

Network Capital

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Abstract

This paper explores the problem of assembling capital for projects. It can be difficult to assemble capital, when it is disaggregated, for a project that exhibits increasing returns. Small investors may be reluctant to participate, as they may question the ability of the project owner to raise the additional capital he requires. This suggests the possibility that agents with blocks of capital (capital that is already aggregated) might earn rents. Similarly, agents with “network capital” — that is, an ability to aggregate the capital of others — may earn rents. In this paper, we develop a simple theory of the rents attached to capital assembly, and discuss the implications for a range of issues from investment, to growth, to inequality.

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1 Introduction

This paper explores the problem of assembling capital for projects. Under the usual economic assumption of decreasing-returns-to-investment, this problem does not arise; but when there are increasing returns over some range, investors may only be willing to invest in projects when they believe others are willing to do so. In such instances, assembling capital (or coordinating investors) is a relevant — and often critical — consideration. This paper addresses the issue by viewing the process of assembling capital as part of the equilibrium, and it explores the consequences of capital assembly for a range of features of investment. One striking implication is that certain agents who possess a privileged network position can use their “network capital” to improve overall investment and they receive outside returns for doing so. Our theory also predicts that investors with blocks of capital will serve as anchor investors for projects and earn higher rates of return than small investors.

Our theory speaks to a fundamental aspect of the investment process that existing models fail to address. In contrast to existing theories, which assume surplus maximization, we emphasize the importance of scarce resources — network capital and block capital — for the execution of valuable projects. This implies that these resources earn rents — potentially large ones — in market equilibrium. It also implies that institutions may be important, as they may affect the supply of these scarce resources, and hence the extent to which valuable projects are implemented.

We analyze a model in which a project owner tries to raise capital for a project that exhibits increasing returns over some range. We first show that by making an anchor investment in the project, a large investor with a block of capital can move the project from a “bad” equilibrium with low investment to a “good” equilibrium with high investment. Since a large investor spurs others to invest by making an anchor investment, he need not finance the entire shift to the good equilibrium himself. We characterize the minimum capital block-size needed to effect a shift to the good equilibrium — as well as the rate of return earned on such an investment. Interestingly, by holding a subordinated claim rather than a senior claim (equity or junior debt), a large investor can move the project to the good equilibrium with a smaller block of capital.¹

¹A subordinated claim facilitates coordination; but a large investor may prefer a senior claim if the

We then consider the possibility of a central network actor generating a rent by assembling the capital of small investors into a larger block; and we discuss how this might be micro-founded.

A key goal of the paper is to develop a *simple* approach to the problem of capital assembly and increasing returns that can be applied in a variety of settings. The details of our analysis, while subtle, can largely be ignored once they have been considered. Our formal model is game-theoretic; but it boils down to a model that is essentially price-theoretic.² We present this simpler, price-theoretic treatment in Section 2 and our formal model in Section 3.

There are many “real-world” examples where people earn enormous sums that seem hard to explain with traditional economic theories.

Warren Buffett’s investment in Goldman Sachs provides a good illustration of the power of blocks of capital. In September of 2008, soon after the collapse of Lehman Brothers, Buffett agreed to provide Goldman with \$5 billion of capital. His investment, plus the additional \$2.5 billion Goldman was able to raise from small investors on the back of his investment, helped Goldman weather the financial crisis. The deal was made on very favorable terms to Buffett. Berkshire Hathaway (Buffett’s company) received a 10% annual dividend on its “perpetual preferred” stock, plus warrants to buy \$5 billion of common stock at 8% percent below the previous day’s closing price.³ By comparison, the investors who provided the additional \$2.5 billion dollars did not receive nearly as favorable terms.

The founding of Federal Express — by entrepreneur Fred Smith — provides a good illustration of the role of network capital. Notably, its establishment required significant capital investment upfront. Before the company could even open its doors, it needed to have in place a fleet of jets, a central hub with sorting facilities, and pickup and delivery operations in twenty-five cities. Furthermore, capital was needed to cover the losses the company could expect to run for the first several years (four, as it turned out) while it

project is risky. See the conclusion for further discussion.

²Bulow and Roberts (1989)’s paper on optimal auctions has a similar aim. They develop a rigorous mechanism-design model but show that their results can be reinterpreted in the language of standard micro theory.

³Bary, Andrew, “Warren Buffett Makes an Offer Goldman Sachs Can’t Refuse,” *The Wall Street Journal* 28 September 2008, Retrieved from <http://www.wsj.com>.

built up demand for its service.

Fred Smith relied heavily on his social connections to coordinate investors on the idea of his company. A graduate of Yale, he had been a member of Skull and Bones, where he befriended both George W. Bush and John Kerry; and he established valuable contacts in the airline industry running a business with his stepfather that bought and sold jets. Smith was also a talented communicator and salesman. As one early FedEx employee put it: “Fred turned on the charm in a way that few others can match.”⁴

The success of the company also depended heavily upon Smith’s ability to assemble a top-notch management team; this task involved coordination as well. For instance, FedEx’s initial COO Roger Frock remarked: “How could I even consider joining Fred in his crazy scheme?...I...knew that Art’s [head of air operations] broad vision and mellow personality would be tremendous assets for Federal Express.”⁵

There is empirical support for the idea that social connections yield substantial returns. Hochberg et al. (2007), for instance, find that socially connected venture capital firms do especially well. The VC industry, in general, is characterized by strong network ties among VC firms that typically syndicate their deals with others. Hochberg et al. (2007) find that the “centrality”⁶ of VCs in their network increase their internal rates of return from 15% to 17% for a one standard-deviation increase in centrality. Similarly, they find that the more central a VC firm, the better the performance of its portfolio companies. A one standard deviation change in VC centrality increases the probability that a portfolio company survives its first funding-round from 66.8% to 72.4%. A possible interpretation of their findings is that VC firms provide startups with network capital in exchange for a share of their returns.

There is a large literature on investment, ranging from so-called “Q-theory” to lumpy-investment models (see Akerlof and Holden (2016), footnotes 3 and 4 for some notable references). Relative to those papers, we focus specifically on a setting in which there are increasing returns and we emphasize the strategic aspects of capital assembly.

Increasing returns naturally brings to mind the trade literature on the subject — especially Krugman (1980, 1981), Helpman (1981), and Helpman and Krugman (1985).

⁴Frock (2006), p. 62.

⁵Ibid., p. 95.

⁶In the sense of “eigenvector centrality” (Bonacich (1972)).

These models focus on a different issue from our paper. They assume, in contrast, that the efficient scale can easily be achieved. They explore the tradeoff between efficiency (which is achieved by industries being large) and variety (for which consumers have a preference).

Closer to us, a different strand of papers considers the possibility that increasing returns can generate multiple equilibria. For instance, Murphy et al. (1989) propose this as a reason for poverty traps.⁷

There is also a literature on contribution games, beginning with Admati and Perry (1991). Andreoni (1998) particularly relates to our paper. In a charitable-giving context, he considers the role of a large contributor or government in achieving successful coordination. Relative to these papers, the novel features of our analysis are our focus on an investment context and our examination of the rents associated with playing a pivotal role in coordination.⁸

There is a large literature in corporate finance on the value of controlling blocks and large shareholders (see Grossman and Hart (1980) and Shleifer and Vishny (1986) for early contributions and Becht et al. (2003) for further discussion and references). In these models, the value of large stakes comes from control rights; but there is scant consideration of the coordination problems involved in raising capital.

The most closely related paper is Akerlof and Holden (2016). In that paper, we consider a specific setting in which a networked agent — a “mover and shaker” — can increase aggregate investment and earn a rent. This paper gives a far more general treatment of the capital assembly problem; it is also much simpler. We accomplish this by stripping out informational considerations and focusing squarely on agents’ coordination problem (using risk dominance rather than global games as a refinement concept). Our methodology does not require us to make *any* assumptions about the shape of pro-

⁷By contrast, Romer (1986) considers increasing returns that come from technological rather than pecuniary externalities. Relatedly, Aghion and Howitt (1992) emphasize the fact that technological innovations improve the quality of products, rendering previous, inferior ones, obsolete.

