When to Drop a Bombshell

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Abstract

At an exogenous deadline, Receiver takes an action, the payoff from which depends on Sender's type. Sender privately observes when a bombshell arrives. Upon arrival, she chooses when to drop it, which starts a public flow of information about her type. Dropping the bombshell earlier exposes it to greater scrutiny, but signals credibility. In all equilibria, Sender delays dropping the bombshell, and completely withholds it with positive probability. Our model provides an explanation for an "October Surprise" effect and generates further predictions about dynamics of information disclosure. We find empirical support for these predictions in the data on US presidential scandals.

Keywords: information disclosure, strategic timing, Bayesian learning, credibility vs. scrutiny.

JEL Classification Numbers: D72, D82, D83.

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1 Introduction

On January 4, 2012 an explosion killed a man in an apartment in the Ukrainian port city of Odessa. Police arrested another occupant. One month later, on February 4, a second man was arrested in connection with the explosion. On February 27—six days before the March 4 Russian presidential election—Russian state controlled television station Channel One broke the story that the two detainees had been part of a plot to assassinate Russian Prime Minister, and presidential candidate, Vladimir Putin. "Channel One said it received information about the assassination attempt 10 days [earlier] but did not explain why it did not release the news sooner."[1]

Political commentators around the world questioned the timing of the disclosure and cast doubt on the allegations themselves. For instance, Dmitri Oreshkin said on Ekho Moskvy Radio: "The timely disclosure of this conspiracy against this leader is a serious addition to the electoral rating of the potential president,"[2] and Danila Lindele wrote on Twitter: "Do I understand correctly that no one believes in the assassination attempt on Putin?"[3]

Two points of this anecdote are noteworthy. First, information about the alleged plot was not released as soon as it was available. Instead, state television dropped the bombshell at a later, strategically-chosen time. Second, voters drew inferences from the timing of the release.

In this paper we analyze a Sender-Receiver game which connects the timing of information release with voters' beliefs prior to elections. Early release of information is more credible, in that it signals that Sender has nothing to hide. On the other hand, such early release exposes the information to scrutiny for a longer period of time—possibly leading to the information being discovered to be false.

This tradeoff is central to the timing of key events leading up to elections. There is a long tradition in US presidential campaigns of scandals being released in the lead-up to the general election, or even important primaries. Gary Hart's infidelities, Bill Clinton's relationship with Gennifer Flowers, Michael Dukakis's granting of weekend release to Willie Horton, and the swift-boat campaign against John Kerry are all notable examples.

The release time is particularly important if scandals can be fabricated. For example, during the 2004 US presidential campaign between George W. Bush and John Kerry, a

[1]The Guardian, February 27 2012: `http://www.theguardian.com/world/2012/feb/27/putin-assassination-plot-denounced`, accessed April 4 2016.

[2]New York Times, February 27 2012: `http://www.nytimes.com/2012/02/28/world/europe/plot-to-kill-vladimir-putin-uncovered.html?r=0`, accessed April 4 2016.

[3]The Guardian, *op cit.*

controversy about Bush's military service was exploding. On September 8, 2004—less than two months before election day—CBS's *60 Minutes II* aired a story supported by four documents concerning Bush's service in the Air National Guard in 1972-3. The documents purported to support the allegations that Bush disobeyed orders in failing to report for duty, and that undue influence was exercised on his superior officers.[4] In the following weeks questions were raised about the authenticity of the documents. On September 20, CBS News reported that their source had lied.

This begs the following question: why did the source not release the documents shortly before election day, when it would be unlikely that they would be discovered to be fabricated in time? Such a late announcement might well have been deemed an "October surprise"[5] and voters would have entertained serious doubts about the authenticity of the documents. Indeed, in the recent Hollywood dramatization of the events, *Truth*, Josh Howard (*60 Minutes II* executive producer) comments "if we go with this [story] we gotta go early. We can't 'October surprise' him."

The same tradeoff between credibility and scrutiny drives the timing of announcements about candidacy, running mates, cabinet members and details of policy platforms. An early announcement exposes the background of the candidate or her team to more scrutiny, but boosts credibility. Beyond announcements, the tradeoff can also determine the timing of policy implementation. For instance, an incumbent may implement policies that are popular in the short run, but pose long-term risks, shortly before a reelection bid. It seems to us that this might provide a rational-agent explanation for the so-called "political business cycle."[6]

In all these situations, (i) biased Sender has information which matters to Receiver; (ii) Receiver must make a choice at a given date; and (iii) Sender privately knows the earliest date at which she can release information to Receiver, but she can choose to release it later. In this paper we introduce and analyze a formal model of precisely these types of dynamic information release problems.

We analyze the credibility-scrutiny tradeoff in a model with three key features: (i) Sender privately knows her binary type, *good* or *bad*, and wants Receiver to take a higher action; (ii) at an exogenous deadline, Receiver chooses his action, which increases in his belief that Sender is good; (iii) Sender privately observes whether and when an opportu-

[4] USA Today published the four documents, along with another two, the day following the broadcast.

[5] The Oxford US English Dictionary: "October Surprise: Any political event orchestrated (or apparently orchestrated) in the month before an election, in the hopes of affecting the outcome" `http://www.oxforddictionaries.com/definition/english/october-surprise`

[6] See Nordhaus (1975) for an early contribution, and Alesina (1987) for the first formal analysis of the phenomenon.

nity to start a public flow of information about her type arrives and chooses when to seize this opportunity.[7] We call this opportunity an *arm* and say that Sender chooses when to *pull* the arm.

In Section 3, we characterize the set of perfect Bayesian equilibria. In all equilibria, bad Sender delays pulling the arm relative to good Sender, despite the fact that pulling the arm has a positive instantaneous effect on Receiver's belief. An immediate implication is that, pulling the arm earlier is more credible in that it induces higher Receiver's belief. Moreover, bad Sender chooses not to pull the arm with strictly positive probability.

We prove that there exists an essentially unique divine equilibrium (Cho and Kreps, 1987).[8] In this equilibrium good Sender *immediately* pulls the arm when it arrives and bad Sender is indifferent between pulling the arm at any time and not pulling it at all. Uniqueness allows us to analyze comparative statics in a tractable way in a special case of our model where the arm arrives according to a Poisson process and pulling the arm starts an "exponential learning process" in the sense of Keller et al. (2005).

We do this in Section 5 and show that the comparative static properties of this equilibrium are very intuitive. Welfare increases with the speed of the learning process and the arrival rate of the arm. A higher probability of good Sender decreases the probability that bad Sender pulls the arm, as Receiver is less likely to believe that Sender is bad, and hence withholding information is less damning. However, this strategic effect does not completely offset the direct effect of the increased probability of good Sender on Receiver's posterior belief, even if no arm is pulled.

We then apply this Poisson model to the strategic release of political scandals in US presidential campaigns. Here, Receiver is the median voter and Sender is a news organization wishing to reduce the incumbent's chances of reelection. At a random time, the news organization may receive some documents implicating the incumbent in a scandal (this corresponds to the arrival of the arm). The news organization has private information about the documents' authenticity and can choose when and whether to run the story (pull the arm). After the story is made public, it becomes the subject of further scrutiny. Therefore, the median voter gradually learns about the authenticity of the documents. If, at the time of the election, the median voter believes the documents to be authentic, the incumbent's chances of reelection are grim.

We show that fabricated scandals are only released sufficiently close to the election.

[7]In Section 4, we generalize the model in several directions allowing for more general utility functions, for Sender to be imperfectly informed, for Sender's type to affect when the arm arrives, and for the deadline at which Receiver takes an action to be stochastic.

[8]The equilibrium is essentially unique in the sense that the probability with which each type of Sender pulls the arm at any time t is uniquely determined.

Also, a higher prior belief that the incumbent is good increases the probability of a scandal being released, provided that the incumbent has a high approval rating. An intuition for this is that the opposition media outlet optimally resorts to fabricated scandals when the incumbent is so popular that only a scandal could undermine her successful reelection. We also show that fewer scandals are released when voters apply more scrutiny to them and when other events make air time scarce. These results are consistent with a number of empirical regularities in US presidential elections.

Perhaps more importantly, we make predictions about the time pattern of campaign events. We show that for a broad range of parameters the probability of release of scandals (authentic or fabricated) is U-shaped, with scandals concentrated towards the beginning and the end of an electoral campaign. We confirm this prediction using data on the release of US presidential scandals and show that presidents are more likely to be alleged to be involved in a scandal at the beginning of their term and just before they are up for reelection. To the best of our knowledge, this is the first empirical evidence about the strategic timing of political scandals relative to the date of elections and a first direct evidence of an "October Surprise" effect.[9] Furthermore, the probability that a released scandal is fabricated increases with the release time. Therefore, the immediate impact on Receiver's belief is generally single-peaked. Consistent with this result, we show that the immediate impact of scandals on the President's approval rate is smaller for scandals released at the beginning of his term and just before he is up for reelection. Nonetheless, the probability that a fabricated scandal is released is single peaked over time, as is the probability that a scandal is revealed to be fabricated. Interestingly, the peak need not be toward the end of the campaign, contrary to what the "October Surprise"-logic would suggest.

1.1 Related Literature

Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) pioneered the study of verifiable information disclosure and established the *unraveling result*: if Sender's preferences are common knowledge and monotonic in Receiver's action (for all types of Sender) then Receiver learns Sender's type in any sequential equilibrium. Dye (1985) first pointed out that the unraveling result fails if Receiver is uncertain about Sender's *information endowment*.[10] When Sender does not disclose information, Receiver is unsure as to why, and thus cannot conclude that the non-disclosure was strategic, and hence does not "assume

[9] See Nyhan (2015) for a recent review.

[10] See also Shin (1994), Jung and Kwon (1988), and Dziuda (2011). The unraveling result migh also fail if disclosure is costly (Jovanovic, 1982) or information acquisition is costly (Shavell, 1994).

the worst" about Sender's type.

Acharya, DeMarzo and Kremer (2011) and Guttman, Kremer and Skrzypacz (2013) explore the strategic timing of information disclosure in a dynamic version of Dye (1985).[11] Acharya et al. (2011) focus on the interaction between the timing of disclosure of private information relative to the arrival of external news, and clustering of the timing of announcements across firms. Guttman et al. (2013) analyze a setting with two periods and two signals and show that, in equilibrium, both *what* is disclosed and *when* it is disclosed matters. Strikingly, the authors show that later disclosures are received more positively.

All these models are unsuited to study either the credibility or the scrutiny sides of our tradeoff, because information in these models is verified instantly and with certainty once disclosed. In our motivating examples, information is not immediately verifiable: when Sender releases the information, Receiver only knows that "time will tell" whether the information released is reliable. To capture this notion of partial verifiability, we model information as being verified stochastically over time in the sense that releasing information starts a learning process for Receiver akin to processes in Bolton and Harris (1999), Keller, Rady and Cripps (2005), and Brocas and Carrillo (2007). In contrast to these papers, in our model Sender is privately informed and she chooses when to start rather than stop the process.[12]

Our application to US presidential scandals also contributes to the literature on the effect of biased media on voters' behavior (e.g., Duggan and Martinelli, 2011; Gentzkow and Shapiro, 2006).[13] DellaVigna and Kaplan (2007) provide evidence that biased media have a significant effect on the vote share in US presidential elections. We focus on when a biased source chooses to release information and show that voters respond differently to information released at different times in the electoral campaign.

[11] Shin (2003, 2006) also study dynamic verifiable information disclosure, but Sender there does not strategically time disclosure. A series of recent papers consider dynamic information disclosure with different focuses to us, including: Che and Hörner (2015); Ely, Frankel and Kamenica (2015); Ely (2015); Grenadier, Malenko and Malenko (2015); Halac, Kartik and Liu (2015); Horner and Skrzypacz (forthcoming).

[12] In our model Sender can influence only the starting time of the experimentation process, but not the design of the process itself. Instead, in the "Bayesian Persuasion" literature (e.g., Rayo and Segal, 2010; Kamenica and Gentzkow, 2011) Sender fully controls the design of the experimentation process.

[13] See also Prat and Stromberg (2013) for a review of this literature in the broader context of the relationship between media and politics.

2 The Model

We begin with a benchmark model in which (i) Sender's payoff is equal to Receiver's belief about Sender's type, (ii) Sender is perfectly informed, (iii) Sender's type does not affect when the arm arrives, (iv) the deadline at which Receiver takes an action is deterministic. Section 4 relaxes each of these assumptions and shows that our main results continue to hold.

2.1 Benchmark Model

There are two players: Sender (she) and Receiver (he). Sender privately knows her binary type θ: good ($\theta = G$) or bad ($\theta = B$). Let $\pi \in (0, 1)$ be the common *prior belief* that Sender is good.

