Timing Information Flows

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Abstract

At an exogenous deadline, Receiver takes an action, the payoff from which depends on Sender's private type. Sender privately observes if and when an opportunity arrives to start a public flow of information about her type. Upon arrival of the opportunity, she chooses when to start the information flow. Starting the information flow earlier allows for greater scrutiny, but signals credibility. We characterize the set of equilibria and show that Sender always delays the information flow, and completely withholds it with strictly positive probability. We derive comparative statics and discuss implications for organizations, politics, and financial markets.

Keywords: information disclosure, strategic timing, Bayesian learning, credibility vs. scrutiny.
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1 Introduction

On 27 February 2012, state-controlled press announced that Russian authorities had foiled an assassination plot against Prime Minister Vladimir Putin. The timing was somewhat curious, as Putin was a candidate for President in an election to be held shortly after, on March 4. Some questioned the timing of the announcement as being designed to favor Putin in the upcoming election. For instance, the New York Times reported:

> Critics questioned the timing of the disclosure. "I think that someone is playing on patriotic sentiments," Sergei M. Mironov, one of Mr. Putin's opponents in the election, told reporters. He said the report could garner "extra votes for Putin." Dmitri Oreshkin, a political commentator often critical of the Kremlin, was more direct. "This is a sign that the real leaders of Mr. Putin's political structure, the people from the Federal Security Service, are trying to mobilize public opinion according to the logic that we are surrounded by enemies and that we have one decisive, effective and intelligent national leader that they want to destroy," he said on Ekho Moskvy Radio. "The timely disclosure of this conspiracy against this leader is a serious addition to the electoral rating of the potential president."[1]

If one takes this argument—that the announcement of the plot was announced to help Putin's candidacy—at face value, the timing is still puzzling. The information was not released the day before the election time when it might have had maximum impact by being most salient in the minds of voters as they went to the polls. Indeed, given the timing of the announcement, the information might have been forgotten by many voters in the intervening period.

One naturally wonders whether an announcement immediately prior to election day might have been seen as particularly suspicious by voters and hence have backfired. At the very least, it seems clear that the timing of the

[1] New York Times, February 27 2012: `http://www.nytimes.com/2012/02/28/world/europe/plot-to-kill-vladimir-putin-uncovered.html?r=0`.

information release was chosen strategically. Indeed, it subsequently became known that the prisoners had been held for two weeks prior to the 27 February announcement. The authorities chose not to make the announcement either earlier or later, even though they could have done so.

What made one week out from election day the optimal timing? The above anecdote highlights a basic tradeoff in strategic dynamic information release. Early release of information is more credible, in that it signals that Sender has nothing to hide. On the other hand, such early release exposes the information to scrutiny for a longer period of time—possibly leading to the information being discovered to be false.

This tradeoff between credibility and scrutiny is central to many economic problems. The seller of a home via auction can schedule a long or short window for potential buyers to inspect the house. The CEO of a company can give her board of directors more or less time to examine draft proposals before the board meets to sign them off. Public figures can announce their intention to run for political posts at different times before the election date. In all these situations, (i) biased Sender has information which matters to Receiver; (ii) Receiver must make a choice at a given date; and (iii) Sender privately knows the earliest date at which she can release information to Receiver, but she can choose to release it later. In this paper we introduce and analyze a formal model of precisely these types of dynamic information release problems.

Understanding the credibility-scrutiny tradeoff is fundamental for the design of a variety of institutions. For instance, should there be media blackout rules a certain time prior to an election, as is the case in some countries? More broadly, what institutional design parameters induce CEOs, media, or politicians to gather more, high quality information and release it faster?

We analyze the credibility-scrutiny tradeoff in a model with three key features: (i) Sender privately knows her binary type, *good* or *bad*, and wants Receiver to take a higher action; (ii) at an exogenous deadline, Receiver chooses his action, which increases in his belief that Sender is good; (iii) Sender privately observes whether and when an opportunity to start a public flow of information about her type arrives and chooses when to seize this opportunity. We call this opportunity an *arm* and say that Sender chooses when to *pull* the arm.

In Section 3, we characterize the set of perfect Bayesian equilibria. In all equilibria, bad Sender delays pulling the arm longer than good Sender, despite the fact that pulling the arm has a positive instantaneous effect on Receiver's belief. An immediate implication is that, all else equal, an arm released earlier induces higher beliefs for Receiver. Thus, *inter alia*, our theory provides a rational foundation for confirmation bias–a greater reliance on information encountered earlier in a sequence (Baron, 2000, pp. 197-200). Moreover, bad Sender chooses not to pull the arm with strictly positive probability. Thus, the tension between credibility and scrutiny provides an opposing force to the Milgrom's (1981) unraveling effect, and helps explain why information is often withheld in organizations.

We prove that there exists an essentially unique divine equilibrium (Cho and Kreps, 1987).[2] In this equilibrium good Sender *immediately* pulls the arm when it arrives and bad Sender is indifferent between pulling the arm at any time and not pulling it at all. Uniqueness allows us to analyze comparative statics in a tractable way in a special case of our model where the arm arrives according to a Poisson process and pulling the arm starts an exponential learning process in the sense of Keller et al. (2005).

We do this in Section 4 and show that the comparative static properties of this equilibrium are very intuitive. Welfare increases with the speed of the learning process, the arrival rate of the arm, and the probability that Sender is good. A higher probability of good Sender also decreases the probability that bad Sender pulls the arm, as Receiver is less likely to believe that Sender is bad, and hence withholding information is less damning. However, this strategic effect does not completely offset the direct effect of the increased probability of good Sender on Receiver's posterior belief, even if no arm is pulled.

We then apply this model to the strategic release of political scandals in US Presidential campaigns. We interpret Sender as a media outlet opposed to the incumbent, and Receiver as the median voter. If the incumbent is worse than alternative candidates, a scandal may arise, but a scandal can also be fabri-

[2] The equilibrium is essentially unique in the sense that the probability with which each type of Sender pulls the arm at any time t is uniquely determined.

cated. We show that fabricated scandals are only released sufficiently close to the election. Also, a higher prior belief that the incumbent is good increases the probability of a scandal being released, provided that the incumbent has a high approval rating. An intuition for this is that the opposition media outlet optimally resorts to fabricated scandals when the incumbent is so popular that only a scandal could undermine her successful reelection. We also show that fewer scandals are released when voters apply more scrutiny to them and when other events make air time scarce. These results are consistent with a number of empirical regularities in US Presidential elections.

We also discuss the time pattern of campaign events. The probability that a released scandal is fabricated increases with the release time. Nonetheless, the probability that a fabricated scandal is released follows an inverted U-shape over time, as does the probability that a scandal is revealed to be fabricated.

In Section 5, we generalize the model in several directions. We then discuss how to extend the model to study the design of institutions. We argue that campaign blackout periods are generally counterproductive for the voters. We further discuss different approaches to the design of institutions that encourage managers to gather more information and report it earlier to the company board. Finally, we discuss a number of further applications captured by our model and conclude.

1.1 Related Literature

Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) pioneered the study of verifiable information disclosure and established the *unraveling result*: if Sender's preferences are common knowledge and monotonic in Receiver's action (for all types of Sender) then Receiver learns Sender's type in any sequential equilibrium. Dye (1985) first pointed out that the unraveling result fails if Receiver is uncertain about Sender's *information endowment*.[3] When Sender does not disclose information, Receiver is unsure as to why, and thus

[3] See also Shin (1994), Jung and Kwon (1988), and Dziuda (2011). The unraveling result migh also fail if disclosure is costly (Jovanovic, 1982) or information acquisition is costly (Shavell, 1994).

cannot conclude that the non-disclosure was strategic, and hence does not "assume the worst" about Sender's type.

Acharya, DeMarzo and Kremer (2011) and Guttman, Kremer and Skrzypacz (2013) explore the strategic timing of information disclosure in a dynamic version of Dye (1985).[4] Acharya et al. (2011) focus on the interaction between the timing of disclosure of private information relative to the arrival of external news, and clustering of the timing of announcements across firms. Guttman et al. (2013) analyze a setting with two periods and two signals and show that, in equilibrium, both *what* is disclosed and *when* it is disclosed matters. Strikingly, the authors show that later disclosures are received more positively.

All these models are unsuited to study either the credibility or the scrutiny sides of our tradeoff, because information in these models is verified instantly and with certainty once disclosed. In our motivating examples, information is not immediately verifiable: when Sender releases the information, Receiver only knows that "time will tell" whether the information released is reliable. To capture this notion of partial verifiability, we model information as being verified stochastically over time in the sense that releasing information starts a learning process for Receiver akin to processes in Bolton and Harris (1999), Keller, Rady and Cripps (2005), and Brocas and Carrillo (2007). In contrast to these papers, in our model Sender is privately informed and she chooses when to start rather than stop the process.[5]

2 The Model

There are two players: Sender (she) and Receiver (he). Sender privately knows her binary type θ: good ($\theta = G$) or bad ($\theta = B$). Let $\pi \in (0,1)$ be the common

[4] Shin (2003, 2006) also study dynamic verifiable information disclosure, but Sender there does not strategically time disclosure. A series of recent papers consider dynamic information disclosure with different focuses to us, including: Che and Hörner (2015); Ely, Frankel and Kamenica (2015); Ely (2015); Grenadier, Malenko and Malenko (2015); Halac, Kartik and Liu (2015); Horner and Skrzypacz (forthcoming).

[5] In our model Sender can influence only the starting time of the experimentation process, but not the design of the process itself. Instead, in the "Bayesian Persuasion" literature (e.g. Rayo and Segal (2010); Kamenica and Gentzkow (2011); Kolotilin (2015)) Sender fully controls the design of the experimentation process.

prior belief that Sender is good.

Time is discrete and indexed by $t \in \{1, 2, \ldots, T+1\}$. At a deadline $t = T$, Receiver must take an action $a \in \mathbb{R}$. (Time $T+1$ combines all future dates after the deadline).

Sender privately observes when an *arm* arrives. The arm arrives at a random time according to a distribution F_θ satisfying Assumption 1.

Assumption 1. *The distribution F_θ satisfies $F_G(t) \leq F_B(t)$ and $0 < F_G(t) < F_G(t+1)$ for all $t \in \{1, 2, .., T\}$.*

This assumption captures both (i) good and bad Sender receiving the arm at the same time and (ii) bad Sender receiving it at time 1 and good Sender receiving it at a later random time. More generally, this assumption allows for the arm to arrive earlier to bad Sender than to good Sender in the first-order stochastic dominance sense. This assumption also ensures that at any time (including $T+1$) the probability that good Sender receives the arm is strictly positive.