⁸Another related paper is Bernstein and Winter (2012). They consider a setting in which a large player imposes positive externalities on smaller players and earns a rent as a result. An example would be a national brand store’s importance for small stores in a shopping mall. Our setting differs since all players impose externalities on all others (those externalities being proportional to players’ size). Bernstein and Winter (2012)’s argument why the large player earns rents does not apply in our setting. In our theory large players earn rents for a different reason: their coordinating role.

duction functions. It reveals the importance of block capital as well as network capital for coordinating investors. Importantly, we are also able to embed our analysis within a market and study the nature of equilibrium: both how block capital and network capital are deployed and the rents they command.

The remainder of the paper is organized as follows. Section 2 gives the simple price-theoretic treatment of our model. Section 3 develops the model more formally. We first examine the role a large investor can play in assembling capital for a project; we then embed our analysis in a market setting (with multiple projects) and analyze the market equilibrium; finally, we consider the role that networked agents can play and examine the market returns to network capital. Section 4 discusses a set of issues related to network capital such as: how it is acquired and whether agents can invest in it. Section 5 considers some possible objections to our approach and contains some concluding remarks.

2 A simple treatment of the capital assembly problem

Imagine a project owner is trying to assemble capital for a project. When k units of capital are invested in the project, it yields a return $f(k)$. Our theory can handle production functions with any shape; for the purposes of illustration, though, assume $f(k)$ has the shape shown in Figure 1.⁹ The production function in Figure 1 exhibits increasing returns for intermediate values of k and decreasing returns for high and low values of k .

Case 1: Small investors only

Suppose there are many small, risk averse investors, each with only a negligible amount of capital. They can invest in the project or earn a market rate of interest, r_{mkt} .

We will show in the next section that there are two equilibria, one good and one bad; however, the bad equilibrium is likely to prevail. In the bad equilibrium, the project owner obtains k_L units of capital at the market interest rate and receives a payoff of Δ_1 . In the good equilibrium, the project owner receives the surplus-maximizing amount of

⁹For a discussion of how our theory generalizes to $f(k)$ of any shape, see Section 3.4.

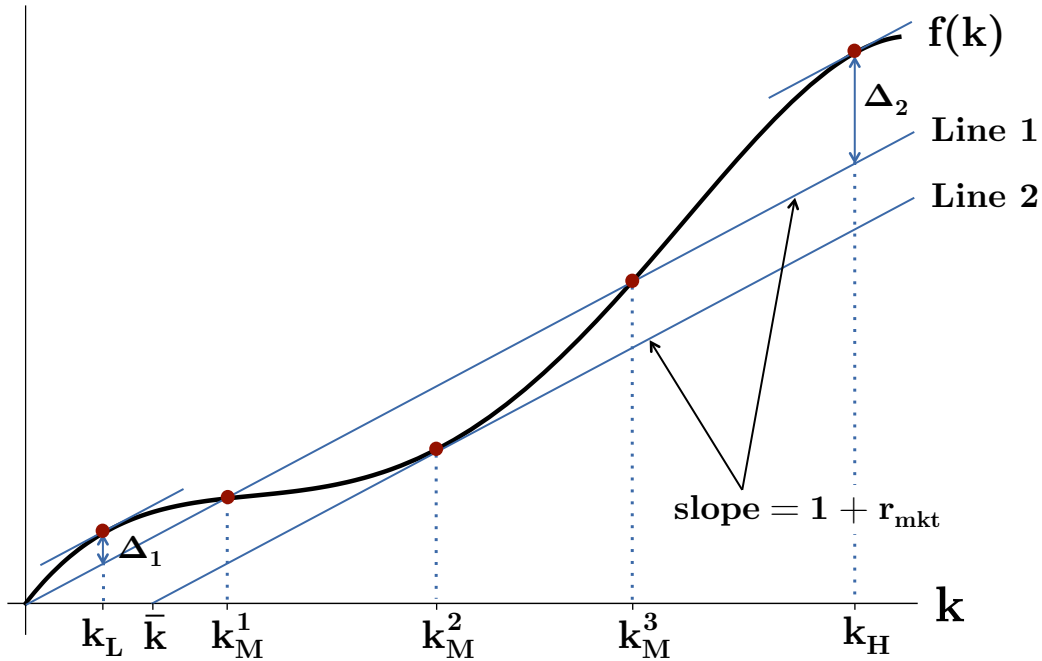


Figure 1 – An example

capital, k_H , at the market interest rate and receives a payoff of $\Delta_2 > \Delta_1$.

Why is the bad equilibrium likely to prevail? Observe that there is a region in which the project is in the “red,” yielding an insufficient return to pay off investors ($f(k) < (1 + r_{mkt})k$). In Figure 1, this is the region between k_M^1 and k_M^3 , in which $f(k)$ dips below Line 1. Investors take a risk when they try to coordinate on lending k_H rather than k_L , since the project may end up in the region in which it is undercapitalized and in the “red.” In game-theoretic terms, the bad equilibrium “risk dominates” the good one. There is a large literature showing that risk dominant equilibria tend to be focal.¹⁰

To summarize, we find that a bad equilibrium, with a k_L -level of investment, is likely to prevail when capital is disaggregated (i.e., investors have only negligible amounts of it).

Case 2: One large investor

Let us assume now that, in addition to small investors, there is a large investor with

¹⁰For notable early contributions see Cooper et al. (1990) and Huyck et al. (1990).

a block of capital of size k_{block} .

If the large investor has enough capital, he can ensure the optimal level of investment (k_H). It is obvious that he can do so if $k_{block} \geq k_H$; but he may be able to bring about the optimal level of investment even if he is unable to fund the entire project. For instance, a block of size $k_M^3 - k_M^1$ is adequate. Small investors are happy to lend when the project is in the “black”; there is only reluctance to lend between k_M^1 and k_M^3 , when the project is in the “red.” A block of size $k_M^3 - k_M^1$ is enough to bridge this gap.

In fact, it turns out that the large investor can bring about the good equilibrium with less capital still. It is sufficient to have a block of size \bar{k} (\bar{k} is graphically represented in Figure 1). Suppose the large investor loans \bar{k} for the project and, additionally, enables the project owner to pay off small investors first. (This could be achieved either by taking junior debt or equity in the project.) Small investors are paid off in this scenario so long as $f(k)$ does not dip below Line 2. $f(k)$ is tangent to Line 2 at k_M^2 but never dips below; hence, small investors are certain to be paid off. Since small investors need not worry about being paid off, they will be willing to provide the project owner with the additional capital he needs to reach the good equilibrium.

Therefore, a large investor with a block of size $k_{block} \geq \bar{k}$ can generate a surplus of size $\Delta_2 - \Delta_1$.

Market rates of return: large versus small investors

Consider next a market setting, with many projects, in which interest rates are endogenous. In a competitive capital market, if block capital is scarce, large investors earn higher rates of return than small investors. Large investors receive, in addition to r_{mkt} , the surplus their blocks help generate.

For example, an investor with a block of size \bar{k} who invests in a project of the type shown in Figure 1 receives $(1 + r_{mkt})\bar{k} + (\Delta_2 - \Delta_1)$.¹¹ He therefore earns a rate of return:

$$\bar{r} = r_{mkt} + \frac{\Delta_2 - \Delta_1}{\bar{k}}.$$

¹¹The project owner receives a payoff of Δ_1 . Because of competition between project owners to obtain block capital, the block investor receives the entire surplus from reaching the good equilibrium ($\Delta_2 - \Delta_1$).

The difference between large investors' and small investors' rates of return is potentially quite significant. A numerical example helps to illustrate. Figure 1 corresponds to a particular numerical example in which $r_{mkt} = 5\%$ and $f(k) = 2.55k - 0.0975k^2 + 0.0016k^3 - 0.0000075k^4$. In the good equilibrium, $k_H = 100$ and $\Delta_2 = 25$; in the bad equilibrium, $k_L = 10$ and $\Delta_1 = 6.775$. The block size needed to reach the good equilibrium is $\bar{k} = 14.881$. It follows that $\bar{r} = 127.5\%$. Therefore, while a small investor earns a return of 5%, an investor with a block of size \bar{k} earns a return of 127.5%.

