -.50 models, and that α in the .007 to .009 range is plausible for the η = -.25 model. The value γ = 0 does not appear plausible in the η = -.75 to -.50 models, while γ = 0 or .25 both appear implausible in the η = -.25 model (although less so with borrowing constraints).

IV.C. Simulation of Effects of Tax Rate Changes

In this Section I use the simple models of Sections III.A and III.B to simulate effects of temporary and permanent tax changes. Tables 3-5 present the results for the models with unconstrained borrowing and lending. Table 3 presents results for models with η = -.75. The left panel of the table shows elasticities with respect to temporary tax changes in period one. The right panel shows elasticities with respect to permanent tax changes (i.e., changes that take effect in both periods one and two). The first three rows show results for α = 0, the case of no human capital accumulation.

Consider the case with γ = .50, which is a commonly assumed value in calibrating real business cycle models. Then, the Marshallian elasticity is $(1+\eta)/(\gamma-\eta)$ = (1-.75)/(0.50+0.75) = 0.20. The compensated substitution (or Hicks) elasticity is $1/(\gamma-\eta)$ = 1/(0.50+0.75) = 0.80. The Frisch elasticity is $1/\gamma$ = 2. As we see in the first three rows of Table 3, these *theoretical* elasticities correspond almost exactly to the *simulated* values of the total and compensated elasticities to permanent tax cuts (which apply in both periods), and to the Frisch elasticity for a temporary tax cut (which applies only in period 1). The latter is calculated as the percentage increase in labor supply from period t=1 to t=2 (-2%) divided by the after-tax wage increase from t=1 to t=2 (-1%). The simulated values for three elasticities reported in the first three rows of Table 3 differ slightly from the theoretical values only because we are taking finite difference derivatives (i.e., we increase (1-τ) by 1%, from .600 to .606, and simulate the corresponding change in labor supply)).

Given that in the baseline (i.e., prior to the tax cut experiments) we have w_1=w_2=100 and $\tau_1 = \tau_2$ =.40, we can use equation (16) to obtain the theoretical value of the labor supply elasticity with respect to a temporary tax change at t=1 in the model with no human capital:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.75}{0.50+.75}\right] - \left[\frac{(-.75)}{.50-(-.75)}\frac{1+.50}{.50}\frac{1}{2+.806}\right] = 0.20 + 0.64 = 0.84$$

This aligns closely with the value of 0.835 obtained in the simulation. Finally, I also report a compensated elasticity with respect to a temporary tax cut of 1.222.

It is necessary to take a detour to explain how the compensated elasticities in Tables 3 to 5 are calculated. There is no direct equivalent to the Slutsky equation in the dynamic case. Thus, I have defined the compensated elasticity as the effect of a wage/tax change holding the optimized value function fixed. In order to determine the amount of initial assets a consumer must be given to compensate for a tax change, I solve the equation:

$$\begin{aligned} &V(\tau_1, \tau_2, 0) = V(\tau_1', \tau_2', A) \approx V(\tau_1', \tau_2', 0) + u'(C)A \\ (52) \quad &A \approx \frac{V(\tau_1, \tau_2, 0) - V(\tau_1', \tau_2', 0)}{u'(C)} \end{aligned}$$

where τ_1' and τ_2' denote the tax rates after the tax change. Giving people the initial asset level defined by A in (52) equates the initial value function $V(\tau_1, \tau_2, 0)$ and the post-tax change value function $V(\tau_1', \tau_2', A)$ to a very high degree of accuracy.

The second panel of Table 3 presents results when the human capital effect α is set at the very weak level of .001. Strikingly, even this very small human capital effect renders the conventional method of estimating the Frisch elasticity – i.e., taking the ratio of hours growth to wage growth – completely unreliable.[10] With α = .001, in the baseline model, the wage rate

[10] Of course, econometric studies that estimate the Frisch elasticity by regressing percentage hours changes on percentage wage changes use more complex instrumental variables techniques, designed to deal with

increases by only a little over 3% between period 1 and period 2 for all values of γ. For instance, if $\gamma = 0.50$ the wage increases by 3.25%. At the same time, hours increase from 101.43 to 102.16, or 0.72%. Thus, taking the ratio we would incorrectly infer that the Frisch elasticity was only 0.72/3.25 = 0.221, and that γ was 4.5 (compared to the true value of 0.50).

One might surmise that the reason the conventional method of calculating the Frisch elasticity is so severely downward biased in this case is that the wage change from t=1 to t=2 in the baseline model is entirely endogenous. That is, it results entirely from human capital investment. There is no source of exogenous variation in the after-tax wage, such as an exogenous tax change or a change in the rental rate on human capital (e.g., a labor demand shift). One might further surmise that if the data contained an event such as a temporary tax cut that shifted the wage path exogenously, one could infer γ more reliably.

Surprisingly, it turns out that this intuitive logic is fundamentally flawed. The left panel of Table 3 reports Frisch elasticities calculated in the conventional manner in a regime with a temporary 1% tax cut in t=1. Looking at the $\gamma = 0.50$ case, we see the estimate is -.478, which is not even the correct sign. What happens in this case is that the human capital effect causes labor supply to increase to 102.18 in period 1, which increases the wage in period 2 to 103.29. But despite this wage increase, labor supply actually declines in period 2 (to 101.11). This is an illustration of how the wage is no longer the opportunity cost of time in the life-cycle model with human capital. Even though the wage is higher at t=2 the opportunity cost of time is lower, because the human capital investment return component is removed.

Another way to look at this is that there is no such thing as a strictly exogenous shift in the wage path in the life-cycle model with human capital. For instance, a higher after-tax wage at t=1 causes hours to increase, but this raises the wage at t=2 via the human capital effect. Thus, a t=1 tax change does not induce an exogenous change in the wage profile, as the wage at t=2 is altered by the behavioral response. This has fundamental implications for the estimation of wage elasticities. That is, if work experience alters wages, methods that rely on exogenous variation in wages will not work. One must model the joint wage/labor supply process, and determine how labor supply responds to the opportunity cost of time.

Next consider the case of $\alpha = .008$, which we determined in Section IV.B generates wage growth roughly consistent with observations. Given this value of α, labor supply at t=1 is 113.94, about 14% higher than in the $\alpha = 0$ case, because the human capital effect raises the

measurement error in wages, heterogeneity in tastes for work, and the fact that part of any wage change may have been unanticipated. We do not have any of those problems here, so the appropriate estimator boils down to just taking the ratio of the percentage hours change to the percentage wage change.

opportunity cost of time. This generates 35.24% wage growth. Labor supply at t=2 is now 121.04, which is a 6.23% increase. Thus, using the baseline data, and using conventional methods, we would estimate the Frisch elasticity as only 6.23/35.25 = 0.177. If, instead of the baseline, we use the data that includes a tax cut in period t=1, we would obtain 0.198.

It is interesting that for α = .001 conventional methods produce estimates of the Frisch elasticity that differ greatly (depending on whether or not the data contain a tax change), while for more plausible larger values like α = .008 the estimates are quite close. This is because at larger values of α wage growth from period t=1 to t=2 is much greater, and this insures that the opportunity cost of time does increase, despite the fact that (i) the human capital component of the opportunity cost vanishes at t=2 and (ii) the tax rate rises.

Next we examine *total* and *compensated* labor supply elasticities, focussing first on the η=-.75, α=.008, γ=.50 case in Table 3. The first thing to note is that both these elasticities drop substantially when human capital is included in the model. Specifically, they fall from .835 and 1.222 in the no human capital case to .312 and .606 in the α=.008 case, more than a factor of two. Thus, even if we knew the true value of γ, ignoring human capital would lead us to a downward biased estimate of η (i.e., to understate the inter-temporal substitution in consumption). This pattern of human capital leading to reduced total and compensated elasticities holds for all values of γ, and, as we will see in Tables 4-5, it holds for a range of values of η. As I noted in Section III.B, this pattern is intuitive: as human capital becomes more important, a temporary tax hits a smaller part of the opportunity cost of time.

Next, compare elasticities with respect to temporary vs. permanent tax cuts. The total elasticity of labor supply at t=1 with respect to a temporary increase in (1-τ_1) is 0.312, while that with respect to a permanent increase in (1-τ) is 0.176. This appears consistent with the conventional wisdom that temporary tax changes have larger effects than permanent changes due to inter-temporal substitution. However, it turns out this is not the case. The *compensated* elasticity of labor supply at t=1 with respect to a temporary tax cut is 0.606, while that with respect to a permanent tax cut is greater, 0.698. Thus, the total elasticity with respect to the permanent tax cut is smaller than that for the temporary tax cut not because of substitution effects, but rather because the income effect of the permanent tax cut is greater. Also, recall from equation (48) that the hurdle that α must exceed for permanent tax cuts to have larger effects is increasing in (-η). When we move to Tables 4-5 we'll see this hurdle being met.

Another key point is that the Frisch elasticity – as conventionally measured – is a factor of 3 to 4 times smaller than the compensated substitution elasticity for both permanent and temporary tax changes. This illustrates a key point: the generally low estimates of the

Frisch elasticity in the literature should not be viewed as an upper bound on compensated substitution elasticities. In fact, the Frisch elasticities do not even give an upper bound on the total elasticities (e.g., the two methods of calculating the Frisch elasticity produce values of 0.177 and 0.198, while the total elasticity for a t=1 tax cut is 0.312).