If the arm has arrived, Sender can pull it immediately or at any time after its arrival (including after the deadline). As Sender can take no action until the arrival of the arm, we can equivalently assume that Sender learns her type either when the game starts or when the arm arrives.

Let τ be the pulling time. If the arm is pulled before the deadline ($\tau \leq T$), Receiver observes realizations of a finite-valued stochastic process

$$L = \{L_\theta(t; \tau), \tau \leq t \leq T\}.$$

The process L can be viewed as a sequence of signals, one per each time from τ to T with the precision of the signal at time t possibly depending on τ and t. Notice that if the arm is pulled at $\tau = T$, Receiver observes the realization $L_\theta(T, T)$ before taking his action.

It is more convenient to work directly with the distribution of beliefs induced by the process L rather than with the process itself. Let p denote Receiver's *interim belief* that Sender is good upon observing that she pulls the arm at time τ and before observing any realizations of L. Likewise, let s denote

Receiver's *posterior belief* that Sender is good after observing all realizations of the process from τ to T. Given τ and p, the process L generates a distribution $H(. \mid \tau, p)$ over Receiver's posterior beliefs s; given τ, p, and θ, the process L generates a distribution $H_\theta(. \mid \tau, p)$ over s. Notice that if the arm is pulled after the deadline ($\tau = T+1$), then the distributions $H_\theta(. \mid \tau, p)$ and $H(. \mid \tau, p)$ assign probability one to $s = p$.

Assumption 2 says that (i) pulling the arm later reveals strictly less information about Sender's type in Blackwell (1953)'s sense and (ii) it is impossible to fully learn Sender's type.[6]

Assumption 2. *(i) For all $\tau, \tau' \in \{1, 2, \ldots, T+1\}$ such that $\tau < \tau'$, $H(. \mid \tau, \pi)$ is a strict mean-preserving spread of $H(. \mid \tau', \pi)$. (ii) The support of $H(. \mid 1, \pi)$ is a subset of $(0,1)$.*

To analyze the model, we need to understand how beliefs evolve over time from Sender's perspective. Lemma 1 presents three statistical properties of the belief evolution. First, a more favorable interim belief results in more favorable posteriors for all types of Sender and for all realizations of the process. Second, from bad Sender's perspective, pulling the arm later results in more favorable and less spread out posteriors (provided that the interim belief does not depend on the pulling time). Third, from good Sender's perspective, pulling the arm earlier results in more favorable and more spread out posteriors (again provided that the interim belief does not depend on the pulling time). The first two properties rely on the standard stochastic orders; the third property relies on a less common stochastic order, which we call second-convex-order stochastic dominance. Formally, distribution H_2 second-convex-order stochastically dominates distribution H_1 if there exists a distribution H such that H_2 first-order stochastically dominates H and H is a mean-preserving spread of H_1.

[6]For example, this assumption holds if $L_\theta(t;\tau) = \widetilde{L}_\theta(t) - \widetilde{L}_\theta(\tau-1)$ or $L_\theta(t;\tau) = \widetilde{L}_\theta(t-(\tau-1))$ for some finite-valued process $\widetilde{L}$ with (i) $\Pr\left(\widetilde{L}_G(t) = y(t)\right) \neq \Pr\left(\widetilde{L}_B(t) = y(t)\right)$ for some $y(t)$ and all $t \in \{1, 2, \ldots, T\}$ and (ii) $\Pr\left(\widetilde{L}_G(t) = y\right) > 0$ equivalent to $\Pr\left(\widetilde{L}_B(t) = y\right) > 0$ for all y and t.

Lemma 1 (Statistical Properties). *For all $\tau, \tau' \in \{1, \ldots, T+1\}$ such that $\tau < \tau'$, and all $p, p' \in (0,1]$ such that $p < p'$,*

1. *$H_\theta(. \mid \tau, p')$ strictly first-order stochastically dominates $H_\theta(. \mid \tau, p)$ for $\theta \in \{G, B\}$;*

2. *$H_B(. \mid \tau', p)$ strictly second-order stochastically dominates $H_B(. \mid \tau, p)$;*

3. *$H_G(. \mid \tau, p)$ strictly second-convex-order stochastically dominates $H_G(. \mid \tau', p)$.*

Proof. In Appendix. □

Sender's and Receiver's payoffs, $v(a, \theta)$ and $u(a, \theta)$, depend on a and θ. We are interested in situations where each type of Sender wishes Receiver to believe that she is good. Formally, Receiver's best response function

$$a^*(s) \equiv \arg\max_a \{su(a, G) + (1-s)u(a, B)\}$$

is well defined for all values of Receiver's posterior belief $s \in [0,1]$. Also, Sender's payoff is a linear strictly increasing function of s in that $v^*_\theta(s) \equiv v(a^*(s), \theta) = \alpha_\theta s + \beta_\theta$ with $\alpha_\theta > 0$.[7]

3 Equilibrium

We characterize the set of perfect Bayesian equilibria, henceforth equilibria. Let $\pi(\tau)$ be Receiver's equilibrium interim belief that Sender is good given that Sender pulls the arm at time $\tau \in \{1, 2, \ldots, T+1\}$. Also, let P_θ denote an equilibrium distribution of pulling time τ given Sender's type θ (with convention $P_\theta(0) = 0$).

We begin by showing that in any equilibrium good Sender strictly prefers to pull the arm at any time at which bad Sender pulls the arm with positive probability.

[7] All results continue to hold if, for example, $v^*_B(s)$ is strictly increasing and concave in s and $v^*_G(s)$ is strictly increasing and convex in s. Otherwise, there may exist only unintuitive equilibria in which good Sender pulls the arm later than bad Sender (see Section 5.1).

Lemma 2 (**Good Sender's Behavior**)**.** *In any equilibrium:*

1. *For all* $\tau, \tau' \in \{1, \ldots, T+1\}$ *such that* $\tau < \tau'$ *and neither* $\pi(\tau) = \pi(\tau') = 0$ *nor* $\pi(\tau) = \pi(\tau') = 1$*, if bad Sender weakly prefers to pull the arm at* τ *than at* τ'*, then* $\pi(\tau) > \pi(\tau')$ *and good Sender strictly prefers to pull the arm at* τ *than at* τ'*;*

2. *For all* $\tau \in \{1, \ldots, T\}$ *in the support of* P_B*, we have* $P_G(\tau) = F_G(\tau)$*.*

Proof. In Appendix. □

The proof relies on the three statistical properties from Lemma 1. To get the intuition, suppose that the interim belief about Sender's type does not depend on the pulling time. Then good Sender would like to pull the arm as soon as it arrives and bad Sender would like to never pull the arm, as pulling the arm earlier reveals more information about Sender's type. Therefore, if bad Sender pulls the arm (with positive probability) at any time τ before the deadline, then the interim belief $\pi(\tau)$ at this time must be higher than $\pi(T+1)$. But then good Sender has an even stronger incentive to pull the arm at τ under these interim beliefs than under constant interim beliefs. Notice that this argument does not imply that good Sender always pulls the arm as soon as it arrives. Indeed, for any $t \leq T$, there always exists an equilibrium in which good Sender never pulls the arm before or at t (i.e., $P_G(t) = 0$) but always pulls it after t (i.e., $P_G(\tau) = F_G(\tau)$ for all $\tau > t$).

Next, we show that good Sender pulls the arm earlier than bad Sender in the first-order stochastic dominance sense. Moreover, good sender pulls the arm strictly earlier unless no type pulls the arm.

Lemma 3 (**Bad Sender's Behavior**)**.** *In any equilibrium, for all* $\tau \in \{1, \ldots, T\}$ *with* $P_G(\tau) > 0$*, we have* $P_B(\tau) < P_G(\tau)$*.*

Proof. In Appendix. □

Intuitively, if bad and good Sender were pulling the arm at the same time, then the interim beliefs would not depend on the pulling time. But as discussed earlier, with constant interim beliefs, bad Sender would never pull the arm. Therefore, good Sender must necessarily pull the arm earlier than bad Sender. Notice that Lemma 3 does not imply that good Sender pulls the arm at a faster rate for all times. Indeed, there exist equilibria with $P_G(t) - P_G(t-1) < P_B(t) - P_B(t-1)$ for some $t \leq T$.

An immediate implication of Lemma 2 is that bad Sender always withholds the arm with positive probability:

Corollary 1 (Bad Sender's Withholding). *In any equilibrium,* $P_B(T) < F_B(T)$.

We now show that bad Sender is indifferent between pulling the arm at any time when good Sender pulls and not pulling the arm at all. Thus, Receiver's beliefs are pinned down by bad Sender's indifference condition (1) and the aggregation condition (2). The aggregation condition requires that the likelihood ratios of bad and good Sender's arms pulled at various times must average out to the prior likelihood ratio of bad and good Sender.

Lemma 4 (*Receiver's Beliefs*). *In any equilibrium, P_G and P_B have the same supports. For τ in the support of P_G, $\pi(\tau) \in (0,1)$ is uniquely determined by the system of equations:*

$$\int v_B^*(s)\, dH_B(s|\tau, \pi(\tau)) = v_B^*(\pi(T+1)), \tag{1}$$

$$\sum_{\tau \in supp(P_G)} \frac{1-\pi(\tau)}{\pi(\tau)} \left(P_G(\tau) - P_G(\tau-1)\right) = \frac{1-\pi}{\pi}. \tag{2}$$

Proof. In Appendix. □

Intuitively, suppose there exists a time $\tau \in \{1, \ldots, T\}$ at which only good Sender pulls the arm with positive probability. Upon observing that the arm is pulled at τ, Receiver must conclude that Sender is good. But then bad Sender should strictly prefer to pull the arm at τ, contradicting our supposition that only good Sender pulls the arm at τ.

We now characterize the set of equilibria. Theorem 1 states that in all equilibria, at any time when good Sender pulls the arm, she pulls it with probability 1 and bad Sender pulls it with strictly positive probability. The probability with which bad Sender pulls the arm at any time is determined by the condition that the induced interim beliefs keep bad Sender exactly indifferent between pulling the arm then and not pulling it at all.

Theorem 1 (All Equilibria). *A pair (P_G, P_B) constitutes an equilibrium if and only if P_G and P_B have the same supports, and for all τ in the support of P_G, $P_G(\tau) = F_G(\tau)$ and*

$$P_B(\tau) = \frac{\pi}{1-\pi} \sum_{t \in supp(P_G) \ s.t. \ t \leq \tau} \frac{1-\pi(t)}{\pi(t)} \left(P_G(t) - P_G(t-1)\right), \tag{3}$$

where $\pi(\tau)$ are uniquely determined by (1) and (2).