Network Capital

If investors are networked, connected agents may be able to play a role in assembling capital. A privileged network position, moreover, can be a source of rents. A connected agent may be able to pool the capital of a group of small investors into a block and earn the difference between the block interest rate and r_{mkt} (the non-block rate).

We will say that agents with the ability to pool others' capital have "network capital" and we will index their network capital by the amount of physical capital they are able to assemble. Observe that n units of network capital generates a rent of $(r_{mkt}(n) - r_{mkt}) \cdot n$, where $r_{mkt}(n)$ denotes the market interest rate on a block of physical capital of size n .

3 The formal model

This section develops the model more formally. It is organized along similar lines to Section 2. Sections 3.1 through 3.4 consider a setting in which a project owner is trying to raise capital from investors. We initially assume that there are only small investors; we then show that a large investor can improve the overall level of investment. Section 3.5 moves to a market setting, with many projects, and examines the market equilibrium. Interest rates are endogenous (in contrast to Sections 3.1 through 3.4). Finally, Section 3.6 discusses network capital and proposes one possible micro-foundation.

3.1 Setup

The owner of a project is trying to raise capital and there is a continuum of potential investors ($i \in [0, K]$). In aggregate, potential investors possess K units of capital; each one has an equal, negligible amount.

At time 1, the project owner decides (i) how much capital he will try to raise ($k_P \geq 0$) and (ii) the interest rate ($r_P \geq 0$) he will pay to those who invest in the project.

At time 2, after observing the project owner's choices, potential investors simultaneously decide under what circumstances they are willing to invest in the project. Each investor chooses $a_i(\kappa) \in \{0, 1\}$ for all $\kappa \in [0, k_P]$. $a_i(\kappa) = 1$ indicates that investor i is willing to invest if the project owner has raised κ units of capital at the point he approaches i .

At time 3, the project owner approaches investors in a random order. Agent i becomes an investor in the project if, when approached, he is willing to invest and the project owner has yet to meet his capital target, k_P . Let k denote the total amount of capital raised at time 3.

At time 4, the project yields a return $f(k)$. The project owner receives $f(k) - (1 + r_P)k$ when the project is in the "black" (that is, when $f(k) - (1 + r_P)k \geq 0$) and 0 when the project is in the "red." Agents who invested in the project receive a rate of return r_P when the project is in the "black"; they receive equal shares of $f(k)$ when the project is in the "red," with an associated rate of return $\frac{f(k)}{k} - 1$. Agents who do not invest in the project receive the market rate of interest, r_{mkt} .

The project owner is risk neutral. Investor i 's utility is given by $u(1 + r_i)$, where r_i denotes investor i 's rate of return.¹² u is strictly increasing and weakly concave: $u' > 0$, $u'' \leq 0$. As a tie-breaking rule, we assume, for ease of later exposition, that investors prefer all else equal to choose $a(\kappa) = 1$ when there is a possibility of being approached by a project owner who has raised κ and $a(\kappa) = 0$ otherwise.

We make a set of simplifying assumptions regarding $f(k)$. Under these assumptions, $f(k)$ resembles the production function in Figure 1. Later, we will discuss how our analysis can be generalized. Let $\pi(k) = f(k) - (1 + r_{mkt})k$. We assume:

¹²We use this utility function to ensure that investors care about their negligible gains/losses.

1. $\pi(k)$ is continuous and $\pi(0) = 0$.
2. $\pi(k)$ has its global maximum at $k_H \leq K$ and $\pi(k_H) = \Delta_2$.
3. $\pi(k)$ also has a local maximum at $k_L < k_H$ and $\pi(k_L) = \Delta_1$.
4. $\pi(k) < 0$ if and only if $k_M^1 < k < k_M^3$, where $k_L < k_M^1 < k_M^3 < k_H$.
5. $\pi(k)$ has its global minimum at k_M^2 and $\pi(k_M^2) = -\Delta_3$.

3.2 Analysis

Let us compare two strategies the project owner might follow. Strategy 1: set out to raise k_L at the market rate of interest ($k_P = k_L$ and $r_P = r_{mkt}$). Strategy 2: set out to raise k_H at the market rate of interest ($k_P = k_H$ and $r_P = r_{mkt}$). (We will later discuss whether there might be a third strategy that is preferable to these two.)

First, consider what happens when the project owner follows Strategy 1.

Proposition 1. *Suppose, at time 1, the project owner sets out to raise k_L at interest rate r_{mkt} . In the unique Nash equilibrium of the time-2 subgame, the project owner successfully raises k_L and receives a payoff of Δ_1 .*

The project owner only seeks to raise k_L and the project is in the black for all $k \leq k_L$. Therefore, the project owner has no trouble raising k_L from investors.

Now, consider what happens when the project owner follows Strategy 2.

Proposition 2. *Suppose, at time 1, the project owner sets out to raise k_H at interest rate r_{mkt} . There are two Nash equilibria of the time-2 subgame:*

1. *In one, the project owner only raises k_M^1 and receives a payoff of 0.*
2. *In the other, the project owner successfully raises k_H and receives a payoff of Δ_2 .*

The time-2 subgame is a coordination game with two equilibria. In Equilibrium 1, investors are willing to invest up to the point the project dips into the red ($a_i(\kappa) = 1$ if and only if $\kappa < k_M^1$); this results in the project owner raising k_M^1 . In Equilibrium 2,

investors are willing to invest even when the project is in the red ($a_i(\kappa) = 1$ for all κ); this results in the project owner raising k_H .

Observe that Strategy 1 yields a higher payoff if Equilibrium 1 prevails while Strategy 2 yields a higher payoff if Equilibrium 2 prevails. As we will see presently, Equilibrium 1 risk dominates Equilibrium 2. There is a large literature showing that, in coordination games, risk dominant equilibria tend to be focal. Consequently, the project owner has good reason to select Strategy 1.

Harsanyi and Selten (1988)'s concept of risk dominance captures the idea that certain equilibria in coordination games may be less risky than others. Suppose a 2x2 coordination game has two pure-strategy Nash equilibria, (U, U) and (D, D) . Players may be uncertain whether the other player intends to play U or D . Harsanyi and Selten say that (U, U) risk dominates (D, D) if players prefer to play U when the other player chooses U with probability $\frac{1}{2}$ and D with probability $\frac{1}{2}$.

Harsanyi and Selten's original paper defines risk dominance for 2x2 games only. There are several papers that propose generalizations to games with n players and more than two actions (see, for instance, Morris et al. (1995) and Kojima (2006)). Below, we define a version of risk dominance that applies to a game with a continuum of players; it closely relates to Kojima (2006).¹³

Definition 1. *Let G be a simultaneous-move game with a continuum of players. Suppose there are two Nash equilibria, a^1 and a^2 , and player i receives a payoff $u(a_i, \theta)$ when he plays a_i , a fraction θ of opponents play their equilibrium-1 strategies, and a fraction $1 - \theta$ play their equilibrium-2 strategies. We will say that a^1 is risk dominant if all players strictly prefer their equilibrium-1 strategies to their equilibrium-2 strategies when θ is uniformly distributed on $[0, 1]$:*

$$E[u(a_i^1, \theta) | \theta \sim U[0, 1]] > E[u(a_i^2, \theta) | \theta \sim U[0, 1]] \text{ for all } i.$$

For ease of exposition, this definition focuses on the case where there are two Nash equilibria. However, it easily generalizes (see the Appendix).

¹³Kojima introduces the equilibrium concept of "u-dominance" for a class of games with *pairwise incentive maximizers* ("PIM games"). An action is u-dominant if it is the (unique) best response to a distribution of actions of other players where the number of opponents using an action is uniformly distributed. For symmetric 2x2 games this is equivalent to risk dominance. Our generalization of risk dominance is very similar to this, but the environment we study is not a PIM game.