Next I turn to Table 4, which presents results for models with η = -.50. Focus again on the γ = 0.50 case. In the model without human capital in the first three rows, we see that the total elasticity with respect to a temporary tax reduction in period one is almost exactly twice as large as that with respect to a permanent tax cut (1.03 vs. 0.50).[11] But if we look at the case of α = .008, we see that the total elasticity with respect to a permanent tax cut is now greater than that with respect to a temporary tax cut in period one (0.445 vs. 0.420). If we move up to the α=.010 case, which is towards the higher end of the plausible range for the human capital effect, the difference grows even larger (0.424 vs. 0.327).

These results illustrate a key point: for plausible parameter values – indeed for what I have argued in Section IV.A are the preferred range of values for α, η and γ – labor supply effects of permanent tax cuts can exceed those of temporary tax changes in the life-cycle model with borrowing/lending and human capital.

The result is even clearer if we look at compensated elasticities. In the α = .008 case the compensated elasticity with respect to a permanent tax cut is 0.884, while that with respect to a temporary tax cut is 0.661. (And recall that in Table 3, where η = -.75, we also found that the compensated elasticity was greater in the permanent case).

Finally, note that in the α=.008 case the conventional method of calculating the Frisch elasticity produces values of .304 and .256 (for cases where the data do or do not contain a temporary tax cut, respectively). These estimates, typical of the low values in prior empirical work, imply values of γ of 3.3 to 3.9. Yet we know the true value is γ=0.50. Strikingly, even the larger (conventional) estimate of the Frisch elasticity is smaller than the Marshallian elasticity with respect to a permanent tax cut (.445) and much smaller than the compensated elasticity (.884).

This illustrates a second key point: the low estimates of the Frisch elasticity obtained in prior literature are consistent not only with low values of γ, but also with quite large values for compensated and uncompensated substitution elasticities. Existing estimates of the Frisch elasticity that ignore human capital should not be viewed as upper bounds on either compensated or uncompensated elasticities.

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[11] Recall from equation (16) that $\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.50}{0.50+.50}\right] - \left[\frac{(-.50)}{.50-(-.50)} \frac{1+.50}{.50} \frac{1}{2+.806}\right] = 0.50 + 0.53 = 1.03$.

Table 5 reports results for η = -.25. In light of the existing literature this is a low value for the degree of curvature of the utility function in consumption. Using this low value magnifies the results from Tables 3 and 4. For instance, if we look at the α = .008 case, the total elasticity with respect to a permanent tax cut is now much greater than that with respect to a temporary tax cut in period one (0.836 vs. 0.557). And the compensated elasticity is much greater as well (1.110 vs. 0.700). In the α=.008, γ=.50 case the (conventional) estimates of the Frisch elasticity are again smaller than compensated and uncompensated elasticities. And the compensated and uncompensated elasticities with respect to temporary tax cuts are again at least a factor of two below their values in the model without human capital.

Table 6 reports results for the model with borrowing constraints. I only report results for the η = -.50 case. This is because Keane and Wolpin (2001) estimated a model with both human capital and liquidity constraints and obtained an estimate of η = -.50.[12] Note that they estimated the extent of liquidity constraints (rather than assuming their existence) and their estimates implied rather tight limits on uncollateralized borrowing.

Of course with no borrowing or lending the inter-temporal substitution mechanism is completely shut down. If taxes are temporarily lowered in the first period it is no longer possible to "make hay while the sun the shins" (Heywood (1547)) and save part of the earnings for the second period. Hence, the Frisch elasticity properly defined does not exist.

It still makes sense, however, to ask what values one would obtain for the Frisch elasticity (and hence what one would infer about γ) if one applied conventional methods in an environment with liquidity constraints. As we see in the first three rows of Table 6, for the case of no human capital (α = 0) one just obtains the Marshallian elasticity. But when human capital is included one typically obtains *negative* values. For example, if α = .008 and γ = 0.5 the "Frisch" elasticity appears to be -.189 or -.119, depending on whether one uses the data that do or do not contain a temporary tax cut, respectively.

As we saw in Section III.A, in a model with human capital but no borrowing/lending (so the inter-temporal substitution mechanism is shut down), the labor supply response to permanent tax changes must exceed that to temporary tax changes. We see this clearly in Table 6. For instance, in the α = .008 and γ = 0.5 case, the total elasticity with respect to a temporary tax change is 0.345 while that with respect to a permanent tax change is 0.469.

[12] While the Keane and Wolpin (2001) model had strict limits on uncollateralized borrowing it did allow saving. But here we rule out saving as well. However, in the simple two period model of the present paper, given wage growth due to human capital, agents will essentially always want to borrow in period one (unless wage growth is trivially small). Thus, in this simple model, eliminating borrowing/lending completely is for all practical purposes equivalent to having tight bounds on uncollateralized borrowing. Hence, the model here can be viewed as a *very* simple version of the Keane and Wolpin (2001) model.

Finally, we consider compensated elasticities. We can no longer use equation (52) to determine the (net) amount of assets to give an agent to compensate him/her for a tax change, because now consumption, and hence the marginal utility of consumption, differs in the two periods. Thus, to compensate for a permanent tax change, I find the asset level A that solves:

$$A \approx \frac{V(\tau_1,\tau_2,0)-V(\tau_1',\tau_2',0)}{\left[u'(C_1)+u'(C_2)\right]/2} \tag{53}$$

and give the agent $\frac{1+r}{2+r}A$ in each period. To compensate for a temporary tax change in period 1, I find the asset level A that solves:

$$A \approx \frac{V(\tau_1,\tau_2,0)-V(\tau_1',\tau_2,0)}{u'(C_1)} \tag{54}$$

and give the agent A in period 1.

Now, looking again at the α = .008 and γ = 0.5 case, we see that the compensated elasticity with respect to a temporary tax change is 0.687 while that with respect to a permanent tax change is 0.958.

In summary, comparing Tables 4 and 6, we see that *with borrowing* total elasticities of labor supply to permanent tax cuts exceed those for temporary tax cuts once $\alpha \geq .008$, and this is true for all values of γ. Compensated elasticities for permanent tax cuts begin to be higher (for some values of γ) for values of α as low as .005, and are consistently higher for all values of γ once $\alpha \geq .006$. As I argued in Section IV.B, α = .008 is at the low end of the plausible range for α, so the cases where elasticities with respect to permanent tax changes exceed those for temporary changes are quite plausible. *With borrowing constraints*, both total and compensated elasticities to permanent tax changes always exceed those for temporary tax changes, and the size of the difference grows with the importance of human capital effects. The models with and without borrowing restrictions are polar cases, with the "truth" presumably somewhere in between. If borrowing constraints are in fact important, it becomes more likely that permanent tax changes have larger effects than temporary ones.

V. "Optimal" Income Tax Rates and the Welfare Losses from Taxation

In this Section I consider how introducing human capital into the life-cycle model affects the (second best) optimal proportional income tax rate, and the welfare losses from distortionary taxes on labor income. Throughout this section I assume a flat rate income tax that is equal in both periods ($\tau_1 = \tau_2 = \tau$). In order to talk about optimal taxation it is necessary to specify that the government provides a public good from which workers derive utility.[13] Let the quantity of the public good be denoted by *P*, and assume that the government provides the same level of *P* in each period. Then the government budget constraint is:

$$(55)\qquad P+\frac{1}{1+r}P=\left\{w_1h_1\tau+\frac{1}{1+r}w_2h_2\tau\right\}$$

Solving (55) for *P* we obtain:

$$(56)\qquad P=\tau\left\{w_1h_1+\frac{1}{1+r}w_2h_2\right\}\frac{1+r}{2+r}$$

Next we modify the value function in equation (28) to include a public good:

$$(57)\qquad \begin{aligned} V&=\lambda f(P)+\frac{[w_1h_1(1-\tau)+b]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}\\ &\quad+\rho\left\{\lambda f(P)+\frac{[w_2h_2(1-\tau)-b(1+r)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}\end{aligned}$$

where $\lambda f(P)$ is the utility that consumers derive from the public good.