Time is discrete and indexed by $t \in \{1, 2, \ldots, T+1\}$. At a deadline $t = T$, Receiver must take an action $a \in \mathbb{R}$. Time $T+1$ combines all future dates after the deadline.

Sender privately observes when an *arm* arrives. The arm arrives to Sender at a random time according to distribution F whose support is $\{1, 2, \ldots, T+1\}$.

If the arm has arrived, Sender can pull it immediately or at any time after its arrival (including after the deadline). Because Sender moves only after the arrival of the arm, it is immaterial for the analysis whether Sender learns her type when the game starts or when the arm arrives.

Pulling the arm starts a learning process for Receiver. Specifically, let τ be the pulling time. If the arm is pulled before the deadline ($\tau \leq T$), Receiver observes realizations of a finite-valued stochastic process

$$L = \{L_\theta(t; \tau), \tau \leq t \leq T\}.$$

The process L can be viewed as a sequence of signals, one per each time from τ to T with the precision of the signal at time t possibly depending on τ, t, and all previous signals. Notice that if the arm is pulled at $\tau = T$, Receiver observes the realization $L_\theta(T, T)$ before taking his action.

It is more convenient to work directly with the distribution of beliefs induced by the process L rather than with the process itself. Let m denote Receiver's *interim belief* that Sender is good upon observing that she pulls the arm at time τ and before observing any realizations of L. Likewise, let s denote Receiver's *posterior belief* that Sender is good after observing all realizations of the process from τ to T. Given τ and m, the process L generates a distribution $H(. \mid \tau, m)$ over Receiver's posterior beliefs s; given τ, m, and θ, the process L generates a distribution $H_\theta(. \mid \tau, m)$ over s. Notice that if the arm is pulled after the deadline ($\tau = T+1$), then the distributions $H_\theta(. \mid \tau, m)$ and $H(. \mid \tau, m)$ assign probability one to $s = m$.

Assumption 1 says that (i) pulling the arm later reveals strictly less information about Sender's type in Blackwell (1953)'s sense and (ii) it is impossible to fully learn Sender's type.

Assumption 1. *(i) For all $\tau, \tau' \in \{1, 2, \ldots, T+1\}$ such that $\tau < \tau'$, $H(. \mid \tau, \pi)$ is a strict mean-preserving spread of $H(. \mid \tau', \pi)$. (ii) The support of $H(. \mid 1, \pi)$ is a subset of $(0, 1)$.*

For example, consider a set of (imperfectly informative) signals $\mathcal{S}$ with some joint distribution and suppose that pulling the arm at τ reveals to Receiver a set of signals $\mathcal{S}_\tau \subset \mathcal{S}$. Assumption 1 holds whenever $\mathcal{S}_\tau$ is a proper subset of $\mathcal{S}_{\tau'}$ for all $\tau' < \tau$.

Sender's and Receiver's payoffs, $v(a, \theta)$ and $u(a, \theta)$, depend on a and θ. We are interested in situations where each type of Sender wishes Receiver to believe that she is good. Formally, for all values of Receiver's posterior belief $s \in [0, 1]$, Receiver's best response function

$$a^*(s) \equiv \arg\max_a \{su(a, G) + (1-s)\, u(a, B)\}$$

is well defined and Sender's payoff is equal to s in that $v^*_\theta(s) \equiv v(a^*(s), \theta) = s$ for $\theta \in \{G, B\}$.

We characterize the set of perfect Bayesian equilibria, henceforth equilibria. Let $\mu(\tau)$ be Receiver's equilibrium interim belief that Sender is good given that Sender pulls the arm at time $\tau \in \{1, 2, \ldots, T+1\}$. Also, let P_θ denote an equilibrium distribution of pulling time τ given Sender's type θ (with the convention that $P_\theta(0) = 0$).

2.2 Discussion

We now pause to interpret key ingredients of our model using our main application—the timing of US presidential scandals in the lead-up to elections. Receiver is the median voter and Sender is a news organization wishing to reduce the incumbent's chances of reelection. At a random time, the news organization may receive some documents implicating the incumbent in a scandal (this corresponds to the arrival of the arm). The news organization has private information about the documents's authenticity and can choose when and whether to run the story (pull the arm). After the story is made public, it becomes the subject of further scrutiny. Therefore, the median voter gradually learns about the authenticity of the documents. If, at the time of the election, the median voter believes the documents to be authentic, the incumbent's chances of reelection are grim.

In this application, Receiver's action is binary. To reconcile this with our continuous-action model, suppose that Sender is uncertain about the ideological position r of the median voter, which is uniformly distributed on the unit interval. If the incumbent is not reelected, the median voter's payoff is normalized to 0. If the incumbent is reelected, the median voter with position r gets payoff $r-1$ if the documents are authentic and payoff r otherwise. Sender gets payoff 0 if the incumbent is reelected and 1 otherwise. Therefore, Sender's expected payoff as a function of posterior belief s is given by

$$v_G^*(s) = v_B^*(s) = \Pr(r \leq s) = s.$$

3 Equilibrium

We begin our analysis by deriving statistical properties of the model that rely only on Receiver being Bayesian. These properties link the pulling time and Receiver's interim belief to the distribution of Receiver's posterior belief. First, from (good and bad) Sender's perspective, keeping the pulling time constant, a higher interim belief results in a higher expected posterior belief. Furthermore, pulling the arm earlier reveals more information about Sender's type. Therefore, from bad (good) Sender's perspective, pulling the arm earlier decreases (increases) the expected posterior belief that Sender is good. In short, Lemma 1 says that credibility is beneficial for both types of Sender, whereas scrutiny is detrimental for bad Sender but beneficial for good Sender.

Lemma 1 (Statistical Properties). *Let $\mathbb{E}[s \mid \tau, m, \theta]$ be the expectation of Receiver's posterior belief s conditional on the pulling time τ, Receiver's interim belief m, and Sender's type θ. For all $\tau, \tau' \in \{1, \ldots, T+1\}$ such that $\tau < \tau'$, and all $m, m' \in (0,1]$ such that $m < m'$,*

1. $\mathbb{E}[s \mid \tau, m', \theta] > \mathbb{E}[s \mid \tau, m, \theta]$ *for* $\theta \in \{G, B\}$;
2. $\mathbb{E}[s \mid \tau', m, B] > \mathbb{E}[s \mid \tau, m, B]$;
3. $\mathbb{E}[s \mid \tau, m, G] > \mathbb{E}[s \mid \tau', m, G]$.

Proof. In Appendix A. □

We now show that in any equilibrium, (i) good Sender strictly prefers to pull the arm whenever bad Sender weakly prefers to do so, and therefore (ii) if the arm has arrived, good Sender pulls it with certainty whenever bad Sender pulls it with positive probability.

Lemma 2 (**Good Sender's Behavior**)**.** *In any equilibrium:*

1. *For all* $\tau, \tau' \in \{1, \ldots, T+1\}$ *such that* $\tau < \tau'$ *and neither* $\mu(\tau) = \mu(\tau') = 0$ *nor* $\mu(\tau) = \mu(\tau') = 1$*, if bad Sender weakly prefers to pull the arm at* τ *than at* τ'*, then* $\mu(\tau) > \mu(\tau')$ *and good Sender strictly prefers to pull the arm at* τ *than at* τ'*;*

2. *For all* $\tau \in \{1, \ldots, T\}$ *in the support of* P_B*, we have* $P_G(\tau) = F(\tau)$*.*

Proof. In Appendix B. □

The proof relies on the three statistical properties from Lemma 1. The key to Lemma 2 is that if bad Sender weakly prefers to pull the arm at some time τ than at $\tau' > \tau$, then Receiver's interim belief $\mu(\tau)$ must be greater than $\mu(\tau')$. Intuitively, bad Sender is willing to endure more scrutiny only if pulling the arm earlier boosts her credibility. Since $\mu(\tau) > \mu(\tau')$, good Sender strictly prefers to pull the arm at the earlier time τ, as she benefits from *both* scrutiny and credibility. Notice that this argument does not imply that good Sender always pulls the arm as soon as it arrives. For example, for any $t \leq T$, there always exists an equilibrium in which good Sender never pulls the arm before or at t (i.e., $P_G(t) = 0$) but always pulls it after t (i.e., $P_G(\tau) = F(\tau)$ for all $\tau > t$).

Next, we show that bad Sender pulls the arm with positive probability whenever good Sender does, but bad Sender pulls the arm later than good Sender in the first-order stochastic dominance sense. Moreover, bad sender pulls the arm strictly later unless no type pulls the arm. An immediate implication is that bad Sender always withholds the arm with positive probability.

Lemma 3 (**Bad Sender's Behavior**)**.** *In any equilibrium,* P_G *and* P_B *have the same supports and, for all* $\tau \in \{1, \ldots, T\}$ *with* $P_G(\tau) > 0$*, we have* $P_B(\tau) < P_G(\tau)$*. Therefore, in any equilibrium,* $P_B(T) < F(T)$*.*

Proof. In Appendix B. □

Intuitively, if there were a time $\tau \in \{1, \ldots, T\}$ at which only good Sender pulled the arm with positive probability, then, upon observing that the arm was pulled at τ, Receiver would conclude that Sender was good. But then, to achieve this perfect credibility, bad Sender would want to mimic good Sender and therefore strictly prefer to pull the arm at τ, contradicting that only good Sender pulled the arm at τ. Nevertheless, bad Sender always delays relative to good Sender. Indeed, if bad and good Sender were to pull the arm at the same time, then Sender's credibility would not depend on the pulling time. But with constant credibility, bad Sender would never pull the arm to avoid scrutiny. Therefore, good Sender must necessarily pull the arm earlier than bad Sender. Notice that Lemma 3 does not imply that good Sender pulls the arm at a faster rate for all times. Indeed, there exist equilibria with $P_G(t) - P_G(t-1) < P_B(t) - P_B(t-1)$ for some $t \leq T$.

We now show that, at any time when good Sender pulls the arm, bad Sender is indifferent between pulling and not pulling the arm. That is, in equilibrium, pulling the arm earlier boosts Sender's credibility as much as to exactly offset the expected cost of longer scrutiny for bad Sender. Thus, Receiver's interim beliefs are pinned down by bad Sender's indifference condition (1) and the aggregation condition (2). The aggregation condition requires that the likelihood ratios of bad and good Sender's arms pulled at various times must average out to the prior likelihood ratio of bad and good Sender.

Lemma 4 (Receiver's Beliefs). *In any equilibrium, for τ in the support of P_G, $\mu(\tau) \in (0,1)$ is uniquely determined by the system of equations:*

$$\int v_B^*(s)\, dH_B(s|\tau, \mu(\tau)) = v_B^*(\mu(T+1)), \tag{1}$$

$$\sum_{\tau \in supp(P_G)} \frac{1-\mu(\tau)}{\mu(\tau)} \left(P_G(\tau) - P_G(\tau-1)\right) = \frac{1-\pi}{\pi}. \tag{2}$$

Proof. In Appendix B. □

We now characterize the set of equilibria. Part 1 of Proposition 1 states that in all equilibria, at any time when good Sender pulls the arm, she pulls it with probability 1 and bad Sender pulls it with strictly positive probability. The probability with which bad Sender pulls the arm at any time is determined by the condition that the induced interim beliefs keep bad Sender exactly indifferent between pulling the arm then and not pulling it at all. Part 2 of Proposition 1 characterizes the set of *divine equilibria* of Banks and Sobel (1987) and Cho and Kreps (1987).[14] In such equilibria, good Sender pulls the arm as soon as it arrives.

Proposition 1 (Equilibrium).

1. *A pair (P_G, P_B) constitutes an equilibrium if and only if P_G and P_B have the same supports, and for all τ in the support of P_G, $P_G(\tau) = F(\tau)$ and*

$$P_B(\tau) = \frac{\pi}{1-\pi} \sum_{t \in supp(P_G)\ s.t.\ t \leq \tau} \frac{1-\mu(t)}{\mu(t)} \left(P_G(t) - P_G(t-1)\right), \tag{3}$$

[14] Divinity is a standard refinement used by the signalling literature. It requires Receiver to attribute a deviation to those types of Sender who would choose it for the widest range of Receiver's interim beliefs. In our setting, the set of divine equilibria coincides with the set of *monotone* equilibria in which Receiver's interim belief about Sender is non-increasing in the pulling time. Specifically, divinity rules out all equilibria in which both types of Sender do not pull the arm at some times, because Receiver's out-of-equilibrium beliefs for those times are sufficiently unfavorable.

where $\mu(\tau)$ is uniquely determined by (1) and (2).