When θ is uniformly distributed on $[0, 1]$, there is a chance the project will end up undercapitalized and in the red. Consequently, investors' equilibrium-1 strategies (which do not involve investing in projects that are in the red) are safer than their equilibrium-2 strategies (which do). Equilibrium 1 therefore risk dominates Equilibrium 2.

Proposition 3. *Equilibrium 1 risk dominates Equilibrium 2.*

It follows that, if the project owner uses risk dominance as the criterion to assess what will happen at time 2, he prefers Strategy 1 to Strategy 2. A remaining question is whether there might be a Strategy 3 that dominates both Strategies 1 and 2. Clearly, it would not be optimal to offer an interest rate below the market rate since this leads to zero investment. It might be optimal, though, to offer a rate greater than r_{mkt} . Doing so might get agents to overcome their fear of investing in the project when it is in the red. Specifically, Strategy 3 would involve offering an interest rate $\tilde{r} > r_{mkt}$ and seeking to raise $\tilde{k} = \arg \max_k [f(k) - (1 + \tilde{r})k]$.

Proposition 4 (stated below) says that, if agents are sufficiently risk averse, Strategy 1 is optimal. There are two reasons for this result. First, if agents are sufficiently risk averse, no above-market interest rate will induce agents to invest when the project is in the red. Second, even if it is possible to induce agents to invest in the project when it is in the red, it may require paying a high interest rate. If \tilde{r} is large, the project owner's payoff from raising \tilde{k} at rate \tilde{r} will be less than the payoff from following Strategy 1 (Δ_1). In other words, the cost to the project owner of paying the higher interest rate may exceed the benefit.

Proposition 4. *Let $\rho(w) = -\frac{u'(w)}{u''(w)}$ denote investors' Arrow-Pratt coefficient of absolute risk aversion. Suppose the project owner uses risk dominance as the criterion to assess what will happen at time 2. There exists a $\bar{\rho}$ such that the project owner prefers to follow Strategy 1 whenever investors' risk aversion exceeds $\bar{\rho}$ ($\rho(w) > \bar{\rho}$ for all w).*

For the remainder of Section 3, we will focus on the case where Strategy 1 is optimal. We focus on this case for simplicity; but a version of our argument regarding the value of block capital goes through even when Strategy 3 is optimal. In that case, block capital is valuable because it reduces the interest rate the project owner needs to pay to small investors.

3.3 A large investor

Suppose that, in addition to small investors, there is one large investor with a block of capital of size $k_{block} > 0$ and the same utility function as small investors. At time 1, the large investor can make a loan to the project owner. A loan contract between the project owner and the large investor specifies five things:

1. The loan size ($k_{large} \leq k_{block}$).
2. The interest rate (r_{large}).
3. Whether the loan is junior seniority or standard seniority.
4. The point at which the loan is to be made (κ_{large}).
5. The amount of capital the project owner will try to raise from small investors (k_P) and the interest rate he will pay them (r_P).

Points 3 and 4 require further elaboration. We assume the loan can either be junior seniority or standard seniority. If it is junior seniority, the large investor gets paid off after small investors. If the loan is standard seniority, the large and small investors have the same seniority; when the project is in the red, the large investor receives a fraction of $f(k)$ proportional to the amount of capital he loaned ($\frac{k_{large}}{k}$).

κ_{large} denotes the point at which the large investor makes a loan. We assume that the large investor puts k_{large} into the project at the point the project owner has raised κ_{large} from small investors. If the project owner never manages to raise κ_{large} from small investors, the large investor does not put capital into the project and he earns the market rate of interest on k_{block} .

Let $r_{mkt}(k)$ denote the market rate of return on a block of capital of size k . r_{mkt} , the rate of return that can be earned by small investors, is equal to $r_{mkt}(0)$. We assume that the project owner and the large investor engage in Nash bargaining over the contract and have equal bargaining power.

Analysis

If a large investor has sufficient capital, he can help the project owner reach k_H . For instance, if $k_{block} \geq k_H$, the large investor can loan the project owner all the capital he needs ($k_{large} = k_H$).

It is not necessarily surplus-maximizing, though, for the project owner to obtain all of his capital from the large investor. If larger blocks of capital are more expensive than smaller blocks of capital (that is, $r_{mkt}(k_1 + k_2) \cdot (k_1 + k_2) > r_{mkt}(k_1) \cdot k_1 + r_{mkt}(k_2) \cdot k_2$ for all k_1, k_2), it is optimal to obtain as much capital as possible from small investors. If the project owner obtains a block, he will want to obtain the *minimal-size* block needed to reach k_H .

This begs the question: how large a block is needed to reach k_H ? First, suppose the large investor makes a standard-seniority loan. A block of size $k_M^3 - k_M^1$ is sufficient to reach k_H if it is invested after the project owner has raised k_M^1 from small investors ($k_{large} = k_M^3 - k_M^1$ and $\kappa_{large} = k_M^1$). A block investment of this type bridges the region where the project is in the red and small investors are unwilling to invest. If the block size is any smaller, though, it is impossible to reach k_H .

Now suppose the large investor makes a junior-seniority loan. To reach k_H , the block only needs to be large enough to ensure that small investors are paid off. It is easily shown that the minimum block-size required to reach k_H is $\bar{k} = \frac{\Delta_3}{1+r_{mkt}}$ and that $\bar{k} < k_M^3 - k_M^1$. The block can be invested at any point before the project dips into the red ($\kappa_{large} \leq k_M^1$).

We conclude that, if the project owner borrows from the large investor, he will obtain a junior-seniority loan of size \bar{k} . He will obtain such a loan if doing so generates a positive surplus (S). The project owner and the large investor will negotiate an interest rate (r_{large}) that ensures an equal division of surplus. The surplus is equal to:

$$S = (\Delta_2 - \Delta_1) - C - (r_{mkt}(\bar{k}) - r_{mkt}(0)) \cdot \bar{k} \quad (1)$$

$\Delta_2 - \Delta_1$ is the benefit associated with reaching k_H rather than k_L . C denotes the cost of breaking up a block of capital of size k_{block} into two pieces. C is equal to: $k_{block} \cdot r_{mkt}(k_{block}) - \bar{k} \cdot r_{mkt}(\bar{k}) - (k_{block} - \bar{k}) \cdot r_{mkt}(k_{block} - \bar{k})$. The final term of S reflects the

interest premium that must be paid on a block of capital of size \bar{k} .

The following proposition summarizes.

Proposition 5. *Suppose the large investor has a block of capital of size \bar{k} or greater; larger blocks of capital are more expensive than smaller blocks of capital ($r_{mkt}(k_1 + k_2) \cdot (k_1 + k_2) > r_{mkt}(k_1) \cdot k_1 + r_{mkt}(k_2) \cdot k_2$ for all k_1, k_2); and $S > 0$. Then, provided investors' risk aversion exceeds $\bar{\rho}$, in equilibrium:*