Given (57), the first order conditions (29)-(31) are modified to become:

$$(58)\qquad \begin{aligned}\frac{\partial V}{\partial h_1}&=\lambda f'(P)\frac{dP}{dh_1}(1+\rho)+\left[w_1h_1(1-\tau)+b\right]^{\eta}w_1(1-\tau)-\beta h_1^{\gamma}\\ &\quad+\rho\left[w_2h_2(1-\tau)-b(1+r)\right]^{\eta}(dw_2/dh_1)h_2(1-\tau)=0\end{aligned}$$

$$(59)\qquad \frac{\partial V}{\partial h_2}=\lambda f'(P)\frac{dP}{dh_2}(1+\rho)+\rho\left\{\left[w_2h_2(1-\tau)-b(1+r)\right]^{\eta}w_2(1-\tau)-\beta h_2^{\gamma}\right\}=0$$

$$(60)\qquad \frac{\partial V}{\partial b}=\left[w_1h_1(1-\tau)+b\right]^{\eta}-\rho\left[w_2h_2(1-\tau)+b(1+r)\right]^{\eta}(1+r)=0$$

where, given the new wage equation (49) we now have $dw_2/dh_1=\alpha-\alpha h_1/200$, and so:

$$\frac{dP}{dh_1}=\tau\left\{w_1+\frac{h_2}{1+r}\frac{dw_2}{dh_1}\right\}\frac{1+r}{2+r}=\tau\left\{w_1+\frac{h_2}{1+r}w_2\left(\alpha-\frac{\alpha}{200}h_1\right)\right\}\frac{1+r}{2+r}\qquad \frac{dP}{dh_2}=\tau\frac{w_2}{2+r}$$

There is also a new first order condition describing the problem of the government:

$$(61)\qquad \frac{\partial V}{\partial\tau}=\lambda f'(P)(1+\rho)\frac{\partial P}{\partial\tau}+C_1^{\eta}\frac{\partial C_1}{\partial\tau}+\rho C_2^{\eta}\frac{\partial C_2}{\partial\tau}=0$$

[13] As we have a representative agent model, the redistributive motive for taxation that is central to work in the tradition of Mirrlees (1971), Sheshinski (1972) and Stern (1976) is not relevant here.

which we can write in more detail as:

$$\text{(62)} \quad \frac{\partial V}{\partial \tau} = \lambda f'(P)(1+\rho)\left[w_1 h_1 + \frac{1}{1+r} w_2 h_2\right]\frac{1+r}{2+r}$$
$$-\left[w_1 h_1(1-\tau)+b\right]^{\eta}(w_1 h_1) - \rho\left[w_2 h_2(1-\tau)-b(1+r)\right]^{\eta}(w_2 h_2) = 0$$

As before I assume $\rho(1+r)=1$ in order to simplify the problem and focus on the key issues. In this case, and with no constraints on borrowing and lending, we get that $C_1 = C_2 = C$. Then, equation (61) and (62) reduce to:

$$\lambda f'(P)(1+\rho)\frac{\partial P}{\partial \tau} + C^{\eta}(1+\rho)\frac{\partial C}{\partial \tau} = 0$$

And, as $\partial P/\partial \tau = -\partial C/\partial \tau$, we just have that:

$$\text{(63)} \quad \lambda f'(P) = C^{\eta}$$

This says that the benevolent government (or social planner) sets the tax rate so as to equate the marginal utility of private consumption to that of consumption of the public good.

To complete the model we must specify the functional form for $f(P)$. Obviously, the choice of $f(P)$ will have a great impact on the results. Thus I consider three alternative functional forms: $f(P) = \log(P)$, $f(P) = 2P^{.5}$ and $f(P) = P$. This allows for three very different degrees of curvature in $f(P)$. The scaling parameter λ is set so the optimal tax rate is 40% when there is no human capital accumulation (i.e., when $\alpha = 0$).

I also consider two variants on the model of equations (58)-(61). Those equations describe a social planner version of the model in which workers, when deciding on hours of work, consider how increased labor supply will lead to increased provision of the public good. We could also consider a "free rider" version of the model where there are many representative workers each solving a problem like (58)-(60), but where each worker assumes that his/her own actions will have a trivial impact on provision of the public good (dP/dh =0). In that case the first term in equations (58)-(59) drops out, and these revert back to being the same as (29)-(30). I also consider a version of the model with borrowing constraints. In that case simply set b=0 in (58)-(59) and drop equation (60). In this model, I still assume that the government can borrow and lend across periods, so (55)-(56) still hold.

Tables 7-10 show how optimal income tax rates vary with the importance of human capital investment (α). As noted earlier, all models are calibrated so that when α=0 the optimal tax rate is 40%. In Table 7, I assume that $f(P) = \log(P)$. Thus, the curvature of the sub utility function for the public good is greater than that for the private good. In this case, it is clear that the optimal tax rate falls as α increases. This effect is stronger in the versions of the model where γ is smaller (i.e., intertemporal substitution is greater). It is also stronger in the (perhaps more plausible) free-rider version of the model. For instance, in the α=.008, γ=.50 case, the optimal tax rate falls from 40% to 33.9%. If we move closer to the high end of the plausible range for human capital effects (α=.010), and adopt the Imai and Keane (2004) estimate that γ = .25, the optimal tax rate falls further to 29.4%.

The bottom panel of Table 7 reports results for the version of the model that does not allow borrowing. The most striking feature of the results is that they differ only slightly from those in the top panel. With no borrowing, optimal tax rates are (very) slightly higher.

Table 8 reports results for the model where $f(P) = 2P^{.5}$ and $\eta = -.50$. In this case (63) is simply $\lambda P^{-0.5}=C^{-0.5}$. The curvature of the sub utility functions for public and private consumption are identical. As $C=I(1-\tau)$ and $P=I\tau$, where I is $(1+r)/(2+r)$ times the present value of lifetime income, we have that $P/C = \tau/(1-\tau)$. This implies that $\lambda^2=\tau/(1-\tau)$ or that $\tau= \lambda^2/(1+ \lambda^2)$. Thus the optimal tax rate is a constant, independent of the level of lifetime income.

As we see in the top panel of Table 8, the optimal tax rate is also independent of the parameter γ that alters labor supply elasticities (Indeed, it is independent of labor supply elasticities in general). However, as we will see below, while the optimal tax rate is invariant to α and $(1/\gamma)$, the welfare cost of a distortionary proportional tax is increasing in both.

The bottom of Table 8 reports results for the model with no borrowing. Here, the above logic for why the optimal tax rate is constant does not hold, because consumption is no longer equal in both periods. But the result that the optimal proportional income tax rate is a constant still holds to a very close approximation, as is evident from the figures in the table.

Table 9 reports results for the model where $f(P) = P$ and $\eta = -.50$. In this case (63) reduces to just $\lambda = C^{\eta}$. That is, the government sets the tax rate so as to keep the marginal utility of private consumption constant. In this (perhaps implausible) case, the marginal utility of public good consumption is constant as P increases, while that of private consumption is diminishing. As α increases people become wealthier, as their t=2 wage is higher. As a result, the government raises τ to keep private consumption constant. This leads to high tax rates at high levels of α. For example, when α=.008 and γ=.50, the optimal tax rate is 68% in the social planner version of the model and 51.9% in the free-rider version.

Again, results are little different in the borrowing constrained case. Here, however, results differ noticeably between the social planner and free-rider versions of the model. This is because, in the free-rider version, agents, knowing they will be subject to a high rate of tax on human capital investment (and not accounting for how their collective actions will increase public good provision) decide to acquire much less human capital.

Finally, Tables 10-12 examine the welfare costs of proportional income taxation, and how this is influenced by parameters that govern labor supply elasticities (γ and η) and the importance of human capital (α). I only report results for the free-rider version of the model with no borrowing constraints.

I report two measures of the welfare loss from the proportional income tax. To obtain these measures, I have also solved a version of the model in which a lump sum tax is used to finance the public good. The lump sum tax is set to the level that would fund the same level of the public good that is obtained in the solution to the free-rider proportional tax version of the model. The first measure of welfare loss, denoted C^* in the tables, is the amount of extra consumption that consumers in the proportional tax world must be given to enable them to attain the same utility level (more precisely, the same level of the optimized value function) they enjoy in the lump sum tax world, expressed as a fraction of consumption in the proportional tax world. The second measure, C^{**}, is the loss in consumption in the lump sum tax world that would bring the consumer down to the utility level he/she has in the proportional tax world, expressed as a fraction of consumption in the lump sum tax world.

Table 10 reports results for the $f(P)=\log(P)$ case. The top panel reports results for the $\eta = -.75$ case and the bottom panel reports results for $\eta = -.50$. Each panel gives results for γ ranging from 0.25 to 4 and for α from 0 to 0.012. I also report the total labor supply elasticity (in the α=0 case) $e=(1+\eta)/(\gamma-\eta)$, because it helps to interpret the results, and because of the important role it plays in the literature on welfare losses from taxation and optimal tax rates. For instance, in a static model without human capital, and abstracting from income effects, Saez et al (2009) give the simple formula that for a flat rate tax the marginal excess burden is $-e\tau/(1-\tau-e\tau)$.[14, 15] Thus, the utility cost of taxation is increasing in e in that framework.

One thing that is evident in both the top and bottom panels is that welfare losses from the proportional tax are strongly inversely related to γ. When γ is large, so that labor supply elasticities are small, welfare losses from the tax are quite small. For example, when $\gamma = 4$, so that e=.11 or .05 for $\eta = -.75$ and $\eta = -.50$, respectively, welfare losses are less than 4% for both the C^* and C^{**} measures. But as we reduce γ the welfare losses become much more

[14] That is, for each extra dollar of tax collected, the utility cost to consumers in dollar terms is $-e\tau/(1-\tau-e\tau)$. For example, if τ=0.40 and e=0.50 this gives 0.50, meaning the cost is 50 cents for each dollar raised.

[15] Similarly, Saez (2001) shows that, in general, optimal tax rates in the Mirrlees (1971) model depend on both compensated and uncompensated elasticities, but his equation (9) shows that only the <u>uncompensated</u> elasticity, and government tastes for redistribution, matter for the optimal <u>flat rate</u> tax (i.e., set the Pareto parameter a=1).

substantial. For instance, consider the case of $\gamma = 0.5$. If $\eta = -.75$ this implies e=0.20, and in the no human capital case (α=0) the welfare loss is 9 to 11%, depending on the measure. If instead $\eta = -.50$ then e=0.50 and the welfare loss is 11 to 16% depending on the measure.