Proof. In Appendix. □

The following theorem characterizes the set of *divine equilibria* of Banks and Sobel (1987) and Cho and Kreps (1987).[8] In such an equilibrium, good Sender pulls the arm as soon as it arrives.

Theorem 2 (Divine Equilibrium). *There exists a divine equilibrium. In any such equilibrium, $P_G(t) = F_G(t)$ for all $t \in \{1, \ldots, T\}$.*

Proof. In Appendix. □

Although there exist a plethora of divine equilibria, in all such equilibria Receiver's beliefs and each type of Sender's pulling probabilities are uniquely determined by (1), (2), $P_G(\tau) = F_G(\tau)$, and

$$P_B(\tau) = \frac{\pi}{1-\pi} \sum_{t \leq \tau} \frac{1-\pi(t)}{\pi(t)} \left(F_G(t) - F_G(t-1)\right) \tag{4}$$

for all $\tau \in \{1, \ldots, T+1\}$.

By Lemma 4, bad Sender is indifferent between pulling the arm at any time when good Sender pulls and not pulling the arm at all. Thus, in every divine equilibrium, bad Sender is indifferent between pulling the arm at any time before the deadline and not pulling the arm at all. Then, by Lemma 2, Receiver's

[8] Divinity is a standard refinement used by the signalling literature. It requires Receiver to attribute a deviation to those types of Sender who would choose it for the widest range of Receiver's interim beliefs. In our setting, the set of divine equilibria coincides with the set of *monotone* equilibria in which Receiver's interim belief about Sender is non-increasing in the pulling time.

interim beliefs $\pi(\tau)$ decrease over time. Furthermore, using $P_G(\tau) = F_G(\tau)$ and (4), we have

$$\frac{1-\pi(\tau)}{\pi(\tau)} = \frac{1-\pi}{\pi}\frac{P_B(\tau)-P_B(\tau-1)}{P_G(\tau)-P_G(\tau-1)}.$$

Therefore the likelihood ratio of an arm being pulled by bad and good Sender increases over time.

Corollary 2 (Equilibrium Dynamics). *In every divine equilibrium, for all* $\tau, \tau' \in \{1, \ldots, T+1\}$ *such that* $\tau < \tau'$, *we have* $\pi(\tau) > \pi(\tau')$ *and*

$$\frac{P_B(\tau)-P_B(\tau-1)}{P_G(\tau)-P_G(\tau-1)} < \frac{P_B(\tau')-P_B(\tau'-1)}{P_G(\tau')-P_G(\tau'-1)}.$$

Pulling the arm is considered good news by Receiver in the sense that Receiver's belief at time τ about Sender's type is higher if Sender pulls the arm than if she does not. Let $\tilde{\pi}(\tau)$ denote Receiver's interim belief that Sender is good given that she does not pull the arm before and including τ. Corollary 3 shows that $\pi(\tau) > \tilde{\pi}(\tau)$. Using (4), we have that for all $\tau \leq T$

$$\begin{aligned}\frac{1-\tilde{\pi}(\tau)}{\tilde{\pi}(\tau)} &= \frac{1-\pi}{\pi}\frac{1-P_B(\tau)}{1-P_G(\tau)} \\ &= \frac{\sum_{t=\tau+1}^{T+1}\frac{1-\pi(t)}{\pi(t)}\left(F_G(t)-F_G(t-1)\right)}{1-F_G(\tau)} \\ &= \mathbb{E}_{F_G}\left[\frac{1-\pi(t)}{\pi(t)} \mid t \geq \tau+1\right].\end{aligned} \tag{5}$$

By Corollary 2 $\pi(t)$ decreases with time, which implies that $\tilde{\pi}(t)$ decreases with time and $\pi(\tau+1) > \tilde{\pi}(\tau)$.

Corollary 3 (Belief Dynamics). *In every divine equilibrium, for all* $\tau, \tau' \in \{1, \ldots, T+1\}$ *such that* $\tau < \tau'$, *we have* $\pi(\tau+1) > \tilde{\pi}(\tau) > \tilde{\pi}(\tau')$.

To understand how primitives of the model affect players' welfare and behavior, in the next section we specialize to a Poisson model. In the Poisson model, however, Assumption 2, part (ii), that it is impossible to fully learn

Sender's type, fails. Nevertheless, Theorem 2 continues to hold even without this assumption, with the caveat that there might exist $\bar{t} \in \{1, \ldots, T\}$ such that $\pi(\tau) = 1$ and $P_B(\tau) = 0$ for all $\tau \leq \bar{t}$.

4 Poisson Model: Comparative Statics

We now specialize to a Poisson model. Time is continuous $t \in [0, T]$.[9] Bad Sender receives the arm at $t = 0$, but good Sender receives the arm at Poisson rate α. After receiving the arm, each type of Sender chooses when to pull it. If the arm is pulled by bad Sender, a *breakdown* occurs at Poisson rate λ. But if the arm is pulled by good Sender, a breakdown never occurs. At a deadline $t = T$, Receiver takes a binary action $a \in \{0, 1\}$.

Each type of Sender gets payoff 1 if $a = 1$ and 0 otherwise. Receiver privately knows her type r, uniformly distributed on the unit interval. If Receiver takes action $a = 0$, he gets payoff r; if he takes action $a = 1$, he gets payoff 1 if Sender is good and 0 otherwise. Therefore, Sender's and Receiver's payoffs as a function of posterior s are given by $v_G^*(s) = v_B^*(s) = s$ and $u^*(s) = (1 + s^2)/2$.

4.1 Divine Equilibrium and Welfare

We begin by explicitly characterizing the divine equilibrium. By Theorem 2 and the discussion at the end of Section 3, the divine equilibrium has the following three properties. First, good Sender pulls the arm as soon as it arrives. Second, bad Sender is indifferent between pulling the arm at any time $t \geq \bar{t} \geq 0$ and not pulling it at all. Third, bad Sender strictly prefers to delay pulling the arm if $t < \bar{t}$. The threshold $\bar{t}$ is uniquely determined by the parameters of the model.

[9] Technically, we use the results from the previous section by treating continuous time as an appropriate limit of discrete time.

In the Poisson model, equations (1) and (2) become

$$\begin{aligned}\frac{\pi(t)\,e^{-\lambda(T-t)}}{\pi(t)+(1-\pi(t))\,e^{-\lambda(T-t)}} &= \pi(T) \text{ for } t \geq \bar{t},\\ \int_0^T \alpha\frac{1-\pi(t)}{\pi(t)}e^{-\alpha t}dt+\frac{1-\pi(T)}{\pi(T)}e^{-\alpha T} &= \frac{1-\pi}{\pi}.\end{aligned}$$

Combining these two equations with the boundary condition $\pi(t) = 1$ for $t < \bar{t}$ yields the explicit solution $\pi(t)$. This completely characterizes the divine equilibrium.

Proposition 1. *In the divine equilibrium, good Sender pulls the arm as soon as it arrives and Receiver's interim belief that Sender is good given pulling time t is:*

$$\pi(t) = \begin{cases}\frac{\pi(T)}{1-\pi(T)\left(e^{\lambda(T-t)}-1\right)} & \textit{if } t \geq \bar{t};\\ 1 & \textit{otherwise,}\end{cases}$$

where $\pi(T)$ is Receiver's posterior belief if the arm is never pulled and

$$\bar{t} = \begin{cases}0 & \textit{if } \pi < \bar{\pi};\\ T-\frac{1}{\lambda}\ln\frac{1}{\pi(T)} & \textit{otherwise,}\end{cases}$$

$$\begin{aligned}\pi(T) &= \begin{cases}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-1} & \textit{if } \pi < \bar{\pi};\\ \left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\lambda}{\alpha+\lambda}} & \textit{otherwise,}\end{cases}\\ \overline{\pi} &= \left[1+\frac{\lambda}{\alpha+\lambda}\left(e^{\lambda T}-e^{-\alpha T}\right)\right]^{-1}.\end{aligned}$$

We define the *probability of withholding*, denoted by q, as the probability that bad Sender never pulls the arm. By Bayes' rule we have

$$\pi(T) = \frac{\pi e^{-\alpha T}}{\pi e^{-\alpha T}+(1-\pi)\,q}, \tag{6}$$

which yields

$$q = \frac{\pi}{1-\pi} \frac{1-\pi(T)}{\pi(T)} e^{-\alpha T}. \tag{7}$$

Proposition 2 presents monotone comparative statics on equilibrium variables.

Proposition 2. *In the divine equilibrium,*

1. *q and $\bar{t}$ increase with π and λ but decrease with α;*

2. *$\pi(T)$ increases with π but decreases with λ and α.*

Proof. In Appendix B. □

Part 1 says that bad Sender pulls the arm later and withholds with a higher probability if the prior belief about Sender is higher, the arrival rate of the breakdown is higher, and the arrival rate of the arm is lower. The intuition is as follows. If the prior belief that Sender is good is high, bad Sender has a lot to lose in case of a breakdown. Similarly, if the arrival rate of the breakdown is high, pulling the arm is likely to reveal that Sender is bad. In both cases, bad Sender is then reluctant to pull the arm. In contrast, if the arrival rate of the arm is high, good Sender is more likely to pull the arm and Receiver will believe that Sender is bad with high probability if she does not pull the arm. In this case, bad Sender is then willing to pull the arm.

Part 2 says that Receiver's posterior belief about Sender if the arm is never pulled is higher if the prior belief about Sender is higher, the arrival rate of the breakdown is lower, and the arrival rate of the arm is lower. Equation (6) suggests that there are direct and strategic effects of the prior belief and the arrival rate of the arm on Receiver's posterior belief. Holding the probability of withholding q constant, a higher prior belief and a lower arrival rate of the arm improve Receiver's posterior belief about Sender if the arm is never pulled. But the strategic effect works in the opposite direction, because the probability of withholding q increases with the prior belief and decreases with the arrival rate of the arm. Part 2 says that the direct effect always dominates the strategic effect in the Poisson model. Finally, a higher arrival rate of the breakdown worsens Receiver's posterior belief about Sender if the arm is never pulled because it increases the probability of withholding 1 but does not affect the behavior of good Sender.

Proposition 3 presents monotone comparative statics on Receiver's and Sender's expected payoffs.