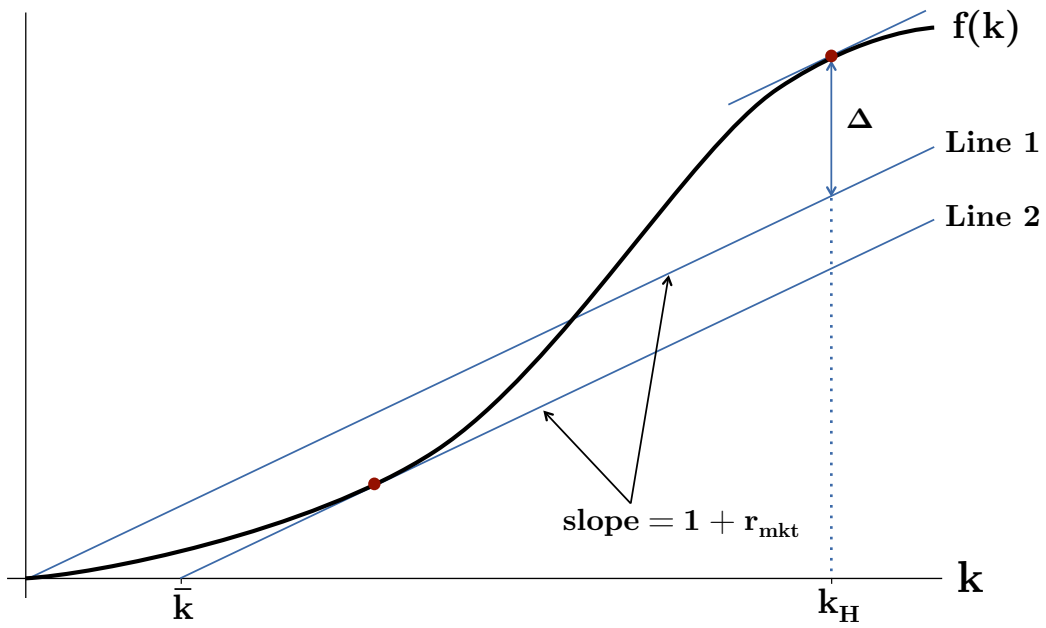
1. *The large investor makes a junior-seniority loan of size \bar{k} to the project owner at an early point ($\kappa_{large} \leq k_M^1$).*
2. *Small investors loan $k_H - \bar{k}$ to the project owner at rate r_{mkt} .*
3. *The large investor's rate of return is: $r_{mkt}(\bar{k}) + \frac{S}{2\bar{k}}$.*
4. *The project owner's payoff is: $\Delta_1 + \frac{S}{2}$.*

3.4 Generalizing

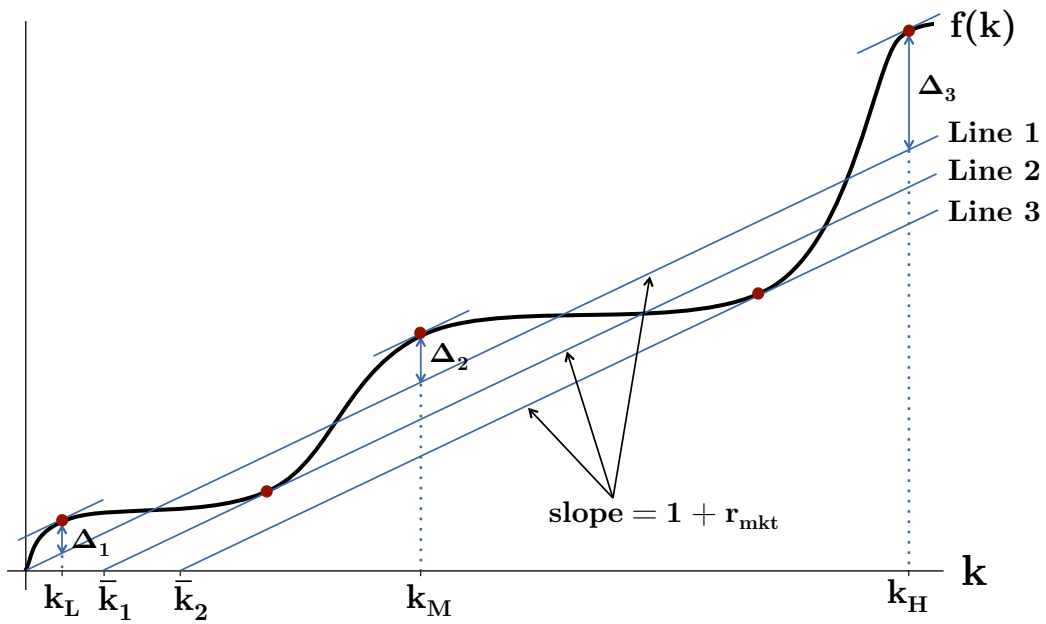
For ease of exposition, our focus thus far has been on production functions resembling the one in Figure 1. Our analysis easily generalizes, though.

For instance, Figure 2a shows a production function that exhibits increasing returns for low values of k rather than intermediate values of k . There is still a “good” equilibrium and a “bad” equilibrium. In the bad equilibrium, *zero* capital is invested in the project. In the good equilibrium, k_H is invested in the project. The good equilibrium generates a surplus of Δ ; a block of capital of size \bar{k} is needed in order to reach it since $f(k)$ dips into the red — down to Line 2 — between $k = 0$ and $k = k_H$.

Figure 2b shows a more complicated production function. $\pi(k) = f(k) - (1 + r_{mkt})k$ has three local maxima — at k_L , k_M , and k_H . The project owner can reach k_L without any help from a block investor because the project is in the black for all $k \leq k_L$. To reach k_M , the project owner must obtain some help from a large investor since the project dips into the red between k_L and k_M . The project dips down to Line 2 and hence a block of size \bar{k}_1 is required to reach k_M . To reach k_H , the project owner must obtain a larger block (of size \bar{k}_2) because the project dips further into the red — down to Line 3 — between k_M and k_H .



(a)



(b)

Figure 2 – Other types of production functions

3.5 Market Equilibrium

Our focus thus far has been on a single project and we have taken market interest rates as exogenous. It is natural at this point to consider a market setting with many projects, in which interest rates are endogenous, and ask what a market equilibrium might look like.

A benchmark case to consider is a market with the following features:

1. There are many different types of projects (where a project's type is defined by its production function); there are many projects of any given type; and each project is owned by a different agent.
2. The supply of block capital is fixed.
3. The supply of non-block capital is increasing in $r_{mkt}(0)$.
4. Breaking up blocks of capital is surplus-destroying.

What can we say about the market equilibrium? First, larger blocks of capital will be more expensive than smaller blocks of capital in the following sense:

$$r_{mkt}(k_1 + k_2) \cdot (k_1 + k_2) > r_{mkt}(k_1) \cdot k_1 + r_{mkt}(k_2) \cdot k_2 \text{ for all } k_1, k_2.$$

This follows from the fact that blocks can always be broken up as well as our assumption that breaking up blocks is surplus-destroying (Assumption 4).

Second, given that larger blocks are more expensive than smaller blocks, Proposition 5 implies that, when project owners obtain blocks, they will obtain the minimum-size blocks they need to effect a shift of investment-level (say, from an investment level of k_L with surplus Δ_1 to an investment level of k_H with surplus Δ_2).

Third, if a block of capital of size \bar{k} is used in equilibrium to increase the level of investment in a project from k_L to k_H :

$$r_{mkt}(\bar{k}) = r_{mkt}(0) + \frac{\Delta_2 - \Delta_1}{\bar{k}}, \quad (2)$$

where $\Delta_2 - \Delta_1$ denotes the surplus associated with a shift from an investment level of k_L

to an investment level of k_H . The logic is as follows. Block capital is scarce (Assumptions 1 and 2) so interest rates will be bid up to the point where S (the surplus from investing the block) is equal to zero. Additionally, blocks do not get broken up in equilibrium given Assumption 4, so $C = 0$. When $S = 0$ and $C = 0$, Equation 1 implies Equation 2.

Finally, blocks will be deployed in equilibrium on the projects that maximize the size of the associated surplus ($\Delta_2 - \Delta_1$). Given the scarcity of block capital, many projects will be undercapitalized in equilibrium. Furthermore, depending upon block interest rates, a project of the type shown in Figure 2b might be funded up to k_M (rather than k_L or k_H).¹⁴

As a final note: Assumptions 1-4 are clearly strong and it is important to remember that they are only meant to serve as a benchmark. In particular, one could imagine settings where there are relatively few projects or where block capital is abundant. In such a setting, $\Delta_2 - \Delta_1$ might be partially or wholly captured by the project owner.