2. *There exists a divine equilibrium. In any such equilibrium, $P_G(\tau) = F(\tau)$ for all $\tau \in \{1, \dots, T\}$.*

Proof. In Appendix B. □

Although there exist a plethora of divine equilibria, in all such equilibria Receiver's beliefs and each type of Sender's pulling probabilities are uniquely determined by (1), (2), $P_G(\tau) = F(\tau)$, and

$$P_B(\tau) = \frac{\pi}{1-\pi} \sum_{t \leq \tau} \frac{1-\mu(t)}{\mu(t)} \left(F(t) - F(t-1)\right) \tag{4}$$

for all $\tau \in \{1, \dots, T+1\}$. In this sense, there exists an essentially unique divine equilibrium.

In the divine equilibrium, Receiver's interim beliefs $\mu(\tau)$ decrease over time and the likelihood ratio of an arm being pulled by bad and good Sender increases over time.

Corollary 1 (Equilibrium Dynamics). *In the divine equilibrium, for all $\tau, \tau' \in \{1, \dots, T+1\}$ such that $\tau < \tau'$, we have $\mu(\tau) > \mu(\tau')$ and*

$$\frac{P_B(\tau) - P_B(\tau-1)}{P_G(\tau) - P_G(\tau-1)} < \frac{P_B(\tau') - P_B(\tau'-1)}{P_G(\tau') - P_G(\tau'-1)}.$$

Proof. By Lemma 4 and part 2 of Proposition 1, bad Sender is indifferent between pulling the arm at any time before the deadline and not pulling the arm at all. Then, by Lemma 2, $\mu(\tau) > \mu(\tau')$. Finally, using $P_G(\tau) = F(\tau)$ and (4), we have

$$\frac{1-\mu(\tau)}{\mu(\tau)} = \frac{1-\pi}{\pi} \frac{P_B(\tau) - P_B(\tau-1)}{P_G(\tau) - P_G(\tau-1)}.$$

□

Pulling the arm boosts credibility in the sense that Receiver's belief at time τ about Sender's type is higher if Sender pulls the arm than if she does not.

Corollary 2 (Belief Dynamics). *Let $\tilde{\mu}(\tau)$ denote Receiver's interim belief that Sender is good given that she has not pulled the arm before or at τ. In the divine equilibrium, for all $\tau, \tau' \in \{1, \dots, T\}$ such that $\tau < \tau'$, we have $\mu(\tau+1) > \tilde{\mu}(\tau) > \tilde{\mu}(\tau')$.*

Proof. Using (4), we have that for all $\tau < T$

$$\begin{aligned}\frac{1-\tilde{\mu}(\tau)}{\tilde{\mu}(\tau)} &= \frac{1-\pi}{\pi}\frac{1-P_B(\tau)}{1-P_G(\tau)} \\ &= \frac{\sum_{t=\tau+1}^{T+1}\frac{1-\mu(t)}{\mu(t)}(F(t)-F(t-1))}{1-F(\tau)} \\ &= \mathbb{E}_F\left[\frac{1-\mu(t)}{\mu(t)} \mid t \geq \tau+1\right].\end{aligned} \tag{5}$$

By Corollary 1, $\mu(t)$ decreases with time, which implies that $\tilde{\mu}(t)$ decreases with time and $\mu(\tau+1) > \tilde{\mu}(\tau)$. □

To understand how primitives of the model affect players' welfare and behavior, in Section 5 we specialize to a Poisson model. In the Poisson model, however, Assumption 1, part (ii), that it is impossible to fully learn Sender's type, fails. Nevertheless, a version of Proposition 1 continues to hold without this assumption. Specifically, there exists $\bar{t} \in \{1, \ldots, T+1\}$ such that Proposition 1 holds for all $\tau \geq \bar{t}$, whereas $\mu(\tau) = 1$ and $P_B(\tau) = 0$ for all $\tau < \bar{t}$.

4 Discussion of Model Assumptions

Before turning to the Poisson model, we discuss how our results change (or do not change) if we relax several of the assumptions made in our benchmark model. We discuss each assumption in a separate subsection. The reader may skip this section without any loss of understanding of subsequent sections.

4.1 Nonlinear Sender's payoff

Our key assumption, which we maintain in this discussion, is that the payoff of both types of Sender is strictly increasing in Receiver's posterior belief, so that both types of Sender want to look good. In the benchmark model, we also assume that Sender's payoff is linear in Receiver's posterior belief: $v_G^*(s) = v_B^*(s) = s$ for all s. In this case, were Sender to be uninformed, she would be exactly indifferent as to when to pull the arm. Thus, our results are driven entirely by the presence of Sender's private information. But in many applications, $v_G^*(s)$ and $v_B^*(s)$ may be different and nonlinear in s, because Sender is not risk-neutral with respect to Receiver's action or because Receiver's optimal action is not linear in his posterior belief.

When Sender's payoff is nonlinear in s, then an uninformed Sender would prefer to pull the arm earlier (later) to increase (decrease) the spread in posterior beliefs s if her payoff is convex (concave) in s. This effect confounds our credibility-scrutiny tradeoff, but some of our analysis extends to this case. In particular, the proof of Proposition 1 (in Appendix B) explcitly allows for good Sender's payoff to be weakly convex and bad Sender's payoff to be weakly concave. In this case, the credibility-scrutiny tradeoff is reinforced and Proposition 1 continues to hold verbatim.

To understand how the shape of the payoff functions $v_G^*(s)$ and $v_B^*(s)$ affects our analysis, we extend the statistical properties of Lemma 1, which describe the evolution of Receiver's posterior belief from an informed Sender perspective. First and not surprisingly, a more favorable interim belief results in more favorable posteriors for all types of Sender and for all realizations of the process. So credibility is beneficial for both types of Sender, regardless of the shape of their payoff functions.

From an uninformed Sender perspective, Receiver's beliefs follow a martingale process (see e.g., Ely et al. (2015)); so pulling the arm earlier results in more spread out posteriors (provided that the interim belief does not depend on the pulling time). We show that from an informed Sender perspective, Receiver's beliefs follow a supermartingale process for bad Sender and a submartingale process for good Sender. Therefore, from bad (good) Sender's perspective, pulling the arm earlier results in more spread out and less (more) favorable posteriors (again provided that the interim belief does not depend on the pulling time). So scrutiny is detrimental for bad Sender if her payoff is not too convex but beneficial for good Sender if her payoff is not too concave. Therefore, for a given process satisfying Assumption 1, Proposition 1 continues to hold if good Sender is not too risk-averse and bad Sender is not too risk-loving.[15]

Lemma $1'$ formalizes the discussed statistical properties, using common (first-order and second-order) stochastic orders and a less common stochastic order, which we call second-convex-order. Formally, distribution H_2 second-convex-order stochastically dominates distribution H_1 if there exists a distribution H such that H_2 first-order stochastically dominates H and H is a mean-preserving spread of H_1.

Lemma $1'$ (Generalized Statistical Properties). *For all $\tau, \tau' \in \{1, \ldots, T+1\}$ such that $\tau < \tau'$, and all $m, m' \in (0,1]$ such that $m < m'$,*

1. *$H_\theta(. \mid \tau, m')$ strictly first-order stochastically dominates $H_\theta(. \mid \tau, m)$ for $\theta \in \{G, B\}$;*
2. *$H_B(. \mid \tau', m)$ strictly second-order stochastically dominates $H_B(. \mid \tau, m)$;*

[15] For the Poisson model of Section 5, Proposition 1 continues to hold for any risk attitude of good Sender and only relies on bad Sender being not too risk-loving.

3. $H_G(. \mid \tau, m)$ *strictly second-convex-order stochastically dominates* $H_G(. \mid \tau', m)$.

Proof. In Appendix A. □

Much less can be said in general if the payoff functions $v_G^*(s)$ and $v_B^*(s)$ have an arbitrary shape. For example, if $v_G^*(s)$ is sufficiently concave, then good Sender can prefer to delay pulling the arm to reduce the spread in posterior beliefs. Likewise, if $v_B^*(s)$ is sufficiently convex, then bad Sender can prefer to pull the arm sooner than good Sender to increase the spread in posterior beliefs. These effects work against our credibility-scrutiny tradeoff and Proposition 1 no longer holds.[16]

4.2 Imperfectly Informed Sender

In many applications, Sender does not know with certainty whether pulling the arm would start a good or bad learning process for Receiver. For example, when Sarah Palin was revealed as McCain's surprise choice for running mate in 2008, McCain's campaign had only cursory knowledge of Ms Palin's character and qualifications.

We generalize our model to allow for Sender to only observe a signal $\sigma \in \{\sigma_B, \sigma_G\}$ about an underlying binary state θ, with $\Pr(\theta = G \mid \sigma_G) > \pi > \Pr(\theta = G \mid \sigma_B)$. The statistical properties of Lemma 1 still hold.

Lemma 1″ (Generalized Statistical Properties). *Let* $\mathbb{E}[s \mid \tau, m, \sigma]$ *be the expectation of Receiver's posterior belief s conditional on the pulling time* τ*, Receiver's interim belief m, and Sender's signal* σ*. For all* $\tau, \tau' \in \{1, \ldots, T+1\}$ *such that* $\tau < \tau'$*, and all* $m, m' \in (0, 1]$ *such that* $m < m'$*,*

1. $\mathbb{E}[s \mid \tau, m', \sigma] > \mathbb{E}[s \mid \tau, m, \sigma]$;
2. $\mathbb{E}[s \mid \tau', m, \sigma_B] > \mathbb{E}[s \mid \tau, m, \sigma_B]$;
3. $\mathbb{E}[s \mid \tau, m, \sigma_G] > \mathbb{E}[s \mid \tau', m, \sigma_G]$.

Proof. In Appendix A. □

These statistical results ensure that credibility is always beneficial for Sender, whereas scrutiny is detrimental for Sender with signal σ_B but beneficial for Sender with signal σ_B. Therefore, all our results carry over.

Moreover, we can extend our analysis to allow for signal σ to be continuously distributed on the interval $[\underline{\sigma}, \bar{\sigma})$, with normalization $\sigma = \Pr(\theta = G \mid \sigma)$. In particular, in

[16]For the special case in which $v_G^*(s) = v_B^*(s) = v^*(s)$ for all s, where $v^*(s)$ is a strictly increasing function, we expect our main insight to hold: bad Sender delays pulling the arm relative to good Sender.

this case, there exists a partition equilibrium with $\bar{\sigma} = \sigma_0 > \sigma_1 > \cdots > \sigma_{T+1} = \underline{\sigma}$ such that Sender $\sigma \in [\sigma_t, \sigma_{t-1})$ pulls the arm as soon as it arrives unless it arrives before time $t \in \{1, \ldots, T+1\}$ (and pulls the arm at time t if it arrives before t).

4.3 Type-Dependent Arrival of the Arm

In many applications, it is more reasonable to assume that the distribution of the arrival of the arm differs for good and bad Sender. This is the case if, for example, Sender is a newly elected politician who, during her campaign, has promised to enact a specific policy. This policy can be of high or low quality, but voters begin to receive information about the quality after the policy is enacted. Both good and bad Sender know their type from the outset. Good Sender is the only player who can enact a high-quality policy, but must wait for the bureaucracy to develop detailed implementation plans before enactment can take place. In this case it may be reasonable to think that good Sender has to wait for the arm to arrive, but bad Sender has the arm from the outset and can delay enactment. Similarly, some scandals may be easy to fabricate at the outset, whereas genuine scandals need time to be discovered.

We generalize the model to allow for different distributions of the arrival of the arm for good and bad Sender. In particular, the arm arrives at a random time according to distributions $F_G = F$ for good Sender and F_B for bad Sender.

The proof of Proposition 1 (in Appendix B) explicitly allows for the arm to arrive (weakly) earlier to bad Sender than to good Sender in the first-order stochastic dominance sense: $F_B(t) \geq F_G(t)$ for all t. This assumption is trivially satisfied if bad Sender has the arm from the outset. More generally, Proposition 1 continues to hold verbatim as long as $F_B(t) \geq P_B(t)$ for all t, where $P_B(t)$ is given by (3). If the arm were to arrive to bad Sender sufficiently slower than to good Sender, such that $F_B(t) < P_B(t)$ for some t, then the full characterization of the set of equilibria is a straightforward but tedious generalization of Proposition 1. In all equilibria, bad Sender would still pull the arm later than good Sender in the first-order stochastic dominance sense, but for some τ such that $F_B(\tau) < F_G(\tau)$ she would do so for a mechanical (rather than strategic) reason. If the arm has arrived, then she would strictly prefer to pull it. But the cumulative probability that bad Sender pulls the arm at or before τ is then given by $F_B(\tau) < P_G(\tau) = F_G(\tau)$.[17]

[17] Notice that the indifference condition (1) would no longer hold at such τ.