Proposition 3. *In the divine equilibrium,*

1. *the expected payoffs of Receiver and good Sender increase with π, λ, and α;*

2. *the expected payoff of bad Sender increases with π but decreases with λ and α.*

Proof. In Appendix. □

There are both direct and strategic effects of parameters on the equilibrium expected payoffs. Just as in Proposition 2, it turns out that direct effects dominate. Specifically, a higher prior probability that Sender is good increases the expected payoffs of all players; a higher arrival rates of the breakdown and the arm allow Receiver to learn more about Sender and take a more appropriate action, which increases the expected payoffs of Receiver and good Sender, but decreases the expected payoff of bad Sender.

4.2 The Pattern of Release of Political Scandals

To interpret the comparative statics results, we use our motivating example of the strategic release of scandals before elections. Our model delivers precise theoretical predictions which we tie to the available empirical evidence on the likelihood of US presidential scandals.

There is an election with two candidates: an incumbent and an opposition candidate. We interpret Sender as an opposition media outlet and θ as the quality of the opposition candidate relative to the incumbent. If $\theta = G$, then the opposition candidate is superior to the incumbent and the opposition media receives a *good* scandal involving the incumbent at Poisson rate α. If $\theta = B$, then the opposition candidate is inferior to the incumbent and the opposition media receives a *bad* or fabricated scandal involving the incumbent at time $t = 0$. We interpret Receiver as the median voter. At the election date T, the voter chooses to elect either the opposition candidate, $a = 1$, or the incumbent $a = 0$.[10]

A breakdown is to be interpreted as the voter's discovery that the scandal is fabricated. Notice that (i) the prior π is the voter's initial belief that the opposition candidate is superior to the incumbent, (ii) the arrival rate of the breakdown λ is the level of scrutiny applied by the voter to a released scandal, and (iii) the arrival rate of the arm α is the level of investigation of the incumbent by the opposition media.

Proposition 2 says that the probability $P_B(T) = 1 - q$ that bad Sender pulls the arm before the deadline decreases with the prior π. Our model then predicts that the probability of release of a bad scandal decreases with voters' initial belief that the opposition candidate is superior to the incumbent. Notice that this does not imply that the total probability that a scandal is released is higher when voters' initial belief is lower. In fact, the total probability of release of a scandal is given by

$$\begin{aligned} R &\equiv \pi \underbrace{\left(1 - e^{-\alpha T}\right)}_{P_G(T)} + (1 - \pi) \underbrace{(1 - q)}_{P_B(T)} \\ &= 1 - \frac{\pi e^{-\alpha T}}{\pi(T)} \end{aligned}$$

As $P_B(T) = 1 - q$ decreases with π, we have two contrasting effects. On one hand, holding the probability of withholding q constant, a marginal increase in π increases the total probability of release R by $P_G(T) - P_B(T)$, which is positive by Lemma 3. This is a direct effect: if the incumbent is expected to be superior (low π), then there are simply fewer good scandals. On the other hand, conditional on a bad scandal, the probability of release $(1 - q)$ decreases with π. This is a strategic effect: if the incumbent is expected to be superior, the opposition media has greater incentives to release bad scandals.

Part 1 of Proposition 4 says that the strategic effect dominates the direct effect when π is sufficiently low.

[10]Recall that Receiver privately knows his type r. Thus, in our model the opposition media is uncertain about the preference parameter r of the median voter.

Proposition 4. *In the divine equilibrium, the total probability that Sender pulls the arm*

1. *decreases with π if*

$$\pi < \frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T} + \lambda \left(e^{\alpha T} - 1\right)} \in (0, 1)$$

and increases with π otherwise;

2. *decreases with λ;*

3. *increases with α.*

Prediction 1. The total probability of release of a scandal:

1. decreases with voters' initial belief that the opposition candidate is superior to the incumbent if voters' initial belief is sufficiently low. Otherwise, it increases with voters' initial belief that the opposition candidate is superior to the incumbent;
2. decreases with the level of scrutiny applied by voters to a released scandal;
3. increases with the level of investigation of the incumbent by the opposition media.

Proof. In Appendix. □

Nyhan (2015) and Raizada (2013) study what factors determine the likelihood of US presidential scandals. Raizada (2013) finds that more scandals involving the incumbent president are released when economic indicators and approval rates suggest that voters approve of the president. From the perspective of the opposition media, this means that the prior belief π that the opposition candidate is superior to the incumbent is low. Thus, this empirical observation is consistent with our finding in Part 1 of Prediction 1 that the strategic effect can dominate the direct effect.

Nyhan (2015) finds that more scandals involving the incumbent president are released when opposition voters are more hostile to the president. The author conjectures that when opposition voters are more hostile to the president, then they are "supportive of scandal allegations against the president and less sensitive to the evidentiary basis for these claims" (p. 6). This mechanism is therefore consistent with Part 2 of Prediction 1.

Nyhan (2015) also finds that more scandals involving the incumbent president are released when the news agenda is less congested. When the news agenda is congested, the opposition media can devote fewer resources to investigating the incumbent, thus reducing the arrival rate of scandals. This empirical observation is therefore consistent with Part 3 of Prediction 1.

As breakdowns are observable, it is also possible to test how the release time of a scandal affects its likelihood to be discovered to be bad before the election. As bad scandals are released later than good ones, then there are two contrasting effects. On one hand, conditional on being bad, a scandal released earlier on is directly more likely to produce a breakdown. On the other hand, bad scandals are strategically more likely to be released later. The following corollary says that the strategic effect dominates if the scandal is released sufficiently early (see Figure 1b).

Corollary 4. *In the divine equilibrium, the probability of a breakdown increases with the pulling time t if*

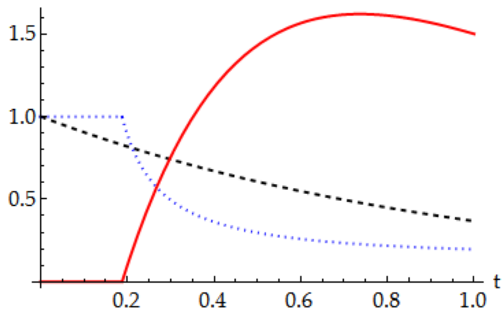
$$t < t_b \equiv T - \frac{1}{\lambda} \ln \left(\frac{1 + \pi(T)}{2\pi(T)} \right) < T$$

and decreases with t otherwise.

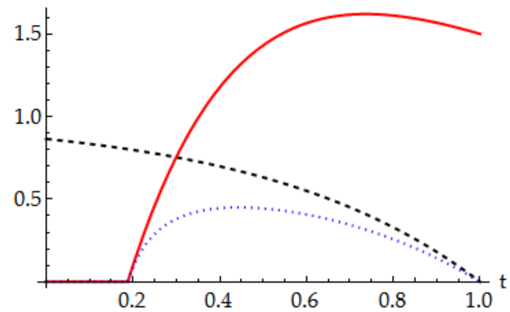
Prediction 2. The probability that a scandal is revealed to be bad increases with its release time if it is released sufficiently early and decreases with its release time otherwise.

Proof. In Appendix. □

Notice that if the interval before the deadline T is sufficiently short or the arrival rate of the arm α is sufficiently small, then t_b is negative and hence the



(a) Pulling density of bad (solid line) and good (dashed line) arms; interim beliefs (dotted line).



(b) Pulling density of bad arm (solid); conditional (i.e., $1 - e^{-\lambda(T-t)}$; dashed) and equilibrium (dotted) probability of a breakdown.

Figure 1: Equilibrium dynamics; $\alpha = 1$, $\lambda = 2$, $\pi = .5$, $T = 1$.

probability of the breakdown monotonically decreases with the pulling time of the arm.

Were it possible to identify ex-post whether a released scandal is bad or good, then one could test whether bad scandals are more likely to be released earlier or later. We can precisely identify the conditions under which bad scandals are more likely to be released later (see Figure 1a).

Corollary 5. *The probability density that bad Sender pulls the arm at time t increases with t if*

$$t < t_p \equiv T - \frac{1}{\lambda} \ln \left(\frac{\alpha}{\alpha + \lambda} \frac{1}{\pi (T)} \right)$$

and decreases with t otherwise.

Prediction 3. The probability of release of a bad scandal increases with time if the scandal is released sufficiently early and decreases with time otherwise.

Proof. In Appendix. □

Notice that if the arrival rate of the arm α is sufficiently small (large), then $t_p > T$ ($t_p < 0$) and hence the probability that bad sender pulls the arm monotonically increases (decreases) with time.

Short-sighted campaigns and political figures might care the most about the instantaneous impact on beliefs upon the release of a scandal at time $t \leq T$, $\pi(t) - \tilde{\pi}(t)$, where (using (5))

$$\tilde{\pi}(t) = \left[1 + \int_t^T \frac{1-\pi(z)}{\pi(z)} \alpha e^{-\alpha(z-t)} dz + \frac{1-\pi(T)}{\pi(T)} e^{-\alpha(T-t)}\right]^{-1}$$

is Receiver's belief at t if Sender has not pulled the arm yet. Corollary 3 says that the instantaneous impact of the release of a scandal is strictly positive for any release time $t < T$. From an empirical perspective, when $\pi(t) - \tilde{\pi}(t)$ is larger, then opinion surveys and voting polls should be more responsive to the release of a scandal. Our model could be used to make predictions about the impact of scandals released at different times. For example, as the election date approaches, $\pi(t) - \tilde{\pi}(t)$ goes to 0, implying that scandals released immediately before an election should have no impact. In contrast, scandals released before the threshold date $\bar{t}$ have greater impact when they are released later.

5 Discussion

5.1 Generalizations

In this section, we discuss how to generalize our model of Section 2 in several directions. We discuss each new aspect in separate paragraphs.

Sender's Risk Attitudes In Section 2 we assumed that good Sender is weakly risk-loving and bad Sender is weakly risk-averse. In contrast, were good Sender very risk-averse, then she could prefer to delay pulling the arm to reduce the spread in posterior beliefs. Likewise, were bad Sender very risk-loving, then she could prefer to pull arm sooner than good Sender to increase the spread in posterior beliefs. However, for a given process satisfying Assumption 2, our results hold if good Sender is not too risk-averse and bad Sender is not too risk-loving. In the Poisson model of Section 4, our results hold for any risk attitude of good Sender and only rely on bad Sender being not too risk-loving.