3.6 Network Capital

If investors are networked, a central network actor may be able to bring together a group of small investors and get them to act collectively — like a large investor. We will say that a central network actor possesses n units of network capital if he is able to bring together a group of small investors with n units of physical capital. A block of network capital of size n is a substitute for a block of physical capital of size n and hence should earn an equivalent rent in a market equilibrium. Specifically, the rent on a block of network capital of size n should be $(r_{mkt}(n) - r_{mkt}(0)) \cdot n$.

While some readers may be happy to “black box” the concept of network capital, others may be interested in understanding the process by which a central network actor can assemble investors and generate a rent. We propose one possible micro-foundation of network capital below.

¹⁴The owner of a project of the type shown in Figure 2b will base his decision of how much capital to obtain on the interest premiums on blocks of capital of size \bar{k}_1 and \bar{k}_2 . If the project owner obtains k_L units of capital, his payoff is $f(k_L) - (1 + r_{mkt}(0))k_L$. If the project owner obtains k_M (k_H) units, his payoff is $f(k_M) - (1 + r_{mkt}(0))k_M - (r_{mkt}(k_M) - r_{mkt}(0))\bar{k}_1$ ($f(k_H) - (1 + r_{mkt}(0))k_H - (r_{mkt}(k_H) - r_{mkt}(0))\bar{k}_2$). The project owner will choose the level of funding so as to maximize his payoff.

A Formal Model

Suppose there is a continuum of small investors ($i \in [0, K]$). In aggregate, they possess K units of capital and each one has an equal, negligible amount. There are no large investors. A network, g , exists among the investors; $g_{ij} = 1$ denotes that investors i and j are connected and $g_{ij} = 0$ denotes that investors i and j are not connected.

Investors play a game with the following timing. At time 1, each investor with at least one connection randomly selects one person to whom he is connected and listens to a take-it-or-leave-it offer from him. Let S_i denote the set of investors who listen to offers from i .

At time 2, investors given the opportunities to make offers make their offers simultaneously. Investor i offers investor $j \in S_i$ an interest rate $r_{offer}^i(j)$ in exchange for allowing i to act as j 's proxy and invest j 's capital on his behalf.

At time 3, each investor who receives an offer simultaneously decides whether to accept. Investors prefer to accept if they are otherwise indifferent. This results in each investor having some capital he is responsible for investing (possibly a combination of his own capital and the capital of others for whom he serves as proxy). Let k_i denote the amount of capital investor i is responsible for investing.

At time 4, each investor i invests k_i in the market and generates a return $R_i = (1 + r_{mkt}(k_i)) \cdot k_i$. Investor i then pays the promised rate-of-return — provided he is able to do so — to those for whom he served as proxy. If he is unable, those for whom he served as proxy equally divide R_i .

Analysis

Let n_i denote the expected mass of investors who receive offers from i . n_i is a function of the network topology (g). In a star network, for instance, $n_i = K$ for the investor at the center of the star and $n_i = 0$ for other investors.

We can think of n_i as investor i 's network capital. Proposition 6 characterizes the return investor i earns on n_i .

Proposition 6. *In equilibrium, investor i receives a monetary payoff of $(r_{mkt}(n_i) - r_{mkt}(0)) \cdot n_i$ with probability 1.*

Proposition 6 is intuitive. In equilibrium, investor i assembles a block of capital of size n_i by promising to pay small investors $r_{mkt}(0)$; he earns $r_{mkt}(n_i)$ on the block; and he pockets the difference between $r_{mkt}(n_i)$ and $r_{mkt}(0)$ $((r_{mkt}(n_i) - r_{mkt}(0)) \cdot n_i)$.¹⁵

It is worth making two comments. First, when investors are more connected, the supply of network capital is not necessarily greater. For instance, the star network maximizes the supply of network capital (since it aggregates investors' capital into a single block). In contrast, in a fully connected network (with all investors linked to all other investors), $n_i = 0$ for all i . It is harder to aggregate capital in a fully connected network because there is no investor in a central position. This result is analogous to Calvo-Armengol and de Marti (2009)'s finding that adding links to a network can impede coordination.

Second, for simplicity, we assumed that all investors in the network have negligible amounts of capital. More generally, the rent investor i earns from his network position depends on whom he assembles capital from as well as how much he assembles. For instance, assembling a block of size n_i from small investors yields a rent of size $(r_{mkt}(n_i) - r_{mkt}(0)) \cdot n_i$ whereas assembling a block of size n_i from two large investors with blocks of size $\frac{n_i}{2}$ yields a rent of size $(r_{mkt}(n_i) - r_{mkt}(\frac{n_i}{2})) \cdot n_i$. The rent is lower in the latter case because block-holders demand a higher interest rate than small investors $(r_{mkt}(\frac{n_i}{2}) \geq r_{mkt}(0))$.

4 Discussion

A large economic literature — pioneered by Mincer (1958) and Becker (1962) — observes that a variety of seemingly disparate activities (for example: on-the-job training or college education) can profitably be thought of as investments in human capital and analyzed using a cohesive framework. This literature has analyzed issues such as: the economic returns to human capital, the optimal level of investment, the incentive of firms to provide it, whether human capital depreciates, and its role in understanding inequality and economic growth.

¹⁵Note that it might be optimal for investor i to offer a higher interest rate than $r_{mkt}(0)$ to an investor j who has a block of network capital himself ($n_j > 0$); however, this has no effect on the return i earns on n_i because there are at most a finite number of investors with blocks of network capital.

In this paper we develop an analogous concept: network capital. We will now make a few remarks.

1. People can invest in network capital, just as they can invest in human capital.¹⁶ Business schools are a notable example. According to *The Economist*: “Business school gives one many advantages...but perhaps the most important of all is a network of other successful people.”¹⁷ Networking is a huge focus of MBA students’ time, and some schools (e.g. London Business School) even offer workshops on how to do so effectively.¹⁸
2. Cultural background and upbringing — which sociologists often refer to as “cultural capital” — affect one’s ability to form ties (see Bourdieu and Passeron (1977)). In this sense, one’s background affects one’s ability to invest in network capital. Therefore, one can think of “cultural capital” as a precursor to network capital.
3. Because relationships need to be tended to, network capital can depreciate over time, in a similar manner to physical and human capital.
4. People can invest in network capital, but they can also have network capital conferred on them (through the investments of others).
5. It may be particularly valuable to connect to highly-connected people (see Akerlof and Holden (2016) for one rationale). Consequently, some people may become and remain highly connected purely as a result of luck.
6. When agents invest in network capital, there are externalities. Such investments affect the overall supply of network capital — not just an agent’s own supply. This raises the question of appropriate policy interventions or institutions to achieve the socially optimal level of investment.

¹⁶Glaeser et al. (2002) provide empirical support. In contrast to Putnam (1993) who pioneered the study of social capital, Glaeser et al. (2002) focus on the investment decisions of individual actors rather than aggregate group outcomes. Glaeser et al. (2002) find, for instance, that individuals put more effort into making social connections in occupations where social skills are more important and that mobility reduces people’s effort at making connections.

¹⁷“Network effects: A ranking of business schools’ alumni,” *The Economist* 6 February 2015, Retrieved from <http://www.economist.com>.

¹⁸Pozniak, Helena, “MBAs and the power of networking,” *The Independent* 10 April 2013, Retrieved from <http://www.independent.co.uk>.

7. If groups A and B rarely interact, an agent with connections to both possesses a rare type of network capital; by virtue of its scarcity, such network capital is likely to command a high market price.¹⁹ Sociologists such as Ronald Burt emphasize the importance of agents who connect disparate groups; in his terminology, they bridge “structural holes” (see Burt (2001) for a review).
8. An interesting issue concerns the incentives in a partnership to share connections. On the one hand, hoarding connections increases one’s value to the firm. On the other, hoarding connections reduces the firm’s value because it makes certain people essential, and use of their connections cannot be compelled.²⁰

5 Concluding remarks

We conclude by considering some issues raised by our theory, in particular whether certain alternative model specifications could remove the benefit that accrues to block- or network capital-holders.