4.4 Stochastic Deadline

In the benchmark model, we assume that the deadline T is fixed and common knowledge. In some applications, the deadline T may be stochastic. In particular, suppose that T is a random variable distributed on $\{\underline{T}, \underline{T}+1, \dots, \bar{T}\}$ where time runs from 1 to $\bar{T}+1$. Now the process L has T as a random variable rather than a constant. For this process, we can define the ex-ante distribution H of posteriors at T, where H depends only on pulling time τ and interim belief m. Notice that Assumption 1 still holds for this ex-ante distribution of posteriors for any $\tau, \tau' \in \{1, \dots, \bar{T}+1\}$. Therefore, from the ex-ante perspective, Sender's problem is identical to the problem with a deterministic deadline and all results carry over.

5 Poisson Model: Comparative Statics

We now specialize to a Poisson model. Time is continuous $t \in [0, T]$.[18] The arm arrives to Sender at Poisson rate α. After receiving the arm, each type of Sender chooses when to pull it. If the arm is pulled by bad Sender, a *breakdown* occurs at Poisson rate λ. But if the arm is pulled by good Sender, a breakdown never occurs. At a deadline $t = T$, Receiver takes a binary action $a \in \{0, 1\}$.

Following our discussion in Section 2.2, each type of Sender gets payoff 1 if $a = 1$ and 0 otherwise. Receiver privately knows her type r, uniformly distributed on the unit interval. If Receiver takes action $a = 1$, he gets payoff 0. If he takes action $a = 0$, he gets payoff $r - 1$ if Sender is good and r otherwise. Therefore, Receiver takes action 1 whenever her posterior belief s is greater than r. It follows that Sender's expected payoff is

$$v_G^*(s) = v_B^*(s) = \Pr(r \leq s) = s,$$

and Receiver's expected payoff $u^*(s)$ is given by

$$u^*(s) = \int_s^1 [s(r-1) + (1-s)r]\, dr = \frac{(1-s)^2}{2}.$$

We begin by explicitly characterizing the divine equilibrium. By Proposition 1 and the discussion at the end of Section 3, the divine equilibrium has the following three properties. First, good Sender pulls the arm as soon as it arrives. Second, bad Sender is

[18] Technically, we use the results from Section 3 by treating continuous time as an appropriate limit of discrete time.

indifferent between pulling the arm at any time $t \geq \bar{t} \geq 0$ and not pulling it at all. Third, bad Sender strictly prefers to delay pulling the arm if $t < \bar{t}$. The threshold $\bar{t}$ is uniquely determined by the parameters of the model.

In the Poisson model, equations (1), (2) become

$$\begin{aligned} \frac{\mu(t) e^{-\lambda(T-t)}}{\mu(t) + (1-\mu(t)) e^{-\lambda(T-t)}} &= \mu(T) \text{ for } t \geq \bar{t}, \\ \int_0^T \alpha \frac{1-\mu(t)}{\mu(t)} e^{-\alpha t} dt + \frac{1-\mu(T)}{\mu(T)} e^{-\alpha T} &= \frac{1-\pi}{\pi}. \end{aligned}$$

Combining these two equations with the boundary condition $\mu(t) = 1$ for $t < \bar{t}$ yields the explicit solution $\mu(t)$. This completely characterizes the divine equilibrium.[19]

Proposition 2. *In the divine equilibrium, good Sender pulls the arm as soon as it arrives and Receiver's interim belief that Sender is good given pulling time t is:*

$$\mu(t) = \begin{cases} \frac{\mu(T)}{1-\mu(T)\left(e^{\lambda(T-t)}-1\right)} & \text{if } t \geq \bar{t}; \\ 1 & \text{otherwise,} \end{cases}$$

where $\mu(T)$ is Receiver's posterior belief if the arm is never pulled and

$$\bar{t} = \begin{cases} 0 & \text{if } \pi < \bar{\pi}; \\ T - \frac{1}{\lambda} \ln \frac{1}{\mu(T)} & \text{otherwise,} \end{cases}$$

$$\begin{aligned} \mu(T) &= \begin{cases} \left[\frac{\alpha e^{\lambda T} + \lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-1} & \text{if } \pi < \bar{\pi}; \\ \left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda \pi} e^{\alpha T} + 1\right]^{-\frac{\lambda}{\alpha+\lambda}} & \text{otherwise,} \end{cases} \\ \bar{\pi} &= \left[1 + \frac{\lambda}{\alpha+\lambda}\left(e^{\lambda T} - e^{-\alpha T}\right)\right]^{-1}. \end{aligned}$$

We define the *probability of withholding*, denoted by q, as the probability that bad Sender

[19] In every divine equilibrium, $P_G(t) = F(t)$ for all $t \in [\bar{t}, T]$ and $\mu(t) = 1$ for all $t \in [0, \bar{t}]$. But for each distribution $\hat{P}$ such that $\hat{P}(t) \leq F(t)$ for all $t \in [0, \bar{t})$ and $\hat{P}(t) = F(t)$ for all $t \in [\bar{t}, T]$, there exists a divine equilibrium with $P_G = \hat{P}$. Hereafter, we focus on the divine equilibrium in which $P_G(t) = F(t)$ for all $t \in [0, T]$.

never pulls the arm. By Bayes' rule we have

$$\mu(T) = \frac{\pi e^{-\alpha T}}{\pi e^{-\alpha T} + (1-\pi) q} \tag{6}$$

which yields

$$q = \frac{\pi}{1-\pi} \frac{1-\mu(T)}{\mu(T)} e^{-\alpha T}. \tag{7}$$

Proposition 3 presents comparative statics on equilibrium variables.

Proposition 3. *In the divine equilibrium,*

1. *q and $\bar{t}$ increase with π and λ but decrease with α;*
2. *$\mu(T)$ increases with π but decreases with λ and α.*

Proof. In Appendix C. □

Part 1 says that bad Sender pulls the arm later and withholds with a higher probability if the prior belief about Sender is higher, the arrival rate of the breakdown is higher, and the arrival rate of the arm is lower. The intuition is as follows. If the prior belief that Sender is good is high, bad Sender has a lot to lose in case of a breakdown. Similarly, if the arrival rate of the breakdown is high, pulling the arm is likely to reveal that Sender is bad. In both cases, bad Sender is then reluctant to pull the arm. In contrast, if the arrival rate of the arm is high, good Sender is more likely to pull the arm and Receiver will believe that Sender is bad with high probability if she does not pull the arm. In this case, bad Sender is more willing to pull the arm.

Part 2 says that Receiver's posterior belief about Sender if the arm is never pulled is higher if the prior belief about Sender is higher, the arrival rate of the breakdown is lower, and the arrival rate of the arm is lower. Equation (6) suggests that there are direct and strategic effects of the prior belief and the arrival rate of the arm on Receiver's posterior belief. Holding the probability of withholding q constant, a higher prior belief and a lower arrival rate of the arm improve Receiver's posterior belief about Sender if the arm is never pulled. But the strategic effect works in the opposite direction, because the probability of withholding q increases with the prior belief and decreases with the arrival rate of the arm. Part 2 says that the direct effect always dominates the strategic effect in the Poisson model. Finally, a higher arrival rate of the breakdown worsens Receiver's posterior belief about Sender if the arm is never pulled because it increases the probability of withholding but does not affect the behavior of good Sender.

Proposition 4 presents comparative statics on Receiver's and Sender's expected payoffs.

Proposition 4. *In the divine equilibrium,*

1. *the expected payoff of bad Sender increases with π but decreases with λ and α;*
2. *the expected payoff of good Sender increases with π, λ, and α;*
3. *the expected payoff of Receiver decreases with π but increases with λ, and α.*

Proof. In Appendix C. □

There are both direct and strategic effects of parameters on the equilibrium expected payoffs. Just as in Proposition 3, it turns out that direct effects dominate. Specifically, a higher prior probability that Sender is good increases the expected payoff of Sender but decreases the expected payoff of Receiver; a higher arrival rates of the breakdown and the arm allow Receiver to learn more about Sender and take a more appropriate action, which increases the expected payoffs of Receiver and good Sender, but decreases the expected payoff of bad Sender.

6 The Pattern of Release of Political Scandals

To interpret the comparative statics results, we use our motivating example of the strategic release of scandals before elections. Scandals have marked the tenures of many recent US presidents and have "forced out (or seriously threatened) [. . .] three of the last eight" (Nyhan, 2015). Yet, to the best of our knowledge, no obvious time pattern of release relative to the date of elections has been uncovered for either presidential or congressmen's scandals (Nyhan, 2015; Peters and Welch, 1980; Welch and Hibbing, 1997).

In this section we use our model to derive clear predictions about the pattern of release of political scandals. Using Nyhan's (2015) dataset, we show how our model can help to understand why previous empirical studies have not found convincing evidence of an October surprise effect: a concentration of scandals towards the end of a term and just before an election.

6.1 Occurrence of Scandals

A first group of comparative statics concerns the cumulative probability of scandals released before the election. Proposition 3 says that the probability $P_B(T) = 1 - q$ that bad

Sender pulls the arm before the deadline decreases with the prior π. Thus our model predicts that when voters hold a higher opinion of the president (low π) then the media outlet is more likely to release a fabricated scandal. Notice that this does not imply that the total probability that a scandal is released is higher when voters' prior belief is lower. In fact, the total probability of release of a scandal is given by

$$\begin{aligned} R &\equiv \pi \underbrace{\left(1 - e^{-\alpha T}\right)}_{P_G(T)} + (1-\pi) \underbrace{(1-q)}_{P_B(T)} \\ &= 1 - \frac{\pi e^{-\alpha T}}{\mu(T)}. \end{aligned}$$

As $P_B(T) = 1 - q$ decreases with π, we have two contrasting effects. On one hand, holding the probability of withholding q constant, a marginal increase in π increases the total probability of release R by $P_G(T) - P_B(T)$, which is positive by Lemma 3. This is a direct effect: if voters hold a high opinion of the incumbent (low π), then there are simply fewer authentic scandals. On the other hand, conditional on a fabricated scandal, the probability of release $(1-q)$ decreases with π. This is a strategic effect: if voters hold a higher opinion of the incumbent, the opposition media has greater incentives to release fabricated scandals.

Part 1 of Proposition 5 says that the strategic effect dominates the direct effect when π is sufficiently low.

Proposition 5. *In the divine equilibrium, the total probability that Sender pulls the arm*

1. *is quasiconvex in π: decreases with π if*

$$\pi < \frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T} + \lambda\left(e^{\alpha T} - 1\right)} \in (0, 1)$$

   and increases with π otherwise;

2. *decreases with λ;*

3. *increases with α.*

Proof. In Appendix C. □

Nyhan (2015) and Sowers and Nelson (2015) study what factors determine the likelihood of US presidential scandals. Sowers and Nelson (2015) finds that more scandals involving the incumbent president are released when economic indicators and approval rates suggest that voters approve of the president. From the perspective of the opposition media, this means that the prior belief π is low. Thus, this empirical observation is consistent with our finding in Part 1 of Prediction 1 that the strategic effect can dominate the direct effect.

Nyhan (2015) finds that more scandals involving the incumbent president are released when opposition voters are more hostile to the president. The author conjectures that when opposition voters are more hostile to the president, then they are "supportive of scandal allegations against the president and less sensitive to the evidentiary basis for these claims" (p. 6). This mechanism is therefore consistent with Part 2 of Proposition 5.

Nyhan (2015) also finds that more scandals involving the incumbent president are released when the news agenda is less congested. When the news agenda is congested, the opposition media has less time to devote to investigate the incumbent and air scandals, thus reducing the arrival rate of scandals. This empirical observation is therefore consistent with Part 3 of Proposition 5.

6.2 Timing of Scandals

Our model also provides dynamic predictions about when scandals are released.