Stochastic Deadline We assumed throughout that the deadline T is fixed and common knowledge. For many applications, it is more realistic to assume that T is stochastic. In particular, suppose that T is a random variable distributed on $\{\underline{T}, \underline{T}+1, \ldots, \bar{T}\}$where time runs from 1 to $\bar{T}+1$. Now the process L has T as a random variable rather than a constant. For this process, we can define the ex-ante distribution H of posteriors at T, where H depends only on pulling time τ and interim belief p. Notice that Assumption 2 still holds for this ex-ante distribution of posteriors for any $\tau, \tau' \in \{1, \ldots, \bar{T}+1\}$. Therefore, from the ex-ante perspective, Sender's problem is identical to the problem with a deterministic deadline and all results carry over.

Imperfectly Informed Sender We assumed throughout that Sender perfectly knows the binary state θ. If instead Sender privately observes a binary signal $\sigma \in \{\sigma_B, \sigma_G\}$ with $\Pr(\theta = G \mid \sigma_G) > \pi > \Pr(\theta = G \mid \sigma_B)$, then from the perspective of Sender σ_B, pulling the arm earlier results in lower posterior beliefs in expectation, but from the perspective of Sender σ_G, pulling the arm earlier results in higher posterior beliefs in expectation. These statistical results are sufficient to establish that in the divine equilibrium, Sender σ_G pulls the arm as soon as it arrives and bad Sender is indifferent between pulling the arm at any time and not pulling it at all. Moreover, we can allow signal σ to be continuously distributed on the interval $[\underline{\sigma}, \bar{\sigma})$ with normalization $\sigma = \Pr(\theta = G \mid \sigma)$. In this case, there exists a partition equilibrium with $\bar{\sigma} = \sigma_0 > \sigma_1 > \cdots > \sigma_{T+1} = \underline{\sigma}$ such that Sender $\sigma \in [\sigma_t, \sigma_{t-1})$ pulls the arm as soon as it arrives unless it arrives before time $t \in \{1, \ldots, T+1\}$ (and pulls the arm at time t if it arrives before t).

5.2 Extensions and Future Directions

In this section, we discuss several extensions to the Poisson model of Section 4.

Blackout Period More than a third of the world's countries mandate a *blackout period* before elections: a ban on political campaigns for one or more days immediately preceding elections.[11] Without such ban, in our model there exists a (non-divine) equilibrium with a blackout period: neither type of Sender pulls the arm between some time $\underline{t}$ and T. This is consistent with a gentlemen agreement between campaigns without external enforcement. Our model can be employed to compare Receiver's expected payoff under a variety of blackout rules. For a wide range of parameter values, we find that Receiver's optimal blackout period is zero.

Information Leaks Sensitive information about candidates also leaks through social networks and independent channels. One naturally wonders whether such leaks are beneficial for the voters. We can incorporate such leaks into the model by assuming that after arrival the arm can be unintentionally pulled at a random time. Receiver observes whether the arm was pulled unintentionally. In the divine equilibrium, if the arm is pulled unintentionally, then Receiver learns that Sender is bad. This is consistent with voters' interpretation of leaked information as evidence that the candidate was suppressing information. Therefore, in this extension of our model, bad Sender will have an additional incentive to pull the arm earlier to preempt unintentional pulling, which suggests that the voters will learn more about the candidates.

Endogenous Parameters In reality, campaigns not only choose when to release information but also actively seek information that can be released; similarly, voters not only cast votes but also investigate the released information. Our model can be extended to allow Sender to choose the arrival rate of the arm α and Receiver to choose the arrival rate of the breakdown λ, perhaps conditional on pulling time τ. In most applications of interest, seeking information is costly for Sender and scrutiny is costly for Receiver. Results for α are sensitive to two dimensions: (i) whether Sender observes the state before the arrival of the arm and (ii) whether Sender's choice is public information. For example, if Sender observes the state only when arm arrives, then the chosen α is greater when it is private; otherwise, the chosen α is greater when it

[11] The 1992 US Supreme Court sentence *Burson v. Freeman*, 504 US 191, forbids such practices as violations of freedom of speech.

is public. When λ is chosen by Receiver, if we keep Sender's pulling strategy fixed, then Receiver would choose a higher λ if the arm has been pulled later. In turn, this would reduce bad Sender's value of delaying pulling the arm, accelerating Sender's release of information.

5.3 Further Applications

The setup we study is sufficiently rich to capture a number of applications beyond the strategic timing of political scandals.

Funds Management Consider a funds manager who claims to have an investment strategy that generates positive risk-adjusted return, or "alpha". The manager knows whether this is the case, but a potential investor does not. The investor must decide whether to invest in the fund by a fixed deadline.[12] Meanwhile, a new investment opportunity may come along. If it does, the manager can invest or not. If she invests then, if she truly has a positive-alpha strategy, the investment will look good. If, however, she does not have such a strategy then with some probability the investment will be revealed to be bad.

Our model speaks directly to the structure of equilibrium in this market and the amount of information about the true ability of the funds manager that is revealed in equilibrium.

Organizations A fundamental design feature of any organization is to provide incentives for information gathering and sharing within the organization. Suppose Sender is an employee and Receiver is a manager with decision-making authority. The manager has many employees and does not generate ideas herself, but decides whether or not to implement ideas brought to her by employees. The employee in question may have an innate "knack" for finding good ideas (the good type), or not (the bad type). Both types of employee simply want their ideas implemented. In many organizational settings in seems counterfactual that full unraveling occurs. Indeed, management practices such

[12] It is standard practice for closed-end funds to announce, well in advance, a date by which commitments to the fund must be finalized.

as choosing which ideas to implement is often seen as an important explanation for differential firm performance for firms within the same industry.

We feel that the temporal dimension of disclosure problems in organizations is a first-order feature in explaining why unraveling fails. Employees are not typically endowed with "ideas"—they arrive over time, perhaps as the result of a search process; and persuading a manager to implement an idea typically involves an evaluation process that takes time and involves potentially both type-I and type-II errors. Our model captures these features of the problem.

Election Candidatures While the time of elections is usually fixed, candidates can announce their intention to run at different times. A US politician can announce his or her intention to run (and file as a candidate with the Federal Election Commission) for President at any time. Some candidates announce their intention to run much before the election. For example, by December 2014, 142 candidates filed for the 2016 Presidential Election and candidate Yinka Abosede Adeshina filed as early as September 9, 2010.[13] As official candidates receive more attention in the media, voters accumulate more information about them if their announcement comes earlier. Earlier announcements, therefore, might signal to voters that the candidate has "nothing to hide." Our results suggest that earlier candidatures should come from less well-known candidates, for whom the prior probability of being a suitable candidate is lower.

5.4 Concluding remarks

The time at which information is released is an important strategic consideration in a wide range of economic problems. In such problems Sender of information faces a tradeoff between *credibility* from early release, but the concomitant *scrutiny* that goes with it.

We have shown that balancing these competing effects induces Sender to

[13]Source: Federal Election Commission: http://fec.gov/press/resources/2016presidential_form2nm.shtml, retrieved December 20, 2014.

strategically delay the release of information to Receiver. This provides a new rationale for why information is withheld, at least for a time, in a variety of settings such as within organizations. We have also offered a setting in which the welfare effects of this effect can be evaluated.

Understanding this credibility-scrutiny tradeoff has important implications for the design of a variety of institutions, and we have highlighted a few of those in this section. We hope our model will serve as a useful framework for addressing these design questions in more depth, and with greater attention to institutional detail, in the future.

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A Omitted Proofs

***Proof of Lemma** 1. Part 1.* By Blackwell (1953), Assumption 2 with $\tau' = T + 1$ implies that releasing the trigger at τ is the same as releasing a finite-valued informative signal y. By Bayes' rule, posterior s is given by:

$$s = \frac{pq\left(y \mid G\right)}{pq\left(y \mid G\right) + \left(1-p\right) q\left(y \mid B\right)}$$

where $q\left(y \mid \theta\right)$ is the probability of y given θ. Therefore,

$$\frac{q\left(y \mid G\right)}{q\left(y \mid B\right)} = \frac{1-p}{p}\frac{s}{1-s}. \tag{8}$$

Writing (8) for interim beliefs p and p', we obtain the following relation for corresponding posterior beliefs s and s':

$$\frac{1-p'}{p'}\frac{s'}{1-s'} = \frac{1-p}{p}\frac{s}{1-s}$$

which implies that $s' > s$ for $p' > p$; so part 1 follows.

Part 2. By Blackwell (1953), Assumption 2 implies that pulling the arm at τ is the same as pulling the arm at τ' and then releasing an additional finite-valued informative signal y. A signal y is informative if there exists y such that $q(y \mid G)$ is not equal to $q(y \mid B)$. Part 2 holds because for any strictly increasing concave v^*, we have

$$\begin{aligned}
\mathbb{E}\left[v^*(s) \mid \tau, p, B\right] &= \mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', p, B\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', s, B\right] \mid \tau', p, B\right] \\
&\leq \mathbb{E}\left[v^*\left(\mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)} \mid \tau', s, B\right]\right) \mid \tau', p, B\right] \\
&< \mathbb{E}\left[v^*\left(\frac{s\mathbb{E}\left[\frac{q(y|G)}{q(y|B)} \mid \tau', s, B\right]}{s\mathbb{E}\left[\frac{q(y|G)}{q(y|B)} \mid \tau', s, B\right] + (1-s)}\right) \mid \tau', p, B\right] \\
&= \mathbb{E}\left[v^*\left(\frac{s\sum \frac{q(y|G)}{q(y|B)} q(y \mid B)}{s\sum \frac{q(y|G)}{q(y|B)} q(y \mid B) + 1 - s}\right) \mid \tau', p, B\right] \\
&= \mathbb{E}\left[v^*(s) \mid \tau', p, B\right],
\end{aligned}$$

where the first line holds by Bayes' rule, the second by the law of iterated expectations, the third by Jensen's inequality applied to concave v^*, the fourth by strict monotonicity of v^* and Jensen's inequality applied to strictly concave function $f(z) \equiv sz/(sz + 1 - s)$, the fifth by definition of expectations, and the last by Kolmogorov's axioms.