A second striking aspect of the results in Table 10 is that, for $\eta = -.75$, welfare losses from the tax are roughly invariant to the level of α. For instance, when $\alpha = .008$ and $\gamma = 0.5$ the welfare losses are still 9 to 11%, depending on the measure, just as I noted for the no human capital case above. It is also worth noting that, as saw back in Table 3, labor supply elasticities with respect to permanent tax or wage changes do not change very much as we alter α in the $\eta = -.75$ case. The total elasticity only changes from .20 when $\alpha = 0$ to .176 when $\alpha = .008$. (In contrast we found that the elasticities with respect to temporary tax changes declined sharply as human capital became more important).

In contrast, for $\eta = -.50$, the welfare cost of the tax, which starts out higher, is falling as α increases. In fact, by the time α reaches the plausible value of .008, the welfare losses are very similar in the $\eta = -.75$ and $\eta = -.50$ cases. This is not explained by any difference in the behavior of labor supply elasticities. Looking back at Table 4, we see that in the $\eta = -.50$ case, just as in the $\eta = -.75$ case, elasticities do not fall very much as α increases. If $\gamma = 0.5$ then the total elasticity only falls from .50 in the $\alpha = 0$ case to .445 in the $\alpha = .008$ case.

The real explanation for why the welfare cost of the tax is decreasing in α in the η = -.50 case is as follows: when $\eta = -.50$, there is less curvature in the sub utility function for private consumption. Hence, the desired ratio of private to public good consumption increases more strongly as α increases. This in turn, causes the optimal tax rate to fall more rapidly. For instance, at $\gamma = 0.5$, as α goes from 0 to .008 the optimal tax rate falls from 40% to 33.9% in the $\eta = -.50$ case (reported in Table 7). But it only falls from 40% to the 38.1% in the $\eta = -.75$ case (not reported in Table 7).

In summary, the reason the welfare loss from proportional taxes falls as α increases in the $\eta = -.50$ case is <u>not</u> that greater importance of human capital reduces the welfare cost of income taxes. In fact, it is just the reverse – as human capital becomes more important the optimal tax rate falls. With a lower tax the welfare cost of the tax is also reduced.

Now consider Table 11, which reports results for the case where $f(P) = 2P^{.5}$. As we discussed earlier, if $\eta = -.50$ then the degree of curvature in the sub utility functions for the public and private good are identical, and hence the optimal tax rate is constant at 40%. But with $\eta = -.75$ the degree of curvature in the sub utility function for the private good is slightly less than that for the public good. Thus, as α increases, making people wealthier, the optimal tax rate increases. However, this effect is very weak for all values of γ. For example, when γ

= 0.5 the optimal tax rate increases to only 42.7% when α increases to .008. Overall, for both η = -.75 and η = -.50, we see that the welfare cost of the proportional tax is increasing as human capital becomes more important. The increase is more pronounced when η = -.75 (the case where taxes are increasing).

But the main result in Table 11 is that same as that of Table 10; namely, that the welfare cost of the proportional income tax can be quite substantial even at fairly modest values of γ and e. For example, consider the case of η = -.75 and γ = 2. In this case the Frisch elasticity is only 1/2 and the Marshallian is only 0.09, yet the welfare loss from the tax is 7 to 8% of consumption. If we reduce γ to 1/2 (which I argued in Section IV.A. is a much more plausible value) then, according to Table 3, the total labor supply elasticity is still a modest 0.176. Yet the welfare loss from the tax is 12 to 15% of consumption, depending on the measure used. Thus, quite modest values of the total labor supply elasticity can be consistent with large welfare losses from proportional taxes.[16]

Note that welfare losses from the income tax would be even greater if the lump sum tax were chosen optimally (achieving the 1st best). For example, in the η = -.50, γ = 1/2, α = .008 case, welfare losses are 12-18% when compared to the constrained lump sum tax (that raises the same revenue), but 17-25% when the optimal lump sum tax is used.

Finally, Table 12 reports results for $f(P) = P$. As I discussed earlier, in this case the optimal tax rate rises with α because as people become wealthier they demand more of the public good. For example, if γ = 1/2 then when α increases from 0 to .008 the optimal tax rate increases from 40% to 51.9% (see Table 9). Note also that the welfare losses from income taxation become quite substantial in this case. For example, when α = .008, η = -.75 and γ = 1/2, the welfare loss is 19 to 28% of consumption, depending on the measure used.

VI. Conclusion

When human capital is added to the standard life-cycle labor supply model, the wage rate is no longer the opportunity cost of time. Rather, the opportunity cost becomes the wage plus a term representing the return on human capital investment. In this paper I have argued that this fact has important (and in my view under appreciated) implications for how workers respond to tax changes, and for the estimation and interpretation of wage elasticities of labor supply. One key result is that permanent tax changes can have larger effects on current labor supply than transitory tax changes. This result, which holds at quite reasonable parameter values, contradicts the widespread presumption that transitory tax (or wage) changes should

[16] According to Table 3, the compensated elasticities are .361 and .698 in the two cases I have discussed here.

have larger effects. The intuition is that a transitory tax change alters only the current after-tax wage (and hence only part of the opportunity cost of time), while a permanent tax change shifts the return on human capital investment as well. For the same reason, elasticities with respect to temporary tax changes decrease as human capital becomes more important.

The second key result is that even a "small" return on human capital investment (in a sense made precise in the paper) can lead to severe downward bias in estimates of the inter-temporal elasticity of substitution in labor supply (the Frisch elasticity). Given plausible values for the return to human capital investment, existing evidence is consistent with a Frisch elasticity of 2 or more. This has additional consequences. In the standard life-cycle model, the Frisch elasticity is an upper bound on Marshallian and Hicks elasticities. Thus, severely downward biased estimates of the Frisch elasticity induced by failure to account for human capital may lead us to incorrectly conclude that uncompensated and compensated elasticities are smaller than they actually are.

Indeed, using simulations of a very simple life-cycle model augmented to include human capital investment, I showed that both permanent and transitory tax changes can have much larger effects on labor supply than the (incorrectly estimated) Frisch elasticity would suggest. The simulations presented here suggest that the compensated substitution effect of a permanent tax change may be several times greater than the conventionally measured intertemporal substitution effect. Hence, the small Frisch elasticities obtained in prior work (ignoring human capital) should <u>not</u> be viewed as an upper bound on plausible compensated substitution effects. Nor are they a bound on uncompensated elasticities.

I also showed that the use of variation in wage rates induced by exogenous tax regime changes to identify labor supply elasticities does not resolve this problem, and can even make the bias greater. The point is that, in a model with human capital, any tax change will induce changes in the incentive to acquire human capital. Thus, any change in the time path of after-tax wages induced by exogenous changes in tax rates will nevertheless be endogenous – as it is contaminated by changes in human capital investment decisions. The only solution to this problem is to model how labor supply responds to changes in the opportunity cost of time (not just the wage), as in Heckman (1973) and Imai and Keane (2004).

I went on to use the simple life-cycle labor supply model with human capital investment to study both optimal proportional tax rates on labor income and the welfare effects of income taxation. The results are summarized in Table 13. The table presents welfare losses from a proportional flat rate income tax, expressed as a fraction of consumption, under a number of parameterizations of the model. The return to work experience is set such that the wage rate grows by roughly 1/3 over the first 20 years of the working life. The benevolent government sets the tax rate optimally to equate marginal utility of consumption of the public and private goods. The top panel presents results with the CRRA curvature parameter for consumption (η) set at -.075, the value estimated by Imai and Keane (2004), while the bottom presents results for the value of -0.50 estimated by Keane and Wolpin (2001).[17] Results are presented for several values of the CRRA curvature parameter in hours (γ), from a value of 4, which implies little inter-temporal substitution in leisure, up to a value of 0.25, which implies an inter-temporal elasticity of substitution of labor supply of 4, close to the Imai and Keane (2004) estimate.

Under the column labeled "uncompensated elasticity" the table reports simulated total labor supply elasticities to permanent tax changes.[18] Note that very high values of the Frisch elasticity ($1/\gamma$) are consistent with very modest uncompensated elasticities. For example, in the η = -.075, γ = 0.25 case, which corresponds to the Imai and Keane (2004) estimates, the simulated uncompensated elasticity is a modest 0.205. But the welfare cost of proportional income taxation is still substantial – i.e., 13% to 35%, depending on the measure.

The welfare cost of income taxation is calculated for three cases: one where utility is log(P), where P is the amount of the private good, one where it is $2P^{.5}$, and one where it is linear in P. This covers a range of degrees of curvature in consumers' utility from the public good, ranging from more than that for the private good to less. The welfare losses in the three cases are equivalent to 13%, 19% and 35% of consumption, respectively.

Even if we reduce ($1/\gamma$) to the much more modest value of 1, in which case the uncompensated elasticity is only 0.133, the welfare losses in the three cases are 9%, 11% and 19% of consumption, respectively. It appears that large welfare losses from income taxation are quite consistent with existing (small) estimates of labor supply elasticities.