Proposition 6. *In the divine equilibrium, the probability density that Sender pulls the arm at time t*

1. *decreases with t from 0 to $\bar{t}$ and is quasiconcave in t on the interval $[\bar{t}, T]$: increases with t if*

$$t \leq T - \frac{1}{\lambda} \ln\left(\frac{\alpha}{\alpha+\lambda} \frac{1+\mu(T)}{\mu(T)}\right)$$

and decreases with t otherwise;

2. *is quasiconvex in t whenever*

$$\frac{\alpha}{\alpha+\lambda} \frac{1+\mu(T)}{\mu(T)} \leq 1.$$

Proof. In the divine equilibrium, the probability density that Sender pulls the arm at time t is given by

$$\begin{aligned} p(t) &= \pi p_G(t) + (1-\pi) p_B(t) \\ &= \begin{cases} \pi \alpha e^{-\alpha t} & \text{if } t < \bar{t} \\ \pi \alpha e^{-\alpha t} + \pi \alpha e^{-\alpha t} \frac{1-\mu(T) e^{\lambda(T-t)}}{\mu(T)} & \text{if } t \geq \bar{t} \end{cases} \end{aligned}$$

where $p_G(t)$ and $p_B(t)$ are the densities for good and bad Sender, respectively. Obviously, if $t < \bar{t}$, then $p(t)$ is decreasing in t. For $t \geq \bar{t}$, differentiating with respect to t we have

$$\frac{dp(t)}{dt} = \pi\alpha e^{-\alpha t}\left[(\alpha+\lambda)e^{\lambda(T-t)} - \alpha\frac{1+\mu(T)}{\mu(T)}\right]$$

which is positive whenever

$$t \leq T - \frac{1}{\lambda}\ln\left(\frac{\alpha}{\alpha+\lambda}\frac{1+\mu(T)}{\mu(T)}\right).$$

We can therefore conclude that $p(t)$ is quasiconcave on the interval $[\bar{t}, T]$. □

As mentioned above, previous empirical studies of US presidential scandals did not find convincing evidence of an October surprise effect. Part (ii) of Proposition 6 says that, for a wide range of parameters, the probability density of scandals is U-shaped: scandals should be more frequent at the beginning of a presidential term and just before the president is up for reelection (see Figure 1a).

We can test this prediction using Nyhan's (2015) data. The data-set contains a weekly binary variable indicating whether a new scandal involving the current US president was first mentioned in the Washington Post during that week, for the period 1977-2008. Although scandals might have first appeared on other outlets, we agree with the author that the Washington Post is likely to have mentioned such scandals immediately thereafter. Therefore we use this variable as a proxy for the date of release of all presidential scandals. As our model concerns scandals involving the incumbent in view of his possible reelection, we focus on all the presidential elections in which the incumbent was a candidate. Therefore we consider only the first term of each president from 1977 to 2008.[20] We consider the first week of January of the year following an election as the *de facto* inauguration date.[21] In all cases, the election was held on the 201st week after this date. We therefore construct the variable *weeks to election* as the difference between 201 and the number of weeks served by the president, with 0 being the week of the election.

Figure 1b depicts the distribution of the first mention of a presidential scandal in the Washington Post as a function of *weeks to election*. The quadratic fit reveals that scandals are more concentrated towards the beginning of the term and when the election is

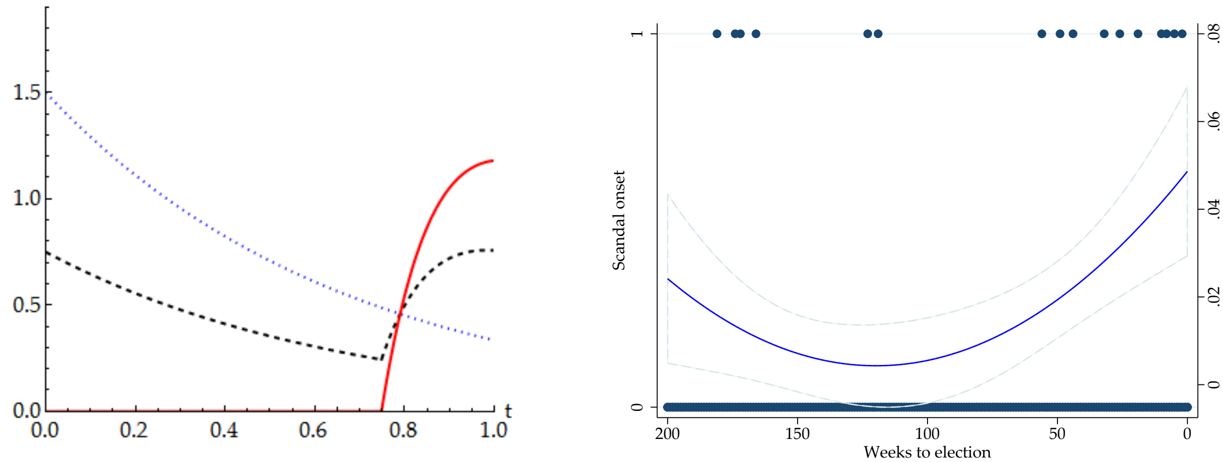
[20]This corresponds to the first terms of five presidents: Jimmy Carter (1976-1980), Ronald Reagan (1980-1984), George H. W. Bush (1988-1992), Bill Clinton (1992-1996), and George W. Bush (2000-2004). Each president run for reelection and three (Reagan, Clinton, and Bush) served two full terms.

[21]Nyhan (2015) does not provide data on scandals involving the president-elect between Election Day and the first week of January of the following year, but it contains data on scandals involving the president-elect between the first week of January and the date of his inauguration: there are no such scandals.

Figure 1: Pulling density and US presidential scandals

(a) Sender's (dashed), good (dotted) and bad (solid) Sender's pulling density; $\alpha = 1.5, \lambda = 6, \pi = .5, T = 1$.

(b) US presidential scandals and weeks to election. Quadratic fit (solid line) and 95% confidence interval (dashed area).



approaching. Column 1 of Table 1 shows the results of a linear regression with the independent variable being whether a scandal was first mentioned by the Washington Post on a determined week. Both linear and quadratic terms for the variable *weeks to election* are statistically significant. The magnitude of the effects we uncover is large when compared to the average probability of a new scandal appearing in the press in a week: 0.016. Towards the beginning of a president's term, any further week reduces the probability of a scandal onset by around 3.2%. This effect reduces to zero just before the mid-point of the term.[22] Thereafter, the probability of a new scandal increases and, during the final election campaign, any new week the probability of a scandal onset increases by around 4.7%.

It is possible that other factors that determine the release of scandals correlate with the president's tenure in office. For example, opposition voters might have a worse opinion of the president when elections get close, because of the effect of the electoral campaign. Or perhaps major events that congest the news agenda are more likely during the middle part of the presidential term. We therefore report in Column 2 of Table 1 the result of a regression including the three other variables that Nyhan (2015) finds to have a strong effect on scandal release, namely a measure of opposition approval, a measure of standardized news pressure, and whether the opposition controlled one or both chambers of Congress. Furthermore, the regression in Column 2 of Table 1 also includes president fixed effects. Both linear and quadratic terms maintain their statistical significance and

[22]On week 82; 119 weeks before the election.

Table 1: Pattern of release of US Predisential scandals

VARIABLES	(1)	(2)
	Scandal onset ($\times 10^4$)	
Weeks to election	-7.40**	-6.97**
	(3.34)	(2.88)
Weeks to election sq.	0.03**	0.03**
	(0.01)	(0.01)
Observations	1,005	1,005
R-squared	0.0087	0.0089
Controls	No	Yes
President fixed effects	No	Yes

Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1. Controls include: divided government, news pressure, lagged opposition approval.

their magnitude is unchanged.[23]

6.3 Further Testable Predictions

As breakdowns are observable, it could be also possible to test how the release time of a scandal affects its likelihood to be discovered to be fabricated before the election. As fabricated scandals are released later than authentic ones, then there are two contrasting effects. On one hand, conditional on being fabricated, a scandal released earlier on is directly more likely to produce a breakdown. On the other hand, fabricated scandals are strategically more likely to be released later. The following proposition says that the strategic effect dominates if the scandal is released sufficiently early (see Figure 2a).

Proposition 7. *In the divine equilibrium, the probability of a breakdown is quasiconcave: increases with the pulling time t if*

$$t < t_b \equiv T - \frac{1}{\lambda} \ln \left(\frac{1 + \mu (T)}{2\mu (T)} \right) < T$$

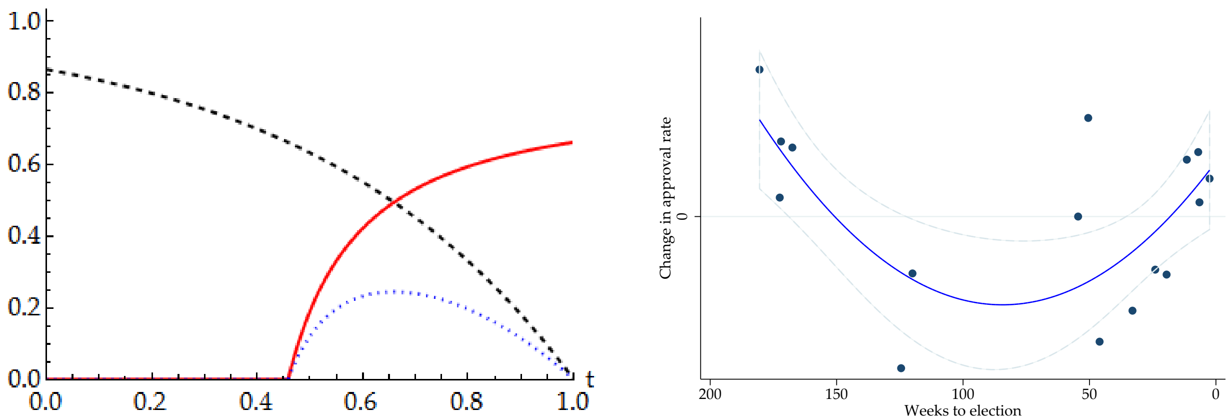
and decreases with t otherwise.

[23]When using data from all elections, even when the incumbent is not running, coefficients have the same sign but lower statistical significance.

Figure 2: Further Testable Predictions

(a) Interim beliefs that Sender is bad (solid); conditional (i.e., $1 - e^{-\lambda(T-t)}$; dashed) and equilibrium (dotted) probability of a breakdown; $\alpha = 1$, $\lambda = 2$, $\pi = .5$, $T = 1$.

(b) The effect of scandals on US President's approval rate. Presidential scandals (dots), quadratic fit (solid line), 95% confidence interval (dashed area).



Proof. In Appendix C. □

Notice that if the interval before the deadline T is sufficiently short or the arrival rate of the arm α is sufficiently small, then t_b is negative and hence the probability of the breakdown monotonically decreases with the pulling time of the arm.

Were it possible to identify ex-post whether a released scandal is fabricated or authentic, then one could test whether fabricated scandals are more likely to be released earlier or later. We can precisely identify the conditions under which fabricated scandals are more likely to be released later.

Proposition 8. *The probability density that bad Sender pulls the arm at time t is quasiconcave: increases with t if*

$$t < t_p \equiv T - \frac{1}{\lambda} \ln \left(\frac{\alpha}{\alpha + \lambda} \frac{1}{\mu(T)} \right)$$

and decreases with t otherwise.

Proof. In Appendix C. □

Notice that if the arrival rate of the arm α is sufficiently small, then $t_p > T$ and hence the probability that bad sender pulls the arm monotonically increases with time (see Figure 1a). When instead α is sufficiently large, then $t_p < 0$ and the probability monotonically decreases with time.

We can also derive the instantaneous impact on beliefs upon the release of a scandal at time $t \leq T$, $\mu(t) - \tilde{\mu}(t)$, where $\tilde{\mu}(t)$ is Receiver's belief at t if Sender has not pulled the arm yet. Corollary 2 says that the instantaneous impact of the release of a scandal is strictly positive for any release time $t < T$. From an empirical perspective, when $\mu(t) - \tilde{\mu}(t)$ is larger, then opinion surveys and voting polls should be more responsive to the release of a scandal. Our model could be used to make predictions about the impact of scandals released at different times. For example, as the election date approaches, $\mu(t) - \tilde{\mu}(t)$ goes to 0, implying that scandals released immediately before an election should have no impact. In contrast, scandals released before the threshold date $\bar{t}$ have greater impact when they are released later. Thus, the instantaneous impact of scandals can be smaller (i.e., less damaging for the president) towards the beginning and the end of an electoral campaign. Figure 2b shows the change in approval rate in each month in which US presidential scandals were first released.[24] Scandals released during the middle part of the President's term appear to be the most damaging to his reputation.

7 Concluding Remarks

This paper analyzes a model in which the timing of information release is driven by the tradeoff between credibility and scrutiny. Our model helps to explain the existing evidence on the frequency of US presidential scandals. The analysis also yields novel predictions about the dynamics of information release. We explore whether these predictions are consistent with available data on the pattern of US presidential scandals and find supporting evidence.

Our model can also be used to deliver normative implications for the design of a variety of institutions. For example, more than a third of the world's countries mandate a *blackout period* before elections: a ban on political campaigns for one or more days immediately preceding elections.[25] We expect Receiver's optimal blackout period to be zero for a wide range of parameter values.

We envision that the credibility-scrutiny tradeoff may be important in other economic applications beyond electoral campaigns. For example, managers can give the board of directors more or less time to examine draft proposals before a board meeting. We hope our model will serve as a useful framework for studying these applications in the future.