Part 3. Analogously to Part 2, Part 3 holds because for any strictly increasing convex v^*, we have

$$\begin{aligned}
\mathbb{E}\left[v^*(s) \mid p, \tau, G\right] &= \mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', p, G\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[v^*\left(\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)}\right) \mid \tau', s, G\right] \mid \tau', p, G\right] \\
&\geq \mathbb{E}\left[v^*\left(\mathbb{E}\left[\frac{sq(y \mid G)}{sq(y \mid G) + (1-s)\,q(y \mid B)} \mid \tau', s, G\right]\right) \mid \tau', p, G\right] \\
&> \mathbb{E}\left[v^*\left(\frac{s}{s + (1-s)\,\mathbb{E}\left[\frac{q(y|B)}{q(y|G)} \mid \tau', s, G\right]}\right) \mid \tau', p, G\right] \\
&= \mathbb{E}\left[v^*\left(\frac{s}{s + (1-s)\sum \frac{q(y|B)}{q(y|G)} q(y \mid G)}\right) \mid \tau', p, G\right] \\
&= \mathbb{E}\left[v^*(s) \mid p, \tau', G\right].
\end{aligned}$$

□

Proof of Lemma 2. *Part 1.* Suppose, on the contrary, that $\pi(\tau) \leq \pi(\tau')$. Then

$$\begin{aligned}
\int v_B^*(s)\, dH_B(s|\tau, \pi(\tau)) &\leq \int v_B^*(s)\, dH_B(s|\tau', \pi(\tau)) \\
&\leq \int v_B^*(s)\, dH_B(s|\tau', \pi(\tau')),
\end{aligned}$$

where the first inequality holds by Lemma 1 part 2 and the second by Lemma 1 part 1. Moreover, at least one inequality is strict. Indeed, if $\pi(\tau) \in (0,1)$, then the first inequality is strict. If $\pi(\tau) = 0$, then $\pi(\tau') > 0$ (because $\pi(\tau) = \pi(\tau') = 0$ is not allowed); so the second inequality is strict. Finally, if $\pi(\tau) = 1$, then $\pi(\tau) \leq \pi(\tau')$ cannot hold (because $\pi(\tau) = \pi(\tau') = 1$ is not allowed). The displayed inequality implies that bad Sender strictly prefers to pull the arm at τ' than at τ. A contradiction.

Good Sender strictly prefers to pull the arm at τ because

$$\begin{aligned}\int v_G^*(s)\,dH_G(s|\tau,\pi(\tau)) &\geq \int v_G^*(s)\,dH_G\left(s|\tau',\pi(\tau)\right)\\ &> \int v_G^*(s)\,dH_G\left(s|\tau',\pi\left(\tau'\right)\right),\end{aligned}$$

where the first inequality holds by Lemma 1 part 3 and the second by $\pi(\tau) > \pi(\tau')$ and Lemma 1 part 1.

Part 2. If τ is in the support of P_B, then bad Sender weakly prefers to pull the arm at τ than at any other $\tau' > \tau$. By Bayes' rule $\pi(\tau) < 1$. Also, $\pi(\tau)$ cannot be zero, otherwise bad Sender would strictly prefer to pull the arm at $T+1$ since $\pi(T+1) > 0$ by $F_G(T) < 1$. Therefore, by part 1 of this lemma, good Sender strictly prefers to pull the arm at τ than at any other $\tau' > \tau$; so $P_G(\tau) = F_G(\tau)$. □

Proof of Lemma 3. Suppose, on the contrary, that there exists τ such that $P_G(\tau) > 0$ and $P_B(\tau) \geq P_G(\tau)$. Because $P_\theta(\tau) = \sum_{t=1}^{\tau}(P_\theta(t) - P_\theta(t-1))$, there exists $\tau' \leq \tau$ in the support of P_B such that $P_B(\tau') - P_B(\tau'-1) \geq P_G(\tau') - P_G(\tau'-1)$. Similarly, because $1 - P_\theta(\tau) = \sum_{t=\tau+1}^{T+1}(P_\theta(t) - P_\theta(t-1))$ and $1 - P_G(\tau) > 0$ by $P_G(T) \leq F_G(T) < 1$, there exists $\tau'' > \tau$ in the support of P_G such that $P_G(\tau'') - P_G(\tau''-1) \geq P_B(\tau'') - P_B(\tau''-1)$. By Bayes' rule,

$$\begin{aligned}\pi\left(\tau'\right) &= \frac{\pi\left(P_G\left(\tau'\right) - P_G\left(\tau'-1\right)\right)}{\pi\left(P_G\left(\tau'\right) - P_G\left(\tau'-1\right)\right) + (1-\pi)\left(P_B\left(\tau'\right) - P_B\left(\tau'-1\right)\right)} \leq \pi\\ &\leq \frac{\pi\left(P_G\left(\tau''\right) - P_G\left(\tau''-1\right)\right)}{\pi\left(P_G\left(\tau''\right) - P_G\left(\tau''-1\right)\right) + (1-\pi)\left(P_B\left(\tau''\right) - P_B\left(\tau''-1\right)\right)} = \pi\left(\tau''\right).\end{aligned}$$

Therefore, by Lemma 2, bad Sender strictly prefers to pull the arm at τ'' than at τ', which implies that τ' cannot be in the support of P_B. A contradiction. □

Proof of Lemma 4. By Lemma 2 part 2, each t' in the support of P_B is also in the support of P_G. Suppose, on the contrary, that there exists t' in the support of P_G but not in the support of P_B. Then, by Bayes' rule $\pi(t') = 1$; so bad Sender who receives the arm at $t \leq t'$ gets the highest possible equilibrium payoff $v_B^*(1)$. Therefore, there exists a period $\tau \geq t'$ at which $\pi(\tau) = 1$ (recall that the support of $H(.|\tau, \pi)$ does not contain $s = 1$) and bad Sender pulls the arm with a positive probability. A contradiction. Therefore, P_G and P_B have the same supports and therefore $\pi(\tau) \in (0, 1)$.

Let the support of P_G be $\{\tau_1, ..., \tau_n\}$. Notice that $\tau_n = T + 1$ because $P_G(T) \leq F_G(T) < 1$. Since τ_{n-1} is in the support of P_B and

$$P_B(\tau_{n-1}) < P_G(\tau_{n-1}) = F_G(\tau_{n-1}) \leq F_B(\tau_{n-1}),$$

where the first inequality holds by Lemma 3, the equality by Lemma 2 part 2, and the last inequality holds by Assumption 1. Therefore, bad Sender who receives the arm at τ_{n-1} must be indifferent between pulling the arm at τ_{n-1} or at τ_n. Analogously, bad Sender who receives the arm at τ_{n-k-1} must be indifferent between τ_{n-k-1} and some $\tau \in \{\tau_{n-k}, \ldots, \tau_n\}$. Thus, by mathematical induction on k, bad Sender is indifferent between all τ in the support of P_G, which proves (1).

By Bayes' rule, for all τ in the support of P_G,

$$\frac{1-\pi}{\pi}\left(P_B(\tau) - P_B(\tau-1)\right) = \frac{1-\pi(\tau)}{\pi(\tau)}\left(P_G(\tau) - P_G(\tau-1)\right). \tag{9}$$

Summing up over τ yields (2). Finally, suppose, on the contrary, that there exist two distinct solutions π' and π'' to (1) and (2). By Lemma 1 part 1, (1) uniquely determines $\pi(\tau)$ for a given $\pi(T+1)$ and $\pi(\tau)$ is increasing in $\pi(T+1)$. Thus, for π' and π'' to be distinct, it must be that $\pi'(T+1) \neq \pi''(T+1)$. Without loss, suppose that $\pi'(T+1) < \pi''(T+1)$, and thus $\pi'(\tau) < \pi''(\tau)$ for all τ in the support of P_G. But then (2) cannot hold for both π' and π''. A contradiction. □

Proof of Theorem 1. Using Lemmas 3 and 4 together with (9) proves the *only if* part of the theorem. Setting $\pi(\tau) = 0$ for τ not in the support of P_G and using Lemma 2 proves the *if* part of the theorem. □

Proof of Theorem 2. First, we notice that, by Theorem 1, there exists an equilibrium with $P_G(t) = F_G(t)$ for all t.

Adopting Cho and Kreps (1987)'s definition to our setting (see e.g., Maskin and Tirole, 1992), we say that an equilibrium is divine if $\pi(t) = 1$ for any $t \notin supp(P_G)$ at which condition D1 holds. D1 holds at t if for all $p \in [0,1]$ that satisfy

$$\int v_B^*(s)\, dH_B(s|t,p) \geq \max_{\tau \in supp(P_G), \tau > t} \int v_B^*(s)\, dH_B(s|\tau, \pi(\tau)) \tag{10}$$

the following inequality holds:

$$\int v_G^*(s)\, dH_G(s|t,p) > \max_{\tau \in supp(P_G), \tau > t} \int v_G^*(s)\, dH_G(s|\tau, \pi(\tau)) . \tag{11}$$

Suppose, on the contrary, that there exists a divine equilibrium in which $P_G(t) < F_G(t)$ for some $t \in \{1, \ldots, T\}$. By Theorem 1, $t \notin supp(P_G)$. Let τ^* denote τ that maximizes the right hand side of (11). By Lemma 4, $\pi(\tau^*) \in (0,1)$ and τ^* maximizes the right hand side of (10). Therefore, by Lemma 2 part 1, D1 holds at t; so $\pi(t) = 1$. But then $t \notin supp(P_G)$ cannot hold, because

$$\int v_G^*(s)\, dH_G(s|t,1) = v_G^*(1) > \max_{\tau \in supp(P_G)} \int v_G^*(s)\, dH_G(s|\tau, \pi(\tau)) .$$

□

Proof of Proposition 3. We first prove part 2. We then separately prove the results about good Sender's and Receiver's expected payoffs in part 1.

Part 2: Bad Sender's expected payoff. Recall that (i) Sender's payoff equals Receiver's posterior belief about Sender at $t = T$ and (ii) in equilibrium, bad Sender (weakly) prefers not to pull the arm at all than pulling it at any time $t \in [0, T]$. Therefore, bad Sender's expected payoff equals Receiver's belief about Sender at $t = T$ if the arm has not been pulled:

$$\mathbb{E}[v_B] = \pi(T) . \tag{12}$$

Part 2 then follows from Proposition 2.

Part 1: Good Sender's expected payoff. By the law of iterated expectations,

$$\begin{aligned}\mathbb{E}[s] &= \pi\mathbb{E}[v_G] + (1-\pi)\mathbb{E}[v_B] = \pi \\ \Rightarrow \mathbb{E}[v_G] &= 1 - \frac{1-\pi}{\pi}\pi(T)\end{aligned}$$

where s is Receiver's posterior belief about Sender at $t = T$ and we used (12) in the last passage. Thus, good Sender's expected payoff increases with α and λ by Proposition 2. Finally, it is easy to see that $\mathbb{E}[v_G]$ increases in π after sustituting $\pi(T)$ in $\mathbb{E}[v_G]$.

Part 1: Receiver's expected payoff. We shall show that in the divine equilibrium

$$\mathbb{E}[u] = \frac{1+\pi\mathbb{E}[v_G]}{2}. \tag{13}$$

Part 1 about Receiver's expected payoff then follows from the result in Part 1 about good Sender's expected payoff.