Historically, Mirrlees (1971) expressed surprise that optimal tax rates were so low (about 20 to 30%) in his model, but Stern (1976) noted that that optimal tax rates would be much higher (i.e., well over 50%) if utility parameters were set to values that implied much less elastic labor supply. He argued this was more consistent with existing empirical work. But, given the downward bias in elasticity estimates induced by failure to account for human capital, the very low elasticity estimates used by Stern may be suspect, while the higher elasticities in Mirrlees' original paper may be more plausible.

[17] These are two structural models that include both labor supply and asset accumulation.
[18] It is important to note that the compensated and uncompensated elasticities reported in Table 13 are not the traditional Marshallian and Hicks elastictities. Instead they are generalizations of these formulas that apply for the dynamic case with human capital, as given by equations (42) and (52).

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Table 1: Baseline Simulation

		η = -.75						η = -.5						η = -.25					
α	**γ**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	h_1		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
	h_2		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
	w_2		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
0.001	h_1	99	102	101	101	101	100	101	103	102	101	101	100	105	105	103	102	101	101
	h_2	109	103	102	101	101	100	113	104	103	102	101	101	127	107	104	102	101	101
	w_2	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103
0.003	h_1	104	105	105	103	102	101	109	109	107	105	103	102	130	116	111	106	104	102
	h_2	120	110	107	104	103	101	135	115	110	106	103	102	211	127	115	108	104	102
	w_2	111	111	111	110	110	110	111	111	111	111	110	110	114	112	112	111	110	110
0.005	h_1	110	109	108	106	104	102	120	116	112	108	105	103	158	130	120	111	106	103
	h_2	132	117	112	108	105	103	166	128	118	110	106	103	390	157	129	115	107	104
	w_2	120	120	119	119	118	118	122	121	120	119	118	118	130	125	122	120	119	118
0.006	h_1	113	112	110	107	105	103	127	120	115	110	106	103	168	138	125	114	108	104
	h_2	139	121	115	109	106	103	186	136	123	113	107	104	518	178	138	118	109	104
	w_2	125	125	124	123	122	122	129	127	126	124	123	122	138	132	129	126	123	122
0.007	h_1	116	114	112	109	106	103	133	124	118	112	107	104	175	146	130	117	109	105
	h_2	147	126	118	111	106	104	209	145	128	116	108	104	670	202	148	122	111	105
	w_2	131	130	129	128	127	126	137	134	132	129	127	126	147	141	136	132	128	126
0.008	h_1	120	117	114	110	107	104	140	128	121	114	108	105	181	153	136	120	111	106
	h_2	155	130	121	113	108	104	236	155	133	118	110	105	849	229	160	127	112	106
	w_2	138	137	135	133	132	130	146	141	139	135	133	131	156	150	144	138	134	131
0.010	h_1	127	122	118	113	109	105	152	138	128	119	111	106	188	166	147	127	114	107
	h_2	174	141	128	117	110	105	299	178	146	125	113	106	1306	295	187	137	116	108
	w_2	154	151	149	145	142	140	166	159	154	149	144	141	175	170	164	154	146	142
0.012	h_1	135	128	123	117	111	106	162	147	136	124	114	107	192	175	158	135	118	109
	h_2	196	152	135	121	112	106	372	206	161	132	116	108	1937	374	220	150	121	109
	w_2	173	168	165	160	155	151	188	180	174	165	157	152	196	193	186	173	161	153

Table 2: Baseline Simulation, Borrowing Constraint

		η = -.75						η = -.5						η = -.25					
α	**γ**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	h_1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	h_2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	w_2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
0.001	h_1	104	103	102	102	101	101	106	104	103	102	101	101	112	106	104	102	101	101
	h_2	101	101	101	100	100	100	103	102	102	101	101	100	112	105	103	102	101	101
	w_2	103	103	103	103	103	103	103	103	103	103	103	103	104	103	103	103	103	103
0.003	h_1	111	108	107	105	103	102	117	112	109	106	103	102	138	118	112	107	104	102
	h_2	104	103	102	101	101	101	112	108	105	103	102	101	152	120	112	106	103	102
	w_2	112	111	111	111	110	110	112	112	111	111	110	110	115	113	112	111	110	110
0.005	h_1	119	114	111	108	105	103	130	120	115	110	106	103	163	133	121	112	107	103
	h_2	107	105	104	103	102	101	125	114	110	106	103	102	221	140	123	112	106	103
	w_2	122	121	120	119	118	118	125	122	121	120	119	118	130	125	123	120	119	118
0.006	h_1	122	117	113	109	106	103	136	124	118	112	107	104	171	140	126	115	108	104
	h_2	109	106	105	103	102	101	132	118	113	108	104	102	265	153	129	115	107	104
	w_2	128	126	125	124	123	122	132	128	127	125	123	122	138	133	129	126	123	122
0.007	h_1	126	119	116	111	107	104	141	128	121	114	108	104	178	147	131	118	110	105
	h_2	110	107	106	104	102	101	140	122	115	109	105	103	318	168	137	118	109	104
	w_2	135	132	131	129	127	126	140	135	133	130	128	126	147	141	137	132	129	127
0.008	h_1	129	122	118	113	108	105	147	132	124	116	109	105	183	154	137	121	111	106
	h_2	112	109	106	104	103	101	148	127	118	111	106	103	379	185	145	122	110	105
	w_2	142	139	137	135	132	131	148	143	140	136	133	131	156	150	145	138	134	131
0.010	h_1	136	128	122	116	110	106	157	140	131	120	112	106	190	166	147	128	115	107
	h_2	117	111	109	106	103	102	167	137	125	114	108	104	537	223	164	130	114	106
	w_2	158	154	151	147	143	140	167	161	156	150	144	141	175	171	164	154	146	142
0.012	h_1	142	133	126	119	112	107	165	148	137	125	114	108	194	175	157	135	118	109
	h_2	121	114	111	107	104	102	189	149	132	118	110	105	755	268	186	139	117	108
	w_2	178	171	167	161	156	151	189	181	174	166	158	152	196	193	186	173	161	153

Table 3: Labor Supply Response to Tax Change, η = -.75

		Tax reduction in period 1						Tax reduction in both periods					
α	**γ**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total elas.		1.570	0.835	0.445	0.235	0.122		0.249	0.199	0.142	0.090	0.052
	Comp. elas.		2.059	1.222	0.721	0.410	0.223		0.990	0.792	0.566	0.361	0.209
	Frisch elas.		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total elas.	7.784	1.278	0.733	0.408	0.220	0.116	0.212	0.236	0.194	0.140	0.090	0.052
	Comp. elas.	8.192	1.731	1.104	0.675	0.392	0.215	0.841	0.935	0.770	0.558	0.358	0.208
	Frisch elas.	-5.203	-0.839	-0.478	-0.263	-0.141	-0.074	3.200	0.430	0.221	0.109	0.053	0.025
0.003	Total elas.	2.267	0.891	0.572	0.341	0.192	0.103	0.223	0.220	0.186	0.137	0.089	0.052
	Comp. elas.	2.663	1.297	0.917	0.596	0.357	0.200	0.883	0.874	0.739	0.546	0.354	0.207
	Frisch elas.	0.814	0.197	0.086	0.027	0.003	-0.002	1.404	0.390	0.208	0.099	0.045	0.020
0.005	Total elas.	1.185	0.645	0.450	0.285	0.166	0.091	0.231	0.213	0.181	0.135	0.088	0.052
	Comp. elas.	1.571	1.020	0.773	0.528	0.326	0.185	0.913	0.843	0.719	0.538	0.352	0.206
	Frisch elas.	0.874	0.314	0.162	0.065	0.020	0.005	1.019	0.359	0.195	0.090	0.038	0.015
0.006	Total elas.	0.912	0.552	0.399	0.259	0.154	0.085	0.233	0.210	0.179	0.135	0.088	0.052
	Comp. elas.	1.289	0.913	0.713	0.498	0.311	0.178	0.920	0.832	0.712	0.535	0.351	0.206
	Frisch elas.	0.846	0.336	0.179	0.074	0.023	0.005	0.915	0.345	0.189	0.086	0.035	0.013
0.007	Total elas.	0.714	0.473	0.353	0.236	0.142	0.079	0.234	0.208	0.178	0.134	0.088	0.052
	Comp. elas.	1.079	0.820	0.657	0.469	0.297	0.171	0.920	0.822	0.705	0.532	0.350	0.206
	Frisch elas.	0.817	0.350	0.190	0.079	0.024	0.005	0.836	0.332	0.183	0.082	0.032	0.011
0.008	Total elas.	0.565	0.405	0.312	0.214	0.131	0.074	0.232	0.205	0.176	0.133	0.088	0.052
	Comp. elas.	0.913	0.738	0.606	0.441	0.283	0.164	0.911	0.811	0.698	0.530	0.350	0.206
	Frisch elas.	0.791	0.358	0.198	0.083	0.025	0.004	0.774	0.319	0.177	0.079	0.029	0.009
0.010	Total elas.	0.358	0.295	0.241	0.174	0.111	0.064	0.221	0.198	0.173	0.132	0.088	0.052
	Comp. elas.	0.663	0.597	0.515	0.391	0.257	0.151	0.865	0.783	0.683	0.525	0.349	0.206
	Frisch elas.	0.752	0.370	0.210	0.088	0.024	0.002	0.682	0.296	0.165	0.072	0.024	0.006
0.012	Total elas.	0.229	0.211	0.183	0.139	0.092	0.054	0.200	0.188	0.168	0.131	0.088	0.052
	Comp. elas.	0.479	0.478	0.434	0.344	0.233	0.139	0.784	0.741	0.663	0.520	0.349	0.207
	Frisch elas.	0.727	0.380	0.220	0.091	0.023	-0.001	0.615	0.276	0.154	0.065	0.019	0.003