[24] Data from Nyhan (2015). Approval rates are available monthly and we do not observe when the approval rate surveys were conducted during the month. Therefore, we use the difference in the approval rate between the month following the scandal and the month preceding it. Alternative measures (current minus preceding and following minus current) also give a U-shaped relationship but the effects are less significant.

[25] The 1992 US Supreme Court sentence *Burson v. Freeman*, 504 US 191, forbids such practices as violations of freedom of speech.

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A Statistical Properties

Proof of Lemma 1. Follows from Lemma 1′. □

Proof of Lemma 1′. *Part 1.* By Blackwell (1953), Assumption 1 with $\tau' = T + 1$ implies that pulling the arm at τ is the same as releasing a finite-valued informative signal y. By Bayes' rule, posterior s is given by:

$$s = \frac{mq\left(y \mid G\right)}{mq\left(y \mid G\right) + \left(1 - m\right) q\left(y \mid B\right)}$$

where $q(y \mid \theta)$ is the probability of y given θ. Therefore,

$$\frac{q(y \mid G)}{q(y \mid B)} = \frac{1-m}{m}\frac{s}{1-s}. \tag{8}$$

Writing (8) for interim beliefs m and m', we obtain the following relation for corresponding posterior beliefs s and s':

$$\frac{1-m'}{m'}\frac{s'}{1-s'} = \frac{1-m}{m}\frac{s}{1-s}$$

which implies that $s' > s$ for $m' > m$; so part 1 follows.

Part 2. By Blackwell (1953), Assumption 1 implies that pulling the arm at τ is the same as pulling the arm at τ' and then releasing an additional finite-valued informative signal y. A signal y is informative if there exists y such that $q(y \mid G)$ is not equal to $q(y \mid B)$. Part 2 holds because for any strictly increasing concave v^*, we have

$$\begin{aligned}
\mathbb{E}\left[v^*(s) \mid \tau, m, B\right] &= \mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)q(y \mid B)}\right) \mid \tau', m, B\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)q(y \mid B)}\right) \mid \tau', s, B\right] \mid \tau', m, B\right] \\
&\leq \mathbb{E}\left[v^*\left(\mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)q(y \mid B)} \mid \tau', s, B\right]\right) \mid \tau', m, B\right] \\
&< \mathbb{E}\left[v^*\left(\frac{s\mathbb{E}\left[\frac{q(y|G)}{q(y|B)} \mid \tau', s, B\right]}{s\mathbb{E}\left[\frac{q(y|G)}{q(y|B)} \mid \tau', s, B\right] + (1-s)}\right) \mid \tau', m, B\right] \\
&= \mathbb{E}\left[v^*\left(\frac{s\sum \frac{q(y|G)}{q(y|B)}q(y \mid B)}{s\sum \frac{q(y|G)}{q(y|B)}q(y \mid B) + 1 - s}\right) \mid \tau', m, B\right] \\
&= \mathbb{E}\left[v^*(s) \mid \tau', m, B\right],
\end{aligned}$$

where the first line holds by Bayes' rule, the second by the law of iterated expectations, the third by Jensen's inequality applied to concave v^*, the fourth by strict monotonicity of v^* and Jensen's inequality applied to strictly concave function $f(z) \equiv sz/(sz + 1 - s)$, the fifth by definition of expectations, and the last by Kolmogorov's axioms.

Part 3. Analogously to Part 2, Part 3 holds because for any strictly increasing convex

v^*, we have

$$\begin{aligned}
\mathbb{E}\left[v^*(s) \mid m, \tau, G\right] &= \mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', m, G\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', s, G\right] \mid \tau', m, G\right] \\
&\geq \mathbb{E}\left[v^*\left(\mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)} \mid \tau', s, G\right]\right) \mid \tau', m, G\right] \\
&> \mathbb{E}\left[v^*\left(\frac{s}{s + (1-s)\,\mathbb{E}\left[\frac{q(y|B)}{q(y|G)} \mid \tau', s, G\right]}\right) \mid \tau', m, G\right] \\
&= \mathbb{E}\left[v^*\left(\frac{s}{s + (1-s)\sum \frac{q(y|B)}{q(y|G)} q(y \mid G)}\right) \mid \tau', m, G\right] \\
&= \mathbb{E}\left[v^*(s) \mid m, \tau', G\right].
\end{aligned}$$

□

Proof of Lemma 1$''$. The proof of part 1 is the same as in Lemma 1$'$. As noted before, pulling the arm at τ is the same as pulling the arm at τ' and then releasing an additional finite-valued informative signal y. Therefore,

$$\begin{aligned}
\mathbb{E}\left[s \mid \tau, m, \sigma\right] &= \mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)} \mid \tau', m, \sigma\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)} \mid \tau', s, \sigma\right] \mid \tau', m, \sigma\right] \\
&= \mathbb{E}\left[\begin{array}{c} s_\sigma \mathbb{E}\left[\frac{sq(y|G)}{sq(y|G)+(1-s)q(y|B)} \mid \tau', s, G\right] + \\ + (1-s_\sigma)\,\mathbb{E}\left[\frac{sq(y|G)}{sq(y|G)+(1-s)q(y|B)} \mid \tau', s, B\right] \end{array} \middle| \tau', m, \sigma\right] \\
&= \mathbb{E}\left[\begin{array}{c} s_\sigma \sum \frac{sq(y|G)}{sq(y|G)+(1-s)q(y|B)} q(y \mid G) + \\ + (1-s_\sigma) \sum \frac{sq(y|G)}{sq(y|G)+(1-s)q(y|B)} q(y \mid B) \end{array} \middle| \tau', m, B\right] \\
&= \mathbb{E}\left[s \sum \frac{s_\sigma + (1-s_\sigma)\frac{q(y|B)}{q(y|G)}}{s + (1-s)\frac{q(y|B)}{q(y|G)}} q(y \mid G) \mid \tau', m, B\right] \\
&\gtreqless \mathbb{E}\left[s \mid \tau', m, B\right] \text{ whenever } s_\sigma \gtreqless s,
\end{aligned}$$

where the first line holds by Bayes' rule, the second by the law of iterated expectations, the third by Proposition 1 of Alonso and Camara (2016) with s_σ given by

$$s_\sigma = \frac{s\frac{\sigma}{\pi}}{s\frac{\sigma}{\pi} + (1-s)\frac{(1-\sigma)}{(1-\pi)}},$$

the fourth by definition of expectations, the fifth by rearrangement, and the last by Jensen's inequality applied to function $h(z) \equiv (s_\sigma + (1-s_\sigma)z) / (sz + (1-s)z)$, which is convex (concave) in z whenever $s_\sigma > s$ ($s_\sigma < s$). Parts 2 and 3 follow because $s_{\sigma_B} < s < s_{\sigma_G}$ whenever $\Pr(\theta = G \mid \sigma_B) < \pi < \Pr(\theta = G \mid \sigma_G)$. □

B Benchmark Model

To facilitate our discussion in Section 4, we prove our results under more general assumptions than in our benchmark model. First, we assume that $v_G^*(s)$ is strictly increasing and (weakly) convex and $v_B^*(s)$ is strictly increasing and (weakly) concave. Second, we assume that the arm arrives at a random time according to distributions $F_G = F$ for good Sender and F_B for bad Sender, where $F_B(t) \geq F_G(t)$ for all t.

Proof of Lemma 2. *Part 1.* Suppose, on the contrary, that $\mu(\tau) \leq \mu(\tau')$. Then

$$\begin{aligned}\int v_B^*(s)\, dH_B(s|\tau, \mu(\tau)) &\leq \int v_B^*(s)\, dH_B(s|\tau', \mu(\tau)) \\ &\leq \int v_B^*(s)\, dH_B(s|\tau', \mu(\tau')),\end{aligned}$$

where the first inequality holds by Lemma 1′ part 2 and the second by Lemma 1′ part 1. Moreover, at least one inequality is strict. Indeed, if $\mu(\tau) \in (0,1)$, then the first inequality is strict. If $\mu(\tau) = 0$, then $\mu(\tau') > 0$ (because $\mu(\tau) = \mu(\tau') = 0$ is not allowed); so the second inequality is strict. Finally, if $\mu(\tau) = 1$, then $\mu(\tau) \leq \mu(\tau')$ cannot hold (because $\mu(\tau) = \mu(\tau') = 1$ is not allowed). The displayed inequality implies that bad Sender strictly prefers to pull the arm at τ' than at τ. A contradiction.

Good Sender strictly prefers to pull the arm at τ because

$$\begin{aligned}\int v_G^*(s)\, dH_G(s|\tau, \mu(\tau)) &\geq \int v_G^*(s)\, dH_G(s|\tau', \mu(\tau)) \\ &> \int v_G^*(s)\, dH_G(s|\tau', \mu(\tau')),\end{aligned}$$

where the first inequality holds by Lemma 1′ part 3 and the second by $\mu(\tau) > \mu(\tau')$ and

Lemma 1′ part 1.

Part 2. If τ is in the support of P_B, then bad Sender weakly prefers to pull the arm at τ than at any other $\tau' > \tau$. By Bayes' rule $\mu(\tau) < 1$. Also, $\mu(\tau)$ cannot be zero, otherwise bad Sender would strictly prefer to pull the arm at $T+1$ since $\mu(T+1) > 0$ by $F_G(T) < 1$. Therefore, by part 1 of this lemma, good Sender strictly prefers to pull the arm at τ than at any other $\tau' > \tau$; so $P_G(\tau) = F_G(\tau)$. □

Proof of Lemma 3. By Lemma 2 part 2, each t' in the support of P_B is also in the support of P_G. We show that each t' in the support of P_G is also in the support of P_B by contradiction. Suppose that there exists t' in the support of P_G but not in the support of P_B. Then, by Bayes' rule $\mu(t') = 1$; so bad Sender who receives the arm at $t \leq t'$ gets the highest possible equilibrium payoff $v_B^*(1)$. Therefore, there exists a period $\tau \geq t'$ at which $\mu(\tau) = 1$ (recall that the support of $H(.|\tau, \pi)$ does not contain $s = 1$) and bad Sender pulls the arm with a positive probability. A contradiction.

Suppose, on the contrary, that there exists τ such that $P_G(\tau) > 0$ and $P_B(\tau) \geq P_G(\tau)$. Because $P_\theta(\tau) = \sum_{t=1}^{\tau}(P_\theta(t) - P_\theta(t-1))$, there exists $\tau' \leq \tau$ in the support of P_B such that $P_B(\tau') - P_B(\tau'-1) \geq P_G(\tau') - P_G(\tau'-1)$. Similarly, because $1 - P_\theta(\tau) = \sum_{t=\tau+1}^{T+1}(P_\theta(t) - P_\theta(t-1))$ and $1 - P_G(\tau) > 0$ by $P_G(T) \leq F_G(T) < 1$, there exists $\tau'' > \tau$ in the support of P_G such that $P_G(\tau'') - P_G(\tau''-1) \geq P_B(\tau'') - P_B(\tau''-1)$. By Bayes' rule,

$$\begin{aligned}\mu(\tau') &= \frac{\pi(P_G(\tau') - P_G(\tau'-1))}{\pi(P_G(\tau') - P_G(\tau'-1)) + (1-\pi)(P_B(\tau') - P_B(\tau'-1))} \leq \pi \\ &\leq \frac{\pi(P_G(\tau'') - P_G(\tau''-1))}{\pi(P_G(\tau'') - P_G(\tau''-1)) + (1-\pi)(P_B(\tau'') - P_B(\tau''-1))} = \mu(\tau'').\end{aligned}$$

Therefore, by Lemma 2, bad Sender strictly prefers to pull the arm at τ'' than at τ', which implies that τ' cannot be in the support of P_B. A contradiction. □

Proof of Lemma 4. By Lemma 3, P_G and P_B have the same supports and therefore $\mu(\tau) \in (0,1)$. Let the support of P_G be $\{\tau_1, ..., \tau_n\}$. Notice that $\tau_n = T+1$ because $P_G(T) \leq F_G(T) < 1$. Since τ_{n-1} is in the support of P_B and

$$P_B(\tau_{n-1}) < P_G(\tau_{n-1}) = F_G(\tau_{n-1}) \leq F_B(\tau_{n-1}),$$

where the first inequality holds by Lemma 3, the equality by Lemma 2 part 2, and the last inequality by assumption $F_B(t) \geq F_G(t)$. Therefore, bad Sender who receives the arm at τ_{n-1} must be indifferent between pulling the arm at τ_{n-1} or at τ_n. Analogously, bad Sender who receives the arm at τ_{n-k-1} must be indifferent between pulling it at τ_{n-k-1} and at some $\tau \in \{\tau_{n-k}, \ldots, \tau_n\}$. Thus, by mathematical induction on k, bad Sender is indifferent between all τ in the support of P_G, which proves (1).