To prove (13), we divide the proof in two cases: $\pi \le \bar{\pi}$ and $\pi > \bar{\pi}$. If $\pi \le \bar{\pi}$, Receiver's expected payoff is given by the sum of four terms: (i) Sender is good and the arm does not arrive; (ii) Sender is good and the arm arrives; (iii) Sender is bad and she does not pull the arm; and (iv) Sender is bad and she pulls the arm. Thus,

$$\begin{aligned}2\mathbb{E}[u] - 1 = {} & \pi e^{-\alpha T}(\pi(T))^2 \\ & +\pi\int_0^T \left(e^{\lambda(T-t)}\pi(T)\right)^2 \alpha e^{-\alpha t}dt \\ & +(1-\pi)q(\pi(T))^2 \\ & +(1-\pi)\int_0^T e^{-\lambda(T-t)}\left(e^{\lambda(T-t)}\pi(T)\right)^2 \frac{\pi}{1-\pi}\left(\frac{1-\pi(t)}{\pi(t)}\right)\alpha e^{-\alpha t}dt.\end{aligned}$$

Solving all integrals and rearranging all common terms we get

$$2\mathbb{E}[u] - 1 = \pi\mathbb{E}[v_G].$$

If $\pi > \bar{\pi}$, Receiver's expected payoff is given by the sum of five terms: (i) Sender is good and the arm does not arrive; (ii) Sender is good and the arm arrives before $\bar{t}$; (iii) Sender is good and the arm arrives between $\bar{t}$ and T; (iv) Sender is bad and she does not pull the arm; (v) Sender is bad and she pulls the arm. Thus,

$$\begin{aligned}
2\mathbb{E}\left[u\right]-1 \quad = \quad & \pi e^{-\alpha T}\left(\pi\left(T\right)\right)^{2} \\
& +\pi\left(1-e^{-\alpha\bar{t}}\right) \\
& +\pi\int_{\bar{t}}^{T}\left(e^{\lambda(T-t)}\pi\left(T\right)\right)^{2}\alpha e^{-\alpha t}dt+ \\
& +\left(1-\pi\right)q\left(\pi\left(T\right)\right)^{2} \\
& +\left(1-\pi\right)\int_{\bar{t}}^{T}e^{-\lambda(T-t)}\left(e^{\lambda(T-t)}\pi\left(T\right)\right)^{2}\frac{\pi}{1-\pi}\left(\frac{1-\pi\left(t\right)}{\pi\left(t\right)}\right)\alpha e^{-\alpha t}dt.
\end{aligned}$$

Solving all integrals and rearranging all common terms we again get

$$2\mathbb{E}\left[u\right]-1 \quad = \quad \pi\mathbb{E}\left[v_G\right].$$

□

Proof of Proposition 4. Part 1: We differentiate R with respect to π:

$$\begin{aligned}
\frac{dR}{d\pi} \quad &= \quad \frac{d}{d\pi}\left(1-\frac{\pi}{\pi\left(T\right)}e^{-\alpha T}\right) \\
&= \quad -e^{-\alpha T}\frac{\pi\left(T\right)-\frac{d\pi(T)}{d\pi}\pi}{\pi\left(T\right)^{2}}.
\end{aligned}$$

Therefore R is non-decreasing in π if and only if

$$\frac{d\pi\left(T\right)}{d\pi}\geq\frac{\pi\left(T\right)}{\pi}.$$

We now show that

$$\frac{d\pi\left(T\right)}{d\pi}\geq\frac{\pi\left(T\right)}{\pi}\iff\pi\geq\frac{\alpha e^{\alpha T}}{\left(\alpha+\lambda\right)e^{\alpha T}-1}.$$

Differentiating $\pi(T)$ with respect to π, we have

$$\frac{d\pi(T)}{d\pi} = \begin{cases} \frac{d}{d\pi}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-1} & \text{if } \pi < \overline{\pi}, \\ \frac{d}{d\pi}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} + 1\right]^{-\frac{\lambda}{\alpha+\lambda}} & \text{otherwise,} \end{cases}$$
$$= \begin{cases} \frac{1}{\pi^2}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda} + \frac{1-\pi}{\pi}\right]^{-2} & \text{if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} + 1\right]^{-\frac{\alpha+2\lambda}{\alpha+\lambda}} & \text{otherwise,} \end{cases}$$
$$= \begin{cases} \frac{\pi(T)^2}{\pi^2} & \text{if } \pi < \bar{\pi}; \\ e^{\alpha T}\frac{\pi(T)^{2+\frac{\alpha}{\lambda}}}{\pi^2} & \text{otherwise.} \end{cases}$$

Case 1: $\pi < \bar{\pi}$.
If $\underline{\pi < \bar{\pi}}$, then $dR/d\pi < 0$ because $\pi(T) < \pi$ and

$$\frac{d\pi(T)}{d\pi} = \frac{\pi(T)^2}{\pi^2} < \frac{\pi(T)}{\pi}.$$

Case 2: $\pi \geq \bar{\pi}$.
If $\pi \geq \bar{\pi}$, then $dR/d\pi < 0$ if and only if

$$\frac{d\pi(T)}{d\pi} = e^{\alpha T}\frac{\pi(T)^{2+\frac{\alpha}{\lambda}}}{\pi^2} < \frac{\pi(T)}{\pi}e^{\alpha T}.$$

Substituting $\pi(T)$, we get that this inequality is equivalent to

$$\pi < \frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T} + \lambda\left(e^{\alpha T} - 1\right)}.$$

It remains to show that

$$\frac{\alpha e^{\alpha T}}{\alpha e^{\alpha T} + \lambda\left(e^{\alpha T} - 1\right)} > \bar{\pi}.$$

Substituting $\bar{\pi}$, we get that this inequality is equivalent to

$$\frac{e^{(\alpha+\lambda)T} - 1}{\alpha + \lambda} > \frac{e^{\alpha T} - 1}{\alpha},$$

which is satisfied because function $(e^x - 1)/x$ is increasing in x.

Part 2: We differentiate R with respect to λ:

$$\begin{aligned}\frac{dR}{d\lambda} &= \frac{d}{d\lambda}\left[\pi\left(1-e^{-\alpha T}\right)+(1-\pi)(1-q)\right] \\ &= -(1-\pi)\frac{dq}{d\lambda} < 0\end{aligned}$$

where the last inequality follows from Proposition 2.

Part 3: We differentiate R with respect to α

$$\begin{aligned}\frac{dR}{d\lambda} &= \frac{d}{d\alpha}\left[\pi\left(1-e^{-\alpha T}\right)+(1-\pi)(1-q)\right] \\ &> -(1-\pi)\frac{dq}{d\alpha} > 0,\end{aligned}$$

where the last inequality follows from Proposition 2. □

Proof of Corollary 4. The unconditional probability of a breakdown of the arm pulled at t is given by

$$\Pr(\text{bd} \mid t) \equiv \left(1-e^{-\lambda(T-t)}\right)[1-\pi(t)].$$

Notice that $\Pr(\text{bd} \mid t)$ is continuous in t because $\pi(t)$ is continuous in t. Also, $\Pr(\text{bd} \mid t)$ equals 0 for $t \leq \bar{t}$, is strictly positive for all $t \in (\bar{t}, T)$, and equals 0 for $t = T$. Substituting $\pi(t)$ and taking the derivative of $\Pr(\text{bd} \mid t)$ with respect to $t \geq \bar{t}$ we have

$$\frac{d\Pr(\text{bd} \mid t)}{dt} = -\lambda\frac{e^{-\lambda(T-t)}(1+\pi(T))-2\pi(T)}{\left[1-\pi(T)\left(1-e^{\lambda(T-t)}\right)\right]^2}$$

which is positive if and only if

$$T - \frac{1}{\lambda}\ln\left(\frac{1+\pi(T)}{2\pi(T)}\right) < T.$$

□

Proof of Corollary 5. The probability density that bad Sender pulls the arm at time t is given by

$$R_B \equiv \frac{dP_B(T)}{dt} = \frac{\alpha e^{-\alpha t}\left(1 - \pi(T)\, e^{\lambda(T-t)}\right)}{\pi(T)}$$

for $t > \bar{t}$ and $R_B = 0$ for $t \leq \bar{t}$. Differentiating with respect to t we get

$$\frac{dR_B}{dt} = \frac{\alpha e^{-\alpha t}}{\pi(T)} \left[(\alpha + \lambda)\, \pi(T)\, e^{\lambda(T-t)} - \alpha\right]$$

which is positive if and only if

$$\bar{t} \leq t < T - \frac{1}{\lambda} \ln\left(\frac{\alpha}{\alpha + \lambda} \frac{1}{\pi(T)}\right).$$

□

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Proof of Proposition 2. We first prove part 2 and then part 1.

Part 2:

For π:

$$\frac{d\pi(T)}{d\pi} = \begin{cases} \frac{d}{d\pi}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-1} \text{ if } \pi < \overline{\pi}, \\ \frac{d}{d\pi}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases}$$

$$= \begin{cases} \frac{1}{\pi^2}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-2} \text{ if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\alpha+2\lambda}{\alpha+\lambda}} \text{ otherwise,} \end{cases}$$

$$= \left\{ \begin{array}{l} \frac{1}{\pi^2}\pi(T)^2 \text{ if } \pi < \overline{\pi}, \\ \frac{e^{\alpha T}}{\pi^2}\pi(T)^{2+\frac{\alpha}{\lambda}} \text{ otherwise,} \end{array} \right\} > 0.$$

For λ:

$$\frac{d\pi(T)}{d\lambda} = \begin{cases} \frac{d}{d\lambda}\left[\frac{\alpha e^{\lambda T}+\lambda e^{-\alpha T}}{\alpha+\lambda}+\frac{1-\pi}{\pi}\right]^{-1} \text{ if } \pi < \overline{\pi}, \\ \frac{d}{d\lambda}\left[\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T}+1\right]^{-\frac{\lambda}{\alpha+\lambda}} \text{ otherwise.} \end{cases}$$

First, when $\pi < \bar{\pi}$, $\frac{d\pi(T)}{d\lambda} < 0$ since $e^{-(\alpha+\lambda)T} > 1 - (\alpha+\lambda)T$ for all $\alpha, \lambda, T > 0$. Second, let

$$\phi(\lambda) \equiv \frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi}e^{\alpha T} > 0.$$

Then, when $\pi > \bar{\pi}$,

$$\frac{d\pi(T)}{d\lambda} = \frac{d}{d\lambda}e^{-\frac{\lambda}{\alpha+\lambda}\ln(1+\phi(\lambda))}.$$

To show $\frac{d\pi(T)}{d\lambda} < 0$ it is then sufficient to note that

$$\frac{d}{d\lambda}\frac{\lambda}{\alpha+\lambda}\ln(1+\phi(\lambda)) = \frac{1}{\alpha+\lambda}\left[\frac{\ln(1+\phi(\lambda))}{\alpha+\lambda} - \frac{1}{\lambda}\frac{1-\pi}{\pi}\frac{1}{1+\phi(\lambda)}\right] > 0$$

where the last passage follows from $(1+\phi(\lambda))\ln(1+\phi(\lambda)) > \phi(\lambda)$.