Table 4: Labor Supply Response to Tax Change, η = -.5

		Tax reduction in period 1						Tax reduction in both periods					
α	γ	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total elas.		1.844	1.030	0.568	0.305	0.160		0.666	0.499	0.332	0.199	0.111
	Comp. elas.		2.279	1.353	0.783	0.434	0.231		1.326	0.994	0.663	0.398	0.221
	Frisch elas.		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total elas.	7.854	1.518	0.915	0.526	0.289	0.153	0.650	0.634	0.487	0.329	0.198	0.110
	Comp. elas.	8.265	1.923	1.226	0.735	0.415	0.223	1.289	1.262	0.971	0.656	0.396	0.220
	Frisch elas.	-3.734	-0.713	-0.431	-0.249	-0.138	-0.073	3.865	0.480	0.238	0.114	0.054	0.026
0.003	Total elas.	2.306	1.083	0.732	0.451	0.257	0.139	0.700	0.599	0.472	0.324	0.197	0.110
	Comp. elas.	2.703	1.447	1.022	0.651	0.379	0.207	1.384	1.191	0.940	0.646	0.393	0.220
	Frisch elas.	1.502	0.346	0.147	0.046	0.009	-0.001	2.064	0.503	0.251	0.112	0.048	0.021
0.005	Total elas.	1.176	0.795	0.589	0.386	0.228	0.125	0.710	0.579	0.462	0.321	0.196	0.110
	Comp. elas.	1.541	1.127	0.861	0.578	0.347	0.192	1.399	1.149	0.920	0.640	0.392	0.220
	Frisch elas.	1.438	0.493	0.242	0.092	0.028	0.006	1.686	0.509	0.256	0.110	0.043	0.016
0.006	Total elas.	0.876	0.682	0.528	0.357	0.215	0.119	0.686	0.567	0.458	0.320	0.196	0.110
	Comp. elas.	1.208	0.997	0.790	0.545	0.331	0.185	1.351	1.125	0.910	0.638	0.392	0.220
	Frisch elas.	1.381	0.528	0.267	0.104	0.032	0.007	1.595	0.508	0.257	0.109	0.041	0.014
0.007	Total elas.	0.654	0.583	0.472	0.329	0.202	0.113	0.641	0.552	0.452	0.319	0.196	0.110
	Comp. elas.	0.945	0.878	0.724	0.513	0.316	0.177	1.259	1.095	0.898	0.635	0.392	0.220
	Frisch elas.	1.334	0.553	0.287	0.114	0.034	0.007	1.536	0.507	0.257	0.107	0.038	0.013
0.008	Total elas.	0.489	0.495	0.420	0.303	0.189	0.107	0.578	0.532	0.445	0.318	0.197	0.110
	Comp. elas.	0.732	0.768	0.661	0.482	0.302	0.171	1.134	1.054	0.884	0.633	0.392	0.220
	Frisch elas.	1.297	0.574	0.304	0.121	0.036	0.007	1.500	0.505	0.256	0.105	0.036	0.011
0.010	Total elas.	0.275	0.349	0.327	0.254	0.165	0.095	0.436	0.475	0.424	0.315	0.197	0.111
	Comp. elas.	0.431	0.570	0.541	0.423	0.274	0.157	0.854	0.938	0.841	0.626	0.393	0.221
	Frisch elas.	1.246	0.610	0.334	0.134	0.038	0.005	1.470	0.502	0.254	0.102	0.032	0.008
0.012	Total elas.	0.162	0.240	0.249	0.210	0.143	0.084	0.315	0.400	0.390	0.309	0.197	0.111
	Comp. elas.	0.255	0.407	0.431	0.367	0.248	0.144	0.618	0.789	0.774	0.614	0.393	0.222
	Frisch elas.	1.213	0.642	0.362	0.145	0.039	0.003	1.470	0.501	0.251	0.098	0.029	0.005

Table 5: Labor Supply Response to Tax Change, η = -.25

		Tax reduction in period 1						Tax reduction in both periods					
α	γ	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total elas.		2.393	1.356	0.741	0.391	0.202		1.504	1.000	0.599	0.332	0.176
	Comp. elas.		2.721	1.572	0.870	0.463	0.240		2.002	1.332	0.798	0.443	0.234
	Frisch elas.		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total elas.	8.040	2.006	1.222	0.693	0.373	0.194	2.058	1.450	0.985	0.595	0.331	0.176
	Comp. elas.	8.454	2.314	1.431	0.819	0.443	0.231	2.726	1.929	1.311	0.793	0.442	0.234
	Frisch elas.	0.191	-0.474	-0.356	-0.230	-0.133	-0.072	5.927	0.580	0.268	0.121	0.055	0.026
0.003	Total elas.	2.163	1.460	1.003	0.608	0.338	0.179	2.043	1.381	0.965	0.591	0.331	0.176
	Comp. elas.	2.493	1.734	1.199	0.729	0.406	0.215	2.687	1.835	1.285	0.788	0.441	0.234
	Frisch elas.	3.121	0.632	0.246	0.073	0.015	0.000	4.464	0.734	0.324	0.131	0.052	0.022
0.005	Total elas.	0.707	1.033	0.816	0.532	0.306	0.164	1.139	1.259	0.941	0.590	0.332	0.176
	Comp. elas.	0.844	1.261	0.997	0.649	0.372	0.200	1.492	1.669	1.251	0.786	0.442	0.234
	Frisch elas.	2.672	0.841	0.374	0.130	0.037	0.008	4.983	0.839	0.362	0.138	0.049	0.018
0.006	Total elas.	0.414	0.840	0.728	0.496	0.290	0.157	0.776	1.148	0.919	0.589	0.332	0.176
	Comp. elas.	0.491	1.036	0.899	0.610	0.356	0.192	1.017	1.520	1.221	0.784	0.443	0.235
	Frisch elas.	2.506	0.904	0.416	0.148	0.042	0.010	5.476	0.888	0.377	0.141	0.048	0.016
0.007	Total elas.	0.254	0.665	0.642	0.461	0.276	0.150	0.538	1.006	0.884	0.587	0.333	0.177
	Comp. elas.	0.299	0.826	0.800	0.572	0.340	0.185	0.706	1.331	1.175	0.781	0.444	0.235
	Frisch elas.	2.363	0.955	0.453	0.163	0.046	0.010	6.030	0.939	0.391	0.143	0.047	0.015
0.008	Total elas.	0.164	0.515	0.557	0.427	0.261	0.144	0.385	0.852	0.836	0.583	0.334	0.177
	Comp. elas.	0.190	0.642	0.700	0.535	0.325	0.178	0.505	1.126	1.110	0.776	0.445	0.236
	Frisch elas.	2.237	0.999	0.487	0.176	0.049	0.010	6.626	0.992	0.405	0.144	0.045	0.013
0.010	Total elas.	0.075	0.303	0.401	0.360	0.233	0.131	0.213	0.575	0.703	0.567	0.336	0.178
	Comp. elas.	0.085	0.375	0.510	0.459	0.295	0.164	0.280	0.760	0.932	0.754	0.447	0.237
	Frisch elas.	2.025	1.068	0.552	0.202	0.054	0.009	7.935	1.106	0.431	0.147	0.043	0.010
0.012	Total elas.	0.038	0.181	0.276	0.295	0.207	0.119	0.127	0.381	0.550	0.536	0.337	0.179
	Comp. elas.	0.041	0.220	0.352	0.382	0.266	0.151	0.167	0.504	0.729	0.712	0.448	0.239
	Frisch elas.	1.857	1.113	0.614	0.228	0.059	0.008	9.440	1.222	0.460	0.149	0.041	0.008