By Bayes' rule, for all τ in the support of P_G,

$$\frac{1-\pi}{\pi}\left(P_B\left(\tau\right)-P_B\left(\tau-1\right)\right)=\frac{1-\mu\left(\tau\right)}{\mu\left(\tau\right)}\left(P_G\left(\tau\right)-P_G\left(\tau-1\right)\right). \tag{9}$$

Summing up over τ yields (2). Finally, suppose, on the contrary, that there exist two distinct solutions π' and π'' to (1) and (2). By Lemma $1'$ part 1, (1) uniquely determines $\mu\left(\tau\right)$ for a given $\mu\left(T+1\right)$ and $\mu\left(\tau\right)$ is increasing in $\mu\left(T+1\right)$. Thus, for π' and π'' to be distinct, it must be that $\mu'\left(T+1\right) \neq \mu''\left(T+1\right)$. Without loss, suppose that $\mu'\left(T+1\right) < \mu''\left(T+1\right)$, and thus $\mu'\left(\tau\right) < \mu''\left(\tau\right)$ for all τ in the support of P_G. But then (2) cannot hold for both π' and π''. A contradiction. □

Proof of Proposition 1. *Part 1.* Using Lemmas 3 and 4 together with (9) proves the *only if* part of part 1. Setting $\mu\left(\tau\right)=0$ for τ not in the support of P_G and using Lemma 2 proves the *if* part of part 1.

Part 2. First, we notice that, by part 1 of Proposition 1, there exists an equilibrium with $P_G\left(\tau\right)=F_G\left(\tau\right)$ for all τ.

Adopting Cho and Kreps (1987)'s definition to our setting (see e.g., Maskin and Tirole, 1992), we say that an equilibrium is divine if $\mu\left(\tau\right)=1$ for any $\tau \notin supp\left(P_G\right)$ at which condition D1 holds. D1 holds at τ if for all $m \in [0,1]$ that satisfy

$$\int v_B^*\left(s\right)dH_B\left(s|\tau,p\right) \geq \max_{t\in supp(P_G),t>\tau}\int v_B^*\left(s\right)dH_B\left(s|t,\mu\left(t\right)\right) \tag{10}$$

the following inequality holds:

$$\int v_G^*\left(s\right)dH_G\left(s|\tau,m\right) > \max_{t\in supp(P_G),t>\tau}\int v_G^*\left(s\right)dH_G\left(s|t,\mu\left(t\right)\right). \tag{11}$$

Suppose, on the contrary, that there exists a divine equilibrium in which $P_G\left(\tau\right) < F_G\left(\tau\right)$ for some $\tau \in \{1,\ldots,T\}$. By part 1 of Proposition 1, $\tau \notin supp\left(P_G\right)$. Let t^* denote t that maximizes the right hand side of (11). By Lemma 4, $\mu\left(t^*\right) \in (0,1)$ and t^* maximizes the right hand side of (10). Therefore, by Lemma 2 part 1, D1 holds at τ; so $\mu\left(\tau\right)=1$. But then $\tau \notin supp\left(P_G\right)$ cannot hold, because

$$\int v_G^*\left(s\right) dH_G\left(s|\tau,1\right) = v_G^*\left(1\right) > \max_{t \in supp(P_G)} \int v_G^*\left(s\right) dH_G\left(s|t,\mu\left(t\right)\right).$$

□

C Poisson model

Proof of Proposition 3. We first prove part 2 and then part 1.

Part 2.

For π:

$$\frac{d\mu\left(T\right)}{d\pi} = \begin{cases} \frac{d}{d\pi}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-1} \text{ if } \pi < \overline{\pi}, \\ \frac{d}{d\pi}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} + 1\right]^{-\frac{\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases}$$
$$= \begin{cases} \frac{1}{\pi^2}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-2} \text{ if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} + 1\right]^{-\frac{\alpha+2\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases}$$
$$= \left\{ \begin{array}{l} \frac{1}{\pi^2}\mu\left(T\right)^2 \text{ if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\mu\left(T\right)^{2+\frac{\alpha}{\lambda}} \text{ otherwise,} \end{array} \right\} > 0.$$

For λ:

$$\frac{d\mu\left(T\right)}{d\lambda} = \begin{cases} \frac{d}{d\lambda}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-1} \text{ if } \pi < \overline{\pi}, \\ \frac{d}{d\lambda}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} + 1\right]^{-\frac{\lambda}{\alpha+\lambda}} \text{ otherwise.} \end{cases}$$

First, when $\pi < \bar{\pi}$, $\frac{d\mu(T)}{d\lambda} < 0$ since $e^{-(\alpha+\lambda)T} > 1 - (\alpha+\lambda)\,T$ for all $\alpha, \lambda, T > 0$. Second, let

$$\phi\left(\lambda\right) \equiv \frac{\left(\alpha+\lambda\right)\left(1-\pi\right)}{\lambda\pi}e^{\alpha T} > 0.$$

Then, when $\pi > \bar{\pi}$,

$$\frac{d\mu\left(T\right)}{d\lambda} = \frac{d}{d\lambda}e^{-\frac{\lambda}{\alpha+\lambda}\ln(1+\phi(\lambda))}.$$

To show $\frac{d\mu(T)}{d\lambda} < 0$ it is then sufficient to note that

$$\frac{d}{d\lambda}\frac{\lambda}{\alpha+\lambda}\ln\left(1+\phi\left(\lambda\right)\right) = \frac{1}{\alpha+\lambda}\left[\frac{\ln\left(1+\phi\left(\lambda\right)\right)}{\alpha+\lambda} - \frac{1}{\lambda}\frac{1-\pi}{\pi}\frac{1}{1+\phi\left(\lambda\right)}\right] > 0$$

where the last passage follows from $(1+\phi(\lambda))\ln(1+\phi(\lambda)) > \phi(\lambda)$.

For α:

If $\pi < \bar{\pi}$, then

$$\begin{aligned} \frac{d\mu(T)}{d\alpha} &= -(\mu(T))^2 \frac{\chi}{(\alpha+\lambda)^2} < 0 \\ \chi &\equiv \lambda\left\{e^{\lambda T} - [1+(\alpha+\lambda)T]e^{-\alpha T}\right\} > 0 \end{aligned}$$

where the last passage follows from $e^{(\alpha+\lambda)T} > 1+(\alpha+\lambda)T$ for all $\alpha, \lambda, T > 0$.

If $\pi \geq \bar{\pi}$, by log-differentiation,

$$\begin{aligned} \frac{d\mu(T)}{d\alpha} &= \mu(T)\frac{\lambda}{\alpha+\lambda}\left[\frac{\ln(1+\xi)}{\alpha+\lambda} - \frac{\frac{d\xi}{d\alpha}}{1+\xi}\right] \\ \xi &\equiv \frac{\alpha+\lambda}{\lambda}\frac{1-\pi}{\pi}e^{\alpha T}. \end{aligned}$$

Thus,

$$\frac{d\mu(T)}{d\alpha} < 0 \iff \frac{(1+\xi)\ln(1+\xi)}{\xi} < 1+T(\alpha+\lambda). \tag{12}$$

For $\pi = \overline{\pi}$, $\xi = e^{(\alpha+\lambda)T} - 1 > 0$; so

$$\frac{d\mu(T)}{d\alpha} < 0 \iff \ln(1+\xi) < \xi,$$

which is true for all $\xi > 0$. If π is greater than $\overline{\pi}$, then ξ is smaller than $e^{(\alpha+\lambda)T} - 1$ and the inequality (12) is stronger because the left hand side is increasing in ξ for $\xi > 0$; so $\frac{d\mu(T)}{d\alpha} < 0$ for $\pi \geq \bar{\pi}$.

Part 1.

For π on q:

$$\begin{aligned} \frac{dq}{d\pi} &= \frac{d}{d\pi}\left[\frac{\pi}{1-\pi}\frac{1-\mu(T)}{\mu(T)}e^{-\alpha T}\right] \\ &= \frac{e^{-\alpha T}}{\mu(T)(1-\pi)} \times \left[\frac{1-\mu(T)}{1-\pi} - \frac{\pi}{\mu(T)}\frac{d\mu(T)}{d\pi}\right], \\ &= \underbrace{\frac{e^{-\alpha T}}{\mu(T)(1-\pi)}}_{>0} \times \left\{ \begin{array}{l} \left[\frac{1-\mu(T)}{1-\pi} - \frac{\mu(T)}{\pi}\right] \text{ if } \pi < \overline{\pi}, \\ \left[\frac{1-\mu(T)}{1-\pi} - e^{\alpha T}\frac{\mu(T)^{1+\frac{\alpha}{\lambda}}}{\pi}\right] \text{ otherwise.} \end{array} \right\} \end{aligned}$$

If $\pi < \bar{\pi}$, then $\frac{dq}{d\pi} > 0$ if and only if $\frac{1-\mu(T)}{\mu(T)} > \frac{1-\pi}{\pi}$, which is satisfied since in equilibrium

$\mu(T) < \pi$.

If $\pi \geq \bar{\pi}$, then $\frac{dq}{d\pi} > 0$ if and only if

$$\frac{1-\mu(T)}{\mu(T)} > \frac{1-\pi}{\pi} e^{\alpha T} \mu(T)^{\frac{\alpha}{\lambda}}, \tag{13}$$

which can be rewritten as

$$\begin{aligned} & 1-(1+\phi(\lambda))^{-\frac{\lambda}{\alpha+\lambda}} > \frac{\lambda}{\alpha+\lambda} \frac{\phi(\lambda)}{1+\phi(\lambda)} \\ \iff & 1+\phi(\lambda)-(1+\phi(\lambda))^{\frac{\alpha}{\alpha+\lambda}} > \frac{\lambda}{\alpha+\lambda}\phi(\lambda) \\ \iff & 1+\frac{\alpha}{\alpha+\lambda}\phi(\lambda) > (1+\phi(\lambda))^{\frac{\alpha}{\alpha+\lambda}}. \end{aligned}$$

To conclude, notice that $1+xb > (1+b)^x$ for $b > 0$ and $x \in (0,1)$.

For π on $\bar{t}$:

For $\pi < \bar{\pi}$, $\bar{t} = 0$, but for $\pi \geq \bar{\pi}$, $\bar{t}$ is increasing in π and decreasing in α because $\mu(T)$ is increasing in π and decreasing in α.

For λ on q:

$$\begin{aligned} \frac{dq}{d\lambda} &= \frac{d}{d\lambda}\left[\frac{\pi}{1-\pi}\frac{1-\mu(T)}{\mu(T)}e^{-\alpha T}\right] \\ &= -\frac{\pi}{1-\pi}\frac{e^{-\alpha T}}{\mu(T)^2}\frac{d\mu(T)}{d\lambda} > 0. \end{aligned}$$

For λ on $\bar{t}$:

For $\pi < \bar{\pi}$, $\bar{t} = 0$, but for $\pi \geq \bar{\pi}$

$$\begin{aligned} \frac{d\bar{t}}{d\lambda} &= -\frac{d}{d\lambda}\left[\frac{1}{\alpha+\lambda}\ln\left(\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right)\right] \\ &= \frac{1}{\alpha+\lambda}\left(\frac{\ln(1+\phi(\lambda))}{\alpha+\lambda} - \frac{d\phi(\lambda)}{d\lambda}\frac{1}{1+\phi(\lambda)}\right) > 0 \end{aligned}$$

where the last passage follows from

$$\frac{d\phi(\lambda)}{d\lambda} = -\frac{\alpha}{\lambda^2}\frac{1-\pi}{\pi}e^{\alpha T} < 0.$$

For α on q:

If $\pi < \overline{\pi}$, then

$$\begin{aligned}
\frac{1-\pi}{\pi}\frac{dq}{d\alpha} &= \frac{(\lambda - \alpha(\alpha+\lambda)T)e^{(\lambda-\alpha)T} - \lambda(1+2(\alpha+\lambda)T)e^{-2\alpha T}}{(\alpha+\lambda)^2} - \left(\frac{1}{\pi}-2\right)Te^{-\alpha T} \\
&< \frac{(\lambda - \alpha(\alpha+\lambda)T)e^{(\lambda-\alpha)T} - \lambda(1+2(\alpha+\lambda)T)e^{-2\alpha T}}{(\alpha+\lambda)^2} - \left(\frac{1}{\overline{\pi}}-2\right)Te^{-\alpha T} \\
&= -\frac{e^{-2\alpha T}}{(\alpha+\lambda)^2}\left(\lambda(1+(\alpha+\lambda)T) + \left((\alpha+\lambda)^2 T - \lambda\right)e^{(\alpha+\lambda)T} - (\alpha+\lambda)^2 Te^{\alpha T}\right).
\end{aligned}$$