For α:

If $\pi < \bar{\pi}$, then

$$\begin{aligned} \frac{d\pi(T)}{d\alpha} &= -(\pi(T))^2 \frac{\chi}{(\alpha+\lambda)^2} < 0 \\ \chi &\equiv \lambda \left\{ e^{\lambda T} - [1 + (\alpha+\lambda) T] e^{-\alpha T} \right\} > 0 \end{aligned}$$

where the last passage follows from $e^{(\alpha+\lambda)T} > 1 + (\alpha+\lambda)T$ for all $\alpha, \lambda, T > 0$.

If $\pi \geq \bar{\pi}$, by log-differentiation,

$$\begin{aligned} \frac{d\pi(T)}{d\alpha} &= \pi(T) \frac{\lambda}{\alpha+\lambda} \left[\frac{\ln(1+\xi)}{\alpha+\lambda} - \frac{\frac{d\xi}{d\alpha}}{1+\xi} \right] \\ \xi &\equiv \frac{\alpha+\lambda}{\lambda} \frac{1-\pi}{\pi} e^{\alpha T}. \end{aligned}$$

Thus,

$$\frac{d\pi(T)}{d\alpha} < 0 \iff \frac{(1+\xi)\ln(1+\xi)}{\xi} < 1 + T(\alpha+\lambda). \tag{14}$$

For $\pi = \overline{\pi}$, $\xi = e^{(\alpha+\lambda)T} - 1 > 0$; so

$$\frac{d\pi(T)}{d\alpha} < 0 \iff \ln(1+\xi) < \xi,$$

which is true for all $\xi > 0$. If π is greater than $\overline{\pi}$, then ξ is smaller than $e^{(\alpha+\lambda)T} - 1$ and the inequality (14) is stronger because the left hand side is increasing in ξ for $\xi > 0$; so $\frac{d\pi(T)}{d\alpha} < 0$ for $\pi \geq \bar{\pi}$.

Part 1:

For π on q:

$$\begin{aligned} \frac{dq}{d\pi} &= \frac{d}{d\pi} \left[\frac{\pi}{1-\pi} \frac{1-\pi(T)}{\pi(T)} e^{-\alpha T} \right] \\ &= \frac{e^{-\alpha T}}{\pi(T)(1-\pi)} \times \left[\frac{1-\pi(T)}{1-\pi} - \frac{\pi}{\pi(T)} \frac{d\pi(T)}{d\pi} \right], \\ &= \underbrace{\frac{e^{-\alpha T}}{\pi(T)(1-\pi)}}_{>0} \times \left\{ \begin{array}{l} \left[\frac{1-\pi(T)}{1-\pi} - \frac{\pi(T)}{\pi} \right] \text{ if } \pi < \overline{\pi}, \\ \left[\frac{1-\pi(T)}{1-\pi} - e^{\alpha T} \frac{\pi(T)^{1+\frac{\alpha}{\lambda}}}{\pi} \right] \text{ otherwise.} \end{array} \right\} \end{aligned}$$

If $\pi < \bar{\pi}$, then $\frac{dq}{d\pi} > 0$ if and only if $\frac{1-\pi(T)}{\pi(T)} > \frac{1-\pi}{\pi}$, which is satisfied since in equilibrium $\pi(T) < \pi$.

If $\pi \geq \bar{\pi}$, then $\frac{dq}{d\pi} > 0$ if and only if

$$\frac{1-\pi(T)}{\pi(T)} > \frac{1-\pi}{\pi} e^{\alpha T} \pi(T)^{\frac{\alpha}{\lambda}}, \tag{15}$$

which can be rewritten as

$$\begin{aligned} 1 - (1+\phi(\lambda))^{-\frac{\lambda}{\alpha+\lambda}} &> \frac{\lambda}{\alpha+\lambda} \frac{\phi(\lambda)}{1+\phi(\lambda)} \\ \iff 1 + \phi(\lambda) - (1+\phi(\lambda))^{\frac{\alpha}{\alpha+\lambda}} &> \frac{\lambda}{\alpha+\lambda} \phi(\lambda) \\ \iff 1 + \frac{\alpha}{\alpha+\lambda} \phi(\lambda) &> (1+\phi(\lambda))^{\frac{\alpha}{\alpha+\lambda}}. \end{aligned}$$

To conclude, notice that $1 + xb > (1+b)^x$ for $b > 0$ and $x \in (0,1)$.

<u>For π on $\bar{t}$:</u>

For $\pi < \bar{\pi}$, $\bar{t} = 0$, but for $\pi \geq \bar{\pi}$, $\bar{t}$ is increasing in π and decreasing in α because $\pi(T)$ is increasing in π and decreasing in α.

<u>For λ on q:</u>

$$\begin{aligned} \frac{dq}{d\lambda} &= \frac{d}{d\lambda} \left[\frac{\pi}{1-\pi} \frac{1-\pi(T)}{\pi(T)} e^{-\alpha T} \right] \\ &= -\frac{\pi}{1-\pi} \frac{e^{-\alpha T}}{\pi(T)^2} \frac{d\pi(T)}{d\lambda} > 0. \end{aligned}$$

<u>For λ on $\bar{t}$:</u>

For $\pi < \bar{\pi}$, $\bar{t} = 0$, but for $\pi \geq \bar{\pi}$

$$\begin{aligned} \frac{d\bar{t}}{d\lambda} &= -\frac{d}{d\lambda} \left[\frac{1}{\alpha+\lambda} \ln \left(\frac{(\alpha+\lambda)(1-\pi)}{\lambda\pi} e^{\alpha T} + 1 \right) \right] \\ &= \frac{1}{\alpha+\lambda} \left(\frac{\ln(1+\phi(\lambda))}{\alpha+\lambda} - \frac{d\phi(\lambda)}{d\lambda} \frac{1}{1+\phi(\lambda)} \right) > 0 \end{aligned}$$

where the last passage follows from

$$\frac{d\phi(\lambda)}{d\lambda} = -\frac{\alpha}{\lambda^2}\frac{1-\pi}{\pi}e^{\alpha T} < 0.$$

<u>For α on q:</u>
If $\pi < \overline{\pi}$, then

$$\begin{aligned}
\frac{1-\pi}{\pi}\frac{dq}{d\alpha} &= \frac{(\lambda - \alpha(\alpha+\lambda)T)e^{(\lambda-\alpha)T} - \lambda(1+2(\alpha+\lambda)T)e^{-2\alpha T}}{(\alpha+\lambda)^2} - \left(\frac{1}{\pi}-2\right)Te^{-\alpha T} \\
&< \frac{(\lambda - \alpha(\alpha+\lambda)T)e^{(\lambda-\alpha)T} - \lambda(1+2(\alpha+\lambda)T)e^{-2\alpha T}}{(\alpha+\lambda)^2} - \left(\frac{1}{\overline{\overline{\pi}}}-2\right)Te^{-\alpha T} \\
&= -\frac{e^{-2\alpha T}}{(\alpha+\lambda)^2}\left(\lambda(1+(\alpha+\lambda)T) + \left((\alpha+\lambda)^2T-\lambda\right)e^{(\alpha+\lambda)T} - (\alpha+\lambda)^2Te^{\alpha T}\right).
\end{aligned}$$

Thus $dq/d\alpha < 0$, because for all positive α and λ

$$\begin{aligned}
f(\alpha,\lambda) &= \lambda(1+(\alpha+\lambda)) + \left((\alpha+\lambda)^2 - \lambda\right)e^{(\lambda+\alpha)} - (\alpha+\lambda)^2 e^{\alpha} \\
&= \sum_{k=3}^{\infty}\left[\frac{(\alpha+\lambda)^k}{(k-2)!} - \lambda\frac{(\alpha+\lambda)^{k-1}}{(k-1)!} - (\alpha+\lambda)^2\frac{\alpha^{k-2}}{(k-2)!}\right] > 0,
\end{aligned}$$

□

where the inequality holds because each term c_k in the sum is positive:

$$\begin{aligned}
c_k &= \frac{(\alpha+\lambda)^2\left((\alpha+\lambda)^{k-2} - \alpha^{k-2}\right)}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} \\
&= \frac{(\alpha+\lambda)^2\lambda\left(\sum_{n=0}^{k-3}(\alpha+\lambda)^{k-3-n}\alpha^n\right)}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} \\
&> \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-2)!} - \frac{(\alpha+\lambda)^2\lambda(\alpha+\lambda)^{k-3}}{(k-1)!} > 0.
\end{aligned}$$

If $\pi \geq \bar{\pi}$, then without loss of generality we can set $T = 1$ and get

$$\frac{1-\pi}{\pi}\frac{dq}{d\alpha} = e^{-\alpha}\left[1 - \frac{1}{\pi(T)}\left(1 + \frac{d\pi(T)}{d\alpha}\pi(T)^{-1}\right)\right] < 0$$

$$\iff \frac{d\pi(T)}{d\alpha} > \pi(T)(\pi(T)-1).$$

This inequality is equivalent to:

$$\frac{1+\xi}{\xi}\left[\ln(1+\xi) + \frac{(\alpha+\lambda)^2}{\lambda}\left(1-(1+\xi)^{-\frac{\alpha+\lambda}{\lambda}}\right)\right] - 1 - \alpha - \lambda > 0$$

The left hand side is increasing in α, treating ξ as a constant. Then the inequality holds because it holds for $\alpha \to 0$:

$$\begin{aligned}
\frac{1+\xi}{\xi}\left[\ln(1+\xi) + \lambda\left(1-(1+\xi)^{-1}\right)\right] - 1 - \lambda &> 0 \\
\frac{1+\xi}{\xi}\left[\ln(1+\xi) + \lambda\frac{\xi}{1+\xi}\right] - 1 - \lambda &> 0 \\
\frac{1+\xi}{\xi}\ln(1+\xi) &> 1.
\end{aligned}$