Table 6: Labor Supply Response to Tax Change, η = -.5, Borrowing Constraint

		Tax reduction in period 1						**Tax reduction in both periods**					
α	γ	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total elas.	1.000	0.666	0.499	0.332	0.199	0.111	1.000	0.666	0.499	0.332	0.199	0.111
	Comp. elas.	1.990	1.326	0.994	0.663	0.398	0.221	1.990	1.326	0.994	0.663	0.398	0.221
	Frisch elas.	1.000	0.666	0.499	0.332	0.199	0.111	1.000	0.666	0.499	0.332	0.199	0.111
0.001	Total elas.	0.943	0.635	0.478	0.320	0.193	0.107	0.996	0.665	0.498	0.332	0.199	0.111
	Comp. elas.	1.877	1.265	0.954	0.639	0.385	0.214	1.985	1.328	0.998	0.666	0.400	0.222
	Frisch elas.	-1.248	-0.906	-0.709	-0.493	-0.306	-0.174	-0.597	-0.433	-0.338	-0.235	-0.146	-0.083
0.003	Total elas.	0.823	0.575	0.439	0.298	0.181	0.101	0.959	0.656	0.496	0.332	0.199	0.111
	Comp. elas.	1.636	1.145	0.876	0.594	0.361	0.202	1.926	1.321	1.000	0.671	0.404	0.224
	Frisch elas.	-0.498	-0.433	-0.367	-0.275	-0.180	-0.106	-0.337	-0.300	-0.257	-0.195	-0.129	-0.076
0.005	Total elas.	0.690	0.513	0.401	0.277	0.169	0.095	0.879	0.636	0.489	0.331	0.199	0.111
	Comp. elas.	1.372	1.023	0.800	0.552	0.338	0.190	1.777	1.289	0.993	0.673	0.407	0.226
	Frisch elas.	-0.243	-0.293	-0.276	-0.226	-0.157	-0.096	-0.159	-0.201	-0.193	-0.161	-0.114	-0.071
0.006	Total elas.	0.621	0.482	0.383	0.266	0.164	0.092	0.823	0.621	0.484	0.330	0.200	0.111
	Comp. elas.	1.235	0.960	0.763	0.532	0.327	0.184	1.670	1.262	0.985	0.673	0.408	0.227
	Frisch elas.	-0.146	-0.241	-0.244	-0.210	-0.150	-0.094	-0.090	-0.160	-0.166	-0.146	-0.108	-0.068
0.007	Total elas.	0.553	0.450	0.364	0.256	0.159	0.090	0.760	0.602	0.477	0.329	0.200	0.111
	Comp. elas.	1.101	0.897	0.725	0.512	0.317	0.179	1.548	1.226	0.974	0.672	0.409	0.228
	Frisch elas.	-0.059	-0.194	-0.215	-0.196	-0.145	-0.092	-0.031	-0.124	-0.141	-0.132	-0.101	-0.065
0.008	Total elas.	0.489	0.418	0.345	0.247	0.154	0.087	0.693	0.579	0.469	0.327	0.200	0.111
	Comp. elas.	0.973	0.833	0.687	0.492	0.307	0.174	1.419	1.183	0.958	0.670	0.410	0.229
	Frisch elas.	0.019	-0.151	-0.189	-0.183	-0.141	-0.091	0.020	-0.092	-0.119	-0.120	-0.095	-0.063
0.010	Total elas.	0.376	0.354	0.307	0.227	0.144	0.082	0.562	0.524	0.447	0.323	0.200	0.111
	Comp. elas.	0.749	0.706	0.612	0.454	0.288	0.164	1.163	1.076	0.916	0.663	0.411	0.230
	Frisch elas.	0.156	-0.072	-0.141	-0.161	-0.133	-0.089	0.102	-0.039	-0.081	-0.097	-0.084	-0.058
0.012	Total elas.	0.287	0.295	0.269	0.208	0.135	0.077	0.448	0.462	0.418	0.317	0.199	0.112
	Comp. elas.	0.573	0.588	0.537	0.416	0.270	0.155	0.940	0.953	0.859	0.651	0.410	0.230
	Frisch elas.	0.273	0.001	-0.096	-0.141	-0.127	-0.088	0.166	0.003	-0.050	-0.077	-0.074	-0.054

Table 7: Optimal Tax Rates: f(P) = log(P), η = -.5

Borrowing/lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.400	0.396	0.386	0.374	0.368	0.361	0.354	0.337	0.318
	Free-rider	0.400	0.392	0.374	0.354	0.343	0.331	0.319	0.294	0.270
0.5	Social planner	0.400	0.396	0.388	0.379	0.373	0.368	0.362	0.349	0.335
	Free-rider	0.400	0.394	0.381	0.365	0.357	0.348	0.339	0.320	0.300
1	Social planner	0.400	0.397	0.390	0.383	0.379	0.375	0.370	0.361	0.350
	Free-rider	0.400	0.396	0.386	0.375	0.370	0.363	0.357	0.344	0.329
2	Social planner	0.400	0.397	0.392	0.386	0.383	0.380	0.377	0.369	0.362
	Free-rider	0.400	0.397	0.390	0.382	0.378	0.374	0.370	0.360	0.351
4	Social planner	0.400	0.398	0.393	0.388	0.386	0.383	0.381	0.375	0.369
	Free-rider	0.400	0.397	0.392	0.386	0.383	0.380	0.377	0.371	0.363
No borrowing lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.400	0.396	0.386	0.376	0.371	0.365	0.360	0.347	0.334
	Free-rider	0.400	0.392	0.376	0.359	0.350	0.341	0.333	0.316	0.300
0.5	Social planner	0.400	0.396	0.388	0.380	0.375	0.371	0.366	0.356	0.345
	Free-rider	0.400	0.394	0.381	0.368	0.361	0.354	0.347	0.332	0.318
1	Social planner	0.400	0.397	0.390	0.384	0.380	0.376	0.372	0.365	0.356
	Free-rider	0.400	0.396	0.386	0.376	0.371	0.366	0.361	0.350	0.338
2	Social planner	0.400	0.397	0.392	0.387	0.384	0.381	0.378	0.372	0.365
	Free-rider	0.400	0.397	0.390	0.383	0.379	0.375	0.372	0.364	0.355
4	Social planner	0.400	0.398	0.393	0.389	0.386	0.384	0.382	0.377	0.371
	Free-rider	0.400	0.397	0.392	0.387	0.384	0.381	0.378	0.372	0.366

Table 8: Optimal Tax Rates: f(P) = 2$P^{.5}$, η = -.5

Borrowing/lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
	Free-rider	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
0.5	Social planner	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
	Free-rider	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
1	Social planner	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
	Free-rider	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
2	Social planner	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
	Free-rider	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
4	Social planner	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
	Free-rider	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
No borrowing lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.4000	0.4000	0.4003	0.4009	0.4013	0.4019	0.4027	0.4049	0.408
	Free-rider	0.4000	0.4000	0.4001	0.4003	0.4006	0.4009	0.4014	0.4029	0.4051
0.5	Social planner	0.4000	0.4000	0.4002	0.4007	0.401	0.4014	0.402	0.4035	0.4056
	Free-rider	0.4000	0.4000	0.4001	0.4003	0.4005	0.4008	0.4012	0.4022	0.4039
1	Social planner	0.4000	0.4000	0.4002	0.4005	0.4008	0.4011	0.4015	0.4025	0.4039
	Free-rider	0.4000	0.4000	0.4001	0.4003	0.4005	0.4007	0.401	0.4018	0.403
2	Social planner	0.4000	0.4000	0.4001	0.4004	0.4006	0.4009	0.4012	0.4019	0.4029
	Free-rider	0.4000	0.4000	0.4001	0.4003	0.4005	0.4007	0.4009	0.4016	0.4024
4	Social planner	0.4000	0.4000	0.4001	0.4004	0.4006	0.4008	0.401	0.4017	0.4025
	Free-rider	0.4000	0.4000	0.4001	0.4003	0.4005	0.4007	0.4009	0.4015	0.4022

Table 9: Optimal Tax Rates: f(P) = P, η = -.5

Borrowing/lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.400	0.460	0.584	0.704	0.758	0.805	0.844	0.904	
	Free-rider	0.400	0.415	0.447	0.480	0.497	0.514	0.532	0.566	0.601
0.5	Social planner	0.400	0.433	0.502	0.574	0.610	0.646	0.680	0.746	0.803
	Free-rider	0.400	0.414	0.443	0.473	0.488	0.503	0.519	0.550	0.582
1	Social planner	0.400	0.420	0.461	0.504	0.526	0.548	0.571	0.616	0.661
	Free-rider	0.400	0.412	0.437	0.463	0.476	0.489	0.503	0.530	0.558
2	Social planner	0.400	0.413	0.441	0.469	0.484	0.498	0.513	0.543	0.574
	Free-rider	0.400	0.410	0.432	0.453	0.465	0.476	0.487	0.510	0.534
4	Social planner	0.400	0.410	0.431	0.452	0.462	0.473	0.484	0.506	0.529
	Free-rider	0.400	0.409	0.427	0.446	0.455	0.465	0.475	0.494	0.514
No borrowing lending		α								
γ		**0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	Social planner	0.400	0.459	0.579	0.702	0.762	0.816	0.861	0.925	0.945
	Free-rider	0.400	0.415	0.446	0.476	0.491	0.506	0.521	0.551	0.580
0.5	Social planner	0.400	0.433	0.501	0.572	0.608	0.645	0.682	0.753	0.816
	Free-rider	0.400	0.414	0.442	0.469	0.483	0.497	0.511	0.539	0.567
1	Social planner	0.400	0.420	0.461	0.504	0.526	0.548	0.571	0.617	0.664
	Free-rider	0.400	0.412	0.436	0.461	0.474	0.486	0.499	0.524	0.550
2	Social planner	0.400	0.413	0.441	0.469	0.484	0.499	0.514	0.545	0.577
	Free-rider	0.400	0.410	0.431	0.453	0.463	0.474	0.485	0.508	0.530
4	Social planner	0.400	0.410	0.431	0.452	0.463	0.474	0.486	0.509	0.533
	Free-rider	0.400	0.409	0.427	0.446	0.455	0.465	0.474	0.494	0.514

Table 10: Welfare Losses from Proportional Income Tax, f(P) = log(P)