First: could project owners and investors write conditional contribution contracts, whereby investors’ capital only goes into a project if the total amount pledged is above a threshold? Such contracts are not common in practice, which suggests that there are theoretical reasons why they are rarely observed. One reason is that it is usually easy to walk away from such pledges. An escrow account might help but such accounts are known to be far from airtight. Furthermore, there is an incentive to wait to contribute to see what other investors will do, which leads to a problem of a “race to the last.” Waiting retains one’s option value; and there is also an informational benefit of waiting.

Second: if the project owner tries to raise k_H but fails, could he put k_L into the project and invest the remainder elsewhere at the market interest rate? Costs of delay in project completion mean that, as a matter of practice, money is invested in projects as it comes in — not held to be invested elsewhere. Project delay may also send a negative signal to

¹⁹Note that in our model, we assume there is only a *single type* of network capital. It is appropriate, however, to think of there being *multiple types* if there are multiple dimension along which agents coordinate. For instance, a project owner might face a problem of coordinating workers as well as investors.

²⁰This resembles the issue raised by Rajan and Zingales (1998). Access to assets (in this case, the connections of the firm) may be preferable to ownership.

investors regarding the project owner's ability to raise capital.²¹

Third: our model assumes that small investors have negligible amounts of capital. If investors instead have small but finite amounts of capital, the "bad" equilibrium can unravel. If the project owner tries to raise k_H , the last investor needed to reach k_H will invest; the second-to-last investor, recognizing this, will invest; by iteration, all investors are prepared to invest. This unraveling argument is fragile, however. For instance, it falls apart if the project's return ($f(k)$) is not strict common knowledge.²²

Fourth: could a large fund play the role of a capital aggregator? As an empirical matter, Vanguard, Fidelity, and other large funds do not invest in big projects like real estate developments — nor do they serve as anchor investors. For moral hazard reasons, there are limits placed on the classes of securities in which they can invest.²³ Large university endowments, like those of Yale and Harvard, that sometimes do make such investments, act like rich individual investors rather than aggregators of funds. They do not allow alumni and donors to co-invest with them, even though they obtain the same kind of returns as block-holders in our model. Private equity and activist hedge funds, by contrast, aggregate capital from a network of dispersed limited partners and make investments where blocks of capital are particularly important — often to control board seats and effect changes to the management team.

Fifth: block investors in our model subordinate their claims to those of small investors (either by taking equity or junior debt in the project). This prediction is stark and may not perfectly reflect what we see in reality. The starkness of this prediction

²¹Furthermore, there might be transaction costs or liquidation costs associated with temporarily investing funds in the market.

²²To illustrate why common knowledge matters, consider a setup as in Section 3.1 with two differences: (1) each investor has a small, finite amount of capital and (2) while each investor knows the value of k_M^3 , k_M^3 (the point where the project moves from "red" back to "black") is not common knowledge. If the project owner has already raised $k \geq k_M^3$, investors will contribute to the project. The last investor needed to reach k_M^3 will also invest in the project given that he can tip the project into the black. However, it does not follow by backward induction that the second-to-last investor needed to reach k_M^3 will be willing to invest. While the second-to-last investor understands that the next investor can tip the project into the black, he does not know whether the next investor will understand this himself (given that k_M^3 is not common knowledge). Hence, the unraveling argument breaks down. One way to think about anchor investments is that they reduce the need for common knowledge.

²³Without limits, the funds manager could take either too little or too much risk from the perspective of the investor. For instance, the manager could invest in almost risk free assets such as government bonds and receive essentially guaranteed carried interest, or gamble with the investors' money in the hope of large upside returns (and hence carried interest), but not take any risk themselves.

is an artifact, though, of our assumption that the project is riskless (i.e., our assumption that $f(k)$ is non-random). Recall that a block investor subordinates his claim in our model because it reduces the amount of capital he needs to invest to bring about the “good equilibrium.” If the project is risky, however, there is an additional consideration. The block investor exposes himself to greater risk if he subordinates his claim. This second consideration might outweigh the first.

Sixth: one might wonder whether it is critical that the good equilibrium never arises in the absence of a block investor. It is not. Our argument regarding the value of block capital only requires that block capital raises the probability of the good equilibrium.

In this paper, we have examined the capital assembly problem, which arises when there are increasing returns to investment. We have argued that holders of block capital play an important role in capital assembly. By serving as anchor investors for projects, they can increase the overall level of investment. Similarly, central network actors are important because they can use their position to pool the capital of small investors into blocks.

The potentially large returns earned by holders of network and block capital have clear implications for income inequality. Our theory also has implications for corporate finance. The problem we study may have a range of further implications. It is common (e.g., in growth theory) to assume that projects/ideas are in short supply. In contrast, the scarce resources in our theory are network and block capital. Our theory therefore shifts the focus from the challenge of generating ideas to the challenge of *implementing* and *executing* them.

6 Appendix

Definition 1 (Generalization). *Let G be a simultaneous-move game with a continuum of players. Suppose there are n Nash equilibria (a^1, a^2, \dots, a^n) and player i receives a payoff $u(a_i, \theta)$ when he plays a_i , and a fraction θ_k of opponents play their equilibrium- k strategies ($k = 1, 2, \dots, n$). We will say that a^1 is risk dominant if all players strictly prefer their equilibrium-1 strategies to*

their other equilibrium strategies when θ is uniformly distributed on the standard $(n-1)$ -simplex:

$$E[u(a_i^1, \theta) | \theta \sim U(\Delta^{n-1})] > E[u(a_i^j, \theta) | \theta \sim U(\Delta^{n-1})] \text{ for all } j \neq 1.$$

Proof of Proposition 1. Suppose the project owner follows Strategy 1 at time 1 (i.e., he chooses $k_P = k_L$ and $r_P = r_{mkt}$). Let us consider the resulting time-2 subgame.

It is clearly an equilibrium for all investors to choose $a(\kappa) = 1$ for all $\kappa < k_P$. Hence, an equilibrium exists in which the project owner raises k_L . Furthermore, this is the unique equilibrium in which the project owner raises k_L since, if the project owner is going to raise k_L , it is optimal for an investor to choose $a(\kappa) = 1$ for all $\kappa < k_P$ (given the tie-breaking rule).

We can prove by contradiction that an equilibrium does not exist in which the project owner raises less than k_L . Suppose the project owner raises $\hat{k} < k_L$ in equilibrium with positive probability. Given that the project is in the “black” for all $k \in [0, k_L]$ and given investors’ tie-breaking rule, investors will all choose $a(\hat{k}) = 1$. Therefore, if the project owner manages to raise \hat{k} , investors always give him additional capital. It follows that the project owner can never raise exactly \hat{k} (which is a contradiction). This completes the proof. \square

Proof of Proposition 2. Suppose the project owner follows Strategy 2 at time 1 (i.e., he chooses $k_P = k_H$ and $r_P = r_{mkt}$). Let us consider the resulting time-2 subgame.

We can prove by contradiction that an equilibrium does not exist in which the project owner raises less than k_M^1 . Suppose the project owner raises $\hat{k} < k_M^1$ in equilibrium with positive probability. Given that the project is in the “black” for all $k \in [0, k_M^1]$ and given investors’ tie-breaking rule, investors will all choose $a(\hat{k}) = 1$. Therefore, if the project owner manages to raise \hat{k} , investors always give him additional capital. It follows that the project owner can never raise exactly \hat{k} (which is a contradiction).

Furthermore, by an analogous argument, an equilibrium does not exist in which the project owner raises $\hat{k} \in [k_M^3, k_H)$ with positive probability.

We can also prove by contradiction that an equilibrium does not exist in which the project owner raises $\hat{k} \in (k_M^1, k_M^3)$ with positive probability. Suppose such an equilibrium exists. Given that the project is in the “red” at \hat{k} , investors’ payoffs are lower

than they would be if they never invested in the project. Hence, investors are not best-responding (which is a contradiction).

To summarize, for all values of \hat{k} except k_M^1 and k^H , we have ruled out that the project owner can raise \hat{k} with positive probability in equilibrium.