Thus $dq/d\alpha < 0$, because for all positive α and λ

$$\begin{aligned}
f(\alpha,\lambda) &= \lambda(1+(\alpha+\lambda)) + \left((\alpha+\lambda)^2 - \lambda\right)e^{(\lambda+\alpha)} - (\alpha+\lambda)^2 e^{\alpha} \\
&= \sum_{k=3}^{\infty}\left[\frac{(\alpha+\lambda)^k}{(k-2)!} - \lambda\frac{(\alpha+\lambda)^{k-1}}{(k-1)!} - (\alpha+\lambda)^2\frac{\alpha^{k-2}}{(k-2)!}\right] > 0,
\end{aligned}$$

□

where the inequality holds because each term c_k in the sum is positive:

$$\begin{aligned}
c_k &= \frac{(\alpha+\lambda)^2\left((\alpha+\lambda)^{k-2} - \alpha^{k-2}\right)}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} \\
&= \frac{(\alpha+\lambda)^2\lambda\left(\sum_{n=0}^{k-3}(\alpha+\lambda)^{k-3-n}\alpha^n\right)}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} \\
&> \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} > 0.
\end{aligned}$$

If $\pi \geq \bar{\pi}$, then without loss of generality we can set $T = 1$ and get

$$\frac{1-\pi}{\pi}\frac{dq}{d\alpha} = e^{-\alpha}\left[1 - \frac{1}{\mu(T)}\left(1 + \frac{d\mu(T)}{d\alpha}\mu(T)^{-1}\right)\right] < 0$$

$$\iff \frac{d\mu(T)}{d\alpha} > \mu(T)(\mu(T)-1).$$

This inequality is equivalent to:

$$\frac{1+\xi}{\xi}\left[\ln(1+\xi) + \frac{(\alpha+\lambda)^2}{\lambda}\left(1-(1+\xi)^{-\frac{\alpha+\lambda}{\lambda}}\right)\right] - 1 - \alpha - \lambda > 0$$

The left hand side is increasing in α, treating ξ as a constant. Then the inequality holds because it holds for $\alpha \to 0$:

$$\begin{aligned}\frac{1+\xi}{\xi}\left[\ln\left(1+\xi\right)+\lambda\left(1-\left(1+\xi\right)^{-1}\right)\right]-1-\lambda &> 0\\ \frac{1+\xi}{\xi}\left[\ln\left(1+\xi\right)+\lambda\frac{\xi}{1+\xi}\right]-1-\lambda &> 0\\ \frac{1+\xi}{\xi}\ln\left(1+\xi\right) &> 1.\end{aligned}$$

Proof of Proposition 4. *Part 1.* Recall that (i) Sender's payoff equals Receiver's posterior belief about Sender at $t = T$ and (ii) in equilibrium, bad Sender (weakly) prefers not to pull the arm at all than pulling it at any time $t \in [0, T]$. Therefore, bad Sender's expected payoff equals Receiver's belief about Sender at $t = T$ if the arm has not been pulled:

$$\mathbb{E}\left[v_B\right] = \mu\left(T\right). \tag{14}$$

Part 1 then follows from Proposition 3.

Part 2. By the law of iterated expectations,

$$\begin{aligned}\mathbb{E}\left[s\right] &= \pi\mathbb{E}\left[v_G\right]+\left(1-\pi\right)\mathbb{E}\left[v_B\right]=\pi\\ \Rightarrow \mathbb{E}\left[v_G\right] &= 1-\frac{1-\pi}{\pi}\mu\left(T\right)\end{aligned}$$

where s is Receiver's posterior belief about Sender at $t = T$ and we use (14) in the last passage. Thus, good Sender's expected payoff increases with α and λ by Proposition 3. Finally, it is easy to see that $\mathbb{E}\left[v_G\right]$ increases in π after sustituting $\mu\left(T\right)$ in $\mathbb{E}\left[v_G\right]$.

Part 3. We shall show that in the divine equilibrium

$$\mathbb{E}\left[u\right] = \frac{\left(1-\pi\right)\left(1-\mu\left(T\right)\right)}{2}. \tag{15}$$

Part 3 then follows from Proposition 3.

Since $u\left(s\right) = \left(1-s\right)^2/2$ and $\mathbb{E}\left[s\right] = \pi$, it is sufficient to prove that $\mathbb{E}\left[s^2\right] = \pi\mathbb{E}\left[v_G\right]$. We divide the proof in two cases: $\pi \leq \bar{\pi}$ and $\pi > \bar{\pi}$. If $\pi \leq \bar{\pi}$, Receiver's expected payoff is given by the sum of four terms: (i) Sender is good and the arm does not arrive; (ii) Sender is good and the arm arrives; (iii) Sender is bad and she does not pull the arm;

and (iv) Sender is bad and she pulls the arm. Thus,

$$\begin{aligned}\mathbb{E}\left[s^2\right] &= \pi e^{-\alpha T}\left(\mu\left(T\right)\right)^2 \\ &+\pi\int_0^T\left(e^{\lambda(T-t)}\mu\left(T\right)\right)^2\alpha e^{-\alpha t}dt \\ &+\left(1-\pi\right)q\left(\mu\left(T\right)\right)^2 \\ &+\left(1-\pi\right)\int_0^T e^{-\lambda(T-t)}\left(e^{\lambda(T-t)}\mu\left(T\right)\right)^2\frac{\pi}{1-\pi}\left(\frac{1-\mu\left(t\right)}{\mu\left(t\right)}\right)\alpha e^{-\alpha t}dt.\end{aligned}$$

Solving all integrals and rearranging all common terms we get

$$\mathbb{E}\left[s^2\right] = \pi\mathbb{E}\left[v_G\right].$$

If $\pi > \bar{\pi}$, Receiver's expected payoff is given by the sum of five terms: (i) Sender is good and the arm does not arrive; (ii) Sender is good and the arm arrives before $\bar{t}$; (iii) Sender is good and the arm arrives between $\bar{t}$ and T; (iv) Sender is bad and she does not pull the arm; (v) Sender is bad and she pulls the arm. Thus,

$$\begin{aligned}\mathbb{E}\left[s^2\right] &= \pi e^{-\alpha T}\left(\mu\left(T\right)\right)^2 \\ &+\pi\left(1-e^{-\alpha\bar{t}}\right) \\ &+\pi\int_{\bar{t}}^T\left(e^{\lambda(T-t)}\mu\left(T\right)\right)^2\alpha e^{-\alpha t}dt+ \\ &+\left(1-\pi\right)q\left(\mu\left(T\right)\right)^2 \\ &+\left(1-\pi\right)\int_{\bar{t}}^T e^{-\lambda(T-t)}\left(e^{\lambda(T-t)}\mu\left(T\right)\right)^2\frac{\pi}{1-\pi}\left(\frac{1-\mu\left(t\right)}{\mu\left(t\right)}\right)\alpha e^{-\alpha t}dt.\end{aligned}$$

Solving all integrals and rearranging all common terms we again get

$$\mathbb{E}\left[s^2\right] = \pi\mathbb{E}\left[v_G\right].$$

□

Proof of Proposition 5. *Part 1.* We differentiate R with respect to π:

$$\begin{aligned}\frac{dR}{d\pi} &= \frac{d}{d\pi}\left(1-\frac{\pi}{\mu\left(T\right)}e^{-\alpha T}\right) \\ &= -e^{-\alpha T}\frac{\mu\left(T\right)-\frac{d\mu(T)}{d\pi}\pi}{\mu\left(T\right)^2}.\end{aligned}$$

Therefore R is non-decreasing in π if and only if

$$\frac{d\mu(T)}{d\pi} \geq \frac{\mu(T)}{\pi}.$$

We now show that

$$\frac{d\mu(T)}{d\pi} \geq \frac{\mu(T)}{\pi} \iff \pi \geq \frac{\alpha e^{\alpha T}}{(\alpha+\lambda)e^{\alpha T}-1}.$$

Differentiating $\mu(T)$ with respect to π, we have

$$\begin{aligned}
\frac{d\mu(T)}{d\pi} &= \begin{cases} \frac{d}{d\pi}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-1} \text{ if } \pi < \overline{\pi}, \\ \frac{d}{d\pi}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases} \\
&= \begin{cases} \frac{1}{\pi^2}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-2} \text{ if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\alpha+2\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases} \\
&= \begin{cases} \frac{\mu(T)^2}{\pi^2} & \text{if } \pi < \bar{\pi}; \\ e^{\alpha T}\frac{\mu(T)^{2+\frac{\alpha}{\lambda}}}{\pi^2} & \text{otherwise.} \end{cases}
\end{aligned}$$

Case 1: $\pi < \bar{\pi}$.
If $\pi < \bar{\pi}$, then $dR/d\pi < 0$ because $\mu(T) < \pi$ and

$$\frac{d\mu(T)}{d\pi} = \frac{\mu(T)^2}{\pi^2} < \frac{\mu(T)}{\pi}.$$

Case 2: $\pi \geq \bar{\pi}$.
If $\pi \geq \bar{\pi}$, then $dR/d\pi < 0$ if and only if

$$\frac{d\mu(T)}{d\pi} = e^{\alpha T}\frac{\mu(T)^{2+\frac{\alpha}{\lambda}}}{\pi^2} < \frac{\mu(T)}{\pi}e^{\alpha T}.$$

Substituting $\mu(T)$, we get that this inequality is equivalent to

$$\pi < \frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T}+\lambda\left(e^{\alpha T}-1\right)}.$$

It remains to show that

$$\frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T}+\lambda\left(e^{\alpha T}-1\right)} > \bar{\pi}.$$

Substituting $\bar{\pi}$, we get that this inequality is equivalent to

$$\frac{e^{(\alpha+\lambda)T}-1}{\alpha+\lambda} > \frac{e^{\alpha T}-1}{\alpha},$$

which is satisfied because function $(e^x - 1)/x$ is increasing in x.

Part 2. We differentiate R with respect to λ:

$$\begin{aligned}\frac{dR}{d\lambda} &= \frac{d}{d\lambda}\left[\pi\left(1-e^{-\alpha T}\right)+(1-\pi)(1-q)\right] \\ &= -(1-\pi)\frac{dq}{d\lambda} < 0\end{aligned}$$

where the last inequality follows from Proposition 3.

Part 3. We differentiate R with respect to α

$$\begin{aligned}\frac{dR}{d\lambda} &= \frac{d}{d\alpha}\left[\pi\left(1-e^{-\alpha T}\right)+(1-\pi)(1-q)\right] \\ &> -(1-\pi)\frac{dq}{d\alpha} > 0,\end{aligned}$$

where the last inequality follows from Proposition 3. □

Proof of Proposition 7. The unconditional probability of a breakdown of the arm pulled at t is given by

$$\Pr(\text{bd} \mid t) \equiv \left(1-e^{-\lambda(T-t)}\right)[1-\mu(t)].$$

Notice that $\Pr(\text{bd} \mid t)$ is continuous in t because $\mu(t)$ is continuous in t. Also, $\Pr(\text{bd} \mid t)$ equals 0 for $t \leq \bar{t}$, is strictly positive for all $t \in (\bar{t}, T)$, and equals 0 for $t = T$. Substituting $\mu(t)$ and taking the derivative of $\Pr(\text{bd} \mid t)$ with respect to $t \geq \bar{t}$ we have

$$\frac{d\Pr(\text{bd} \mid t)}{dt} = -\lambda\frac{e^{-\lambda(T-t)}(1+\mu(T))-2\mu(T)}{\left[1-\mu(T)\left(1-e^{\lambda(T-t)}\right)\right]^2}$$

which is positive if and only if

$$T - \frac{1}{\lambda}\ln\left(\frac{1+\mu(T)}{2\mu(T)}\right) < T.$$

□

Proof of Proposition 8. The probability density that bad Sender pulls the arm at time t is given by

$$p_B(t) \equiv \frac{dP_B(t)}{dt} = \frac{\pi}{1-\pi} \frac{\alpha e^{-\alpha t}\left(1-\mu(T)\, e^{\lambda(T-t)}\right)}{\mu(T)}$$

for $t > \bar{t}$ and $r_B(t) = 0$ for $t \leq \bar{t}$. Differentiating with respect to t we get

$$\frac{dp_B(t)}{dt} = \frac{\pi}{1-\pi} \frac{\alpha e^{-\alpha t}}{\mu(T)} \left[(\alpha+\lambda)\, \mu(T)\, e^{\lambda(T-t)} - \alpha\right]$$

which is positive if and only if

$$\bar{t} \leq t < T - \frac{1}{\lambda} \ln\left(\frac{\alpha}{\alpha+\lambda} \frac{1}{\mu(T)}\right).$$

□