η = -.75

γ	$\frac{1+\eta}{\gamma-\eta}$		α								
			0	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	0.25	C*	13.40	13.39	13.46	13.51	13.49	13.45	13.35	13.03	12.50
		C**	-10.66	-10.65	-10.68	-10.69	-10.67	-10.63	-10.56	-10.33	-9.97
0.5	0.20	C*	11.25	11.27	11.34	11.41	11.43	11.44	11.42	11.31	11.07
		C**	-9.26	-9.27	-9.31	-9.34	-9.35	-9.35	-9.33	-9.24	-9.06
1	0.14	C*	8.56	8.60	8.69	8.79	8.84	8.89	8.92	8.97	8.97
		C**	-7.35	-7.38	-7.45	-7.52	-7.55	-7.58	-7.60	-7.63	-7.61
2	0.09	C*	5.81	5.86	5.95	6.06	6.11	6.16	6.22	6.32	6.42
		C**	-5.23	-5.27	-5.34	-5.42	-5.47	-5.51	-5.55	-5.63	-5.70
4	0.05	C*	3.56	3.59	3.66	3.74	3.78	3.83	3.87	3.96	4.05
		C**	-3.33	-3.36	-3.42	-3.49	-3.53	-3.56	-3.60	-3.68	-3.75

η = -.5

γ	$\frac{1+\eta}{\gamma-\eta}$		α								
			0	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	0.67	C*	20.02	19.23	17.66	15.73	14.61	13.40	12.16	9.77	7.74
		C**	-12.63	-12.26	-11.47	-10.47	-9.87	-9.22	-8.54	-7.15	-5.89
0.5	0.50	C*	15.56	15.16	14.32	13.33	12.74	12.09	11.38	9.83	8.25
		C**	-10.74	-10.51	-10.03	-9.44	-9.10	-8.71	-8.29	-7.35	-6.34
1	0.33	C*	10.83	10.69	10.38	10.02	9.81	9.57	9.30	8.68	7.92
		C**	-8.28	-8.17	-7.96	-7.70	-7.55	-7.39	-7.21	-6.78	-6.26
2	0.20	C*	6.79	6.75	6.68	6.60	6.55	6.49	6.43	6.28	6.09
		C**	-5.69	-5.66	-5.60	-5.52	-5.48	-5.43	-5.38	-5.26	-5.10
4	0.11	C*	3.90	3.90	3.90	3.89	3.89	3.89	3.88	3.87	3.85
		C**	-3.51	-3.51	-3.51	-3.50	-3.49	-3.49	-3.48	-3.46	-3.44

Note: C* = consumption gain needed to compensate for tax distortion (starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)

Table 11: Welfare Losses from Proportional Income Tax, f(P) = 2$P^{.5}$

η = -.75

γ	$\frac{1+\eta}{\gamma-\eta}$		α: **0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	0.25	C*	13.40	13.88	15.09	16.54	17.33	18.16	19.03	20.83	22.70
		C**	-10.66	-10.98	-11.75	-12.65	-13.13	-13.63	-14.14	-15.18	-16.23
0.5	0.20	C*	11.25	11.63	12.54	13.60	14.19	14.81	15.47	16.86	18.34
		C**	-9.26	-9.53	-10.14	-10.86	-11.24	-11.65	-12.06	-12.94	-13.85
1	0.14	C*	8.56	8.83	9.46	10.18	10.58	11.01	11.46	12.43	13.50
		C**	-7.35	-7.56	-8.03	-8.56	-8.84	-9.15	-9.47	-10.14	-10.87
2	0.09	C*	5.81	5.99	6.38	6.83	7.08	7.34	7.62	8.23	8.91
		C**	-5.23	-5.38	-5.70	-6.06	-6.26	-6.46	-6.68	-7.15	-7.67
4	0.05	C*	3.56	3.66	3.89	4.14	4.28	4.43	4.59	4.93	5.31
		C**	-3.33	-3.42	-3.62	-3.84	-3.96	-4.09	-4.23	-4.52	-4.84

η = -.5

γ	$\frac{1+\eta}{\gamma-\eta}$		α: **0**	**0.001**	**0.003**	**0.005**	**0.006**	**0.007**	**0.008**	**0.01**	**0.012**
0.25	0.67	C*	20.02	20.36	21.31	22.27	22.65	22.93	23.08	23.01	22.57
		C**	-12.63	-12.79	-13.19	-13.58	-13.74	-13.85	-13.92	-13.90	-13.73
0.5	0.50	C*	15.56	15.84	16.51	17.23	17.56	17.86	18.11	18.41	18.40
		C**	-10.74	-10.88	-11.21	-11.55	-11.71	-11.86	-11.98	-12.12	-12.13
1	0.33	C*	10.83	11.04	11.51	12.03	12.30	12.57	12.83	13.32	13.72
		C**	-8.28	-8.40	-8.67	-8.97	-9.12	-9.27	-9.42	-9.69	-9.91
2	0.20	C*	6.79	6.92	7.21	7.53	7.71	7.89	8.07	8.46	8.87
		C**	-5.69	-5.79	-5.99	-6.21	-6.33	-6.45	-6.58	-6.83	-7.09
4	0.11	C*	3.90	3.97	4.14	4.32	4.41	4.51	4.62	4.84	5.08
		C**	-3.51	-3.57	-3.70	-3.85	-3.92	-4.00	-4.09	-4.26	-4.45

Note: C* = consumption gain needed to compensate for tax distortion (starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)

Table 12: Welfare Losses from Proportional Income Tax, f(P) = P

η = -.75

γ	$\frac{1+\eta}{\gamma-\eta}$		α: 0	0.001	0.003	0.005	0.006	0.007	0.008	0.01	0.012
0.25	0.25	C*	13.40	15.06	19.21	24.56	27.76	31.33	35.33	44.70	56.16
		C**	-10.66	-11.75	-14.31	-17.33	-18.99	-20.75	-22.60	-26.53	-30.69
0.5	0.20	C*	11.25	12.55	15.68	19.66	22.01	24.64	27.57	34.41	42.76
		C**	-9.26	-10.16	-12.25	-14.71	-16.08	-17.54	-19.09	-22.43	-26.05
1	0.14	C*	8.56	9.45	11.57	14.21	15.75	17.46	19.36	23.79	29.17
		C**	-7.35	-8.03	-9.58	-11.40	-12.43	-13.52	-14.70	-17.28	-20.15
2	0.09	C*	5.81	6.36	7.63	9.19	10.09	11.08	12.18	14.72	17.79
		C**	-5.23	-5.68	-6.71	-7.92	-8.60	-9.33	-10.12	-11.88	-13.89
4	0.05	C*	3.56	3.86	4.56	5.40	5.88	6.40	6.98	8.31	9.89
		C**	-3.33	-3.60	-4.21	-4.93	-5.33	-5.77	-6.24	-7.30	-8.53

η = -.5

γ	$\frac{1+\eta}{\gamma-\eta}$		α: 0	0.001	0.003	0.005	0.006	0.007	0.008	0.01	0.012
0.25	0.67	C*	20.02	22.71	29.73	39.23	45.06	51.71	59.27	77.46	100.47
		C**	-12.63	-13.86	-16.71	-19.98	-21.75	-23.58	-25.47	-29.36	-33.33
0.5	0.50	C*	15.56	17.49	22.30	28.61	32.42	36.74	41.62	53.27	67.86
		C**	-10.74	-11.75	-14.08	-16.78	-18.26	-19.82	-21.46	-24.92	-28.57
1	0.33	C*	10.83	12.05	14.98	18.71	20.93	23.43	26.23	32.87	41.14
		C**	-8.28	-9.03	-10.75	-12.76	-13.88	-15.07	-16.34	-19.09	-22.11
2	0.20	C*	6.79	7.46	9.05	11.02	12.18	13.47	14.90	18.27	22.43
		C**	-5.69	-6.18	-7.30	-8.62	-9.36	-10.16	-11.02	-12.92	-15.07
4	0.11	C*	3.90	4.25	5.05	6.02	6.58	7.20	7.88	9.46	11.38
		C**	-3.51	-3.80	-4.45	-5.22	-5.65	-6.12	-6.62	-7.75	-9.06

Note: C* = consumption gain needed to compensate for tax distortion (starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)

Table 13: Summary Results for Welfare Losses From Proportional Income Taxes

	γ	Uncompensated elasticity	Compensated elasticity	Welfare loss (C*) f(P) = log(P)	f(P) = $2P^{.5}$	f(P) = P
η = -.75	**0.25**	0.205	0.811	13.35	19.03	35.33
	0.5	0.176	0.698	11.42	15.47	27.57
	1	0.133	0.530	8.92	11.46	19.36
	2	0.088	0.350	6.22	7.62	12.18
	4	0.052	0.206	3.87	4.59	6.98
η = -.5	**0.25**	0.532	1.054	12.16	23.08	59.27
	0.5	0.445	0.884	11.38	18.11	41.62
	1	0.318	0.633	9.30	12.83	26.23
	2	0.197	0.392	6.43	8.07	14.90
	4	0.110	0.220	3.88	4.62	7.88

Note: All results are for α = .008. C* = percentage consumption gain needed to compensate for tax distortion (starting from proportional tax world).