Now, suppose the project owner raises k_M^1 with positive probability. Given that the project dips into the red when $k \in (k_M^1, k_M^3)$, investors all prefer to choose $a(k_M^1) = 0$. If investors all choose $a(k_M^1) = 0$, the project owner cannot raise more than k_M^1 . Furthermore, we have already shown that an equilibrium does not exist in which the project owner raises less than k_M^1 with positive probability. Hence, if the project owner raises k_M^1 with positive probability in equilibrium, he raises k_M^1 with probability 1 in equilibrium.

At this point, we have shown that at most two types of equilibria exist: (1) an equilibrium in which the project owner raises k_M^1 with probability 1, and (2) an equilibrium in which the project owner raises k^H with probability 1. Let us now show existence of such equilibria.

It is clearly an equilibrium for all investors to choose $a(\kappa) = 1$ for $\kappa < k_M^1$ and $a(\kappa) = 0$ for $\kappa \geq k_M^1$. This results in the project owner raising k_M^1 . Furthermore, this is the unique equilibrium in which the project owner raises k_M^1 since, if the project owner is going to raise k_M^1 , it is optimal for investors to choose $a(\kappa) = 1$ for $\kappa < k_M^1$ and $a(\kappa) = 0$ for $\kappa \geq k_M^1$ (given their tie-breaking rule).

It is also clearly an equilibrium for investors to choose $a(\kappa) = 1$ for all κ . This results in the project owner raising k^H . Furthermore, this is the unique equilibrium in which the project owner raises k^H since, if the project owner is going to raise k^H , it is optimal for investors to choose $a(\kappa) = 1$ for κ (given their tie-breaking rule). This completes the proof. \square

Proof of Proposition 3. Let a_i^1 denote investor i 's equilibrium-1 strategy and a_i^2 denote investor i 's equilibrium-2 strategy. Investor i 's equilibrium-1 strategy involves choosing $a_i(\kappa) = 1$ if and only if $\kappa < k_M^1$; investor i 's equilibrium-2 strategy involves choosing $a_i(\kappa) = 1$ for all κ . Let $u(a_i^1, \theta)$ denote i 's expected payoff when a fraction θ of other players follow their equilibrium-1 strategies and a fraction $1 - \theta$ follow their equilibrium-2 strategies. We need to show that $E[u(a_i^1, \theta) | \theta \sim U[0, 1]] > E[u(a_i^2, \theta) | \theta \sim U[0, 1]]$ for all i .

Observe that the difference between investor i 's equilibrium-1 and equilibrium-2 strategies is that, in equilibrium 1, investor i chooses $a_i(\kappa) = 0$ for $\kappa \geq k_M^1$ and, in equilibrium 2, investor i chooses $a_i(\kappa) = 1$ for $\kappa \geq k_M^1$. It is therefore sufficient to show that investor i does strictly worse by choosing $a_i(\kappa) = 1$ for $\kappa \geq k_M^1$.

If $0 < \theta < \frac{k_M^3 - k_M^1}{K - k_M^1}$, the project owner raises $k \in (k_M^1, k_M^3)$; investors in this case receive a rate of return below r_{mkt} . $\theta \sim U[0, 1]$ so there is a positive probability that $0 < \theta < \frac{k_M^3 - k_M^1}{K - k_M^1}$. Therefore, if an investor chooses to invest when $\kappa \geq k_M^1$, he receives a rate of return below r_{mkt} with positive probability. Investors therefore strictly prefer to choose $a_i(\kappa) = 0$ for $\kappa \geq k_M^1$. Hence, equilibrium-1 risk dominates equilibrium-2. This completes the proof. \square

Proof of Proposition 4. Suppose the project owner follows a "Strategy 3" of the form $r_P = \tilde{r}$ and $k_P = \tilde{k}$, where $\tilde{r} > r_{mkt}$ and $\tilde{k} = \arg \max_k [f(k) - (1 + \tilde{r})k]$. We will show that, if investors are sufficiently risk averse, Strategy 1 yields a higher payoff to the project owner than Strategy 3.

First, observe that Strategy 3 yields a lower payoff to the project owner than Strategy 1 if $\tilde{k} \leq k_M^3$. Therefore, we can restrict attention to the case where $\tilde{k} > k_M^3$.

Let us examine the time-2 subgame that results from following Strategy 3 at time 1. Following a logic analogous to that given in the proof of Proposition 2, the time-2 subgame has two equilibria. In equilibrium 1, investors supply the project owner with k_M^1 units of capital (choosing $a(\kappa) = 1$ if and only if $\kappa < k_M^1$); in equilibrium 2, investors supply the project owner with \tilde{k} units of capital (choosing $a(\kappa) = 1$ for all κ).

Observe that, in equilibrium 1, the payoff to the project owner is 0. Therefore, the project owner prefers to follow Strategy 1 if equilibrium 1 prevails. We will now show that equilibrium 1 risk dominates equilibrium 2 if investors are sufficiently risk averse (and hence, equilibrium 1 prevails).

To prove risk dominance, we need to show that when investors are risk averse, $E[u(a_i^1, \theta) | \theta \sim U[0, 1]] > E[u(a_i^2, \theta) | \theta \sim U[0, 1]]$ for all i . The difference between investor i 's equilibrium-1 and equilibrium-2 strategies is that, in equilibrium 1, investor i chooses $a_i(\kappa) = 0$ for $\kappa \geq k_M^1$ and, in equilibrium 2, investor i chooses $a_i(\kappa) = 1$ for $\kappa \geq k_M^1$. Therefore, it is sufficient to show that investor i does strictly worse by choosing $a_i(\kappa) = 1$ for $\kappa \geq k_M^1$.

If $0 < \theta < \frac{k_M^3 - k_M^1}{K - k_M^1}$, the project owner raises $k \in (k_M^1, k_M^3)$; investors in this case receive a rate of return below r_{mkt} . On the other hand, if $\theta > \frac{k_M^3 - k_M^1}{K - k_M^1}$, the project owner raises $k > k_M^3$ and investors receive a rate of return above r_{mkt} . Therefore, when $\theta \sim U[0, 1]$, there is both an upside and a downside risk associated with choosing $a_i(\kappa) = 1$ for $\kappa \geq k_M^1$. Provided investors are sufficiently risk averse, though, the downside risk will dominate and investors will prefer to choose $a_i(\kappa) = 0$ for $\kappa \geq k_M^1$. Hence, equilibrium-1 risk dominates when investors are sufficiently risk averse. This completes the proof. \square

Proof of Proposition 5. The proof of Proposition 5 is given in the text of Section 3.3. \square

Proof of Proposition 6. Let n_i denote the expected mass of investors who receive an offer from i . We will show that i 's expected monetary payoff is equal to $(r_{mkt}(n_i) - r_{mkt}(0)) \cdot n_i$.

Suppose $j \in S_i$ is one investor who receives an offer from i and suppose $n_j = 0$. Given that $n_j = 0$, investor j is unable to form a block of capital himself (by serving as a proxy for other investors). Therefore, if j rejects i 's offer, he can only obtain the non-block rate of return on his own capital ($r_{mkt}(0)$). It follows that investor i will only offer j the non-block rate of return ($r_{mkt}(0)$); investor j will accept i 's offer and let i invest on his behalf.

There can be at most a finite number of investors $j \in S_i$ for whom $n_j > 0$. Therefore, i will offer the non-block rate of return ($r_{mkt}(0)$) to all investors he is connected to — except possibly a set of measure zero — and these investors will accept his offer. Hence, with probability 1, i obtains a block of capital of size n_i . From serving as a proxy, he earns the difference between the block rate of return and the non-block rate of return: $(r_{mkt}(n_i) - r_{mkt}(0)) \cdot n_i$.

In addition to earning a payoff from serving as a proxy, investor i receives a return on his own capital (which he may invest himself or hand over to another investor to invest on his behalf). Given that investor i has only a negligible amount of capital, he earns at most a negligible amount from his own capital. Hence, we can ignore this contribution to i 's monetary payoff. This completes the proof. \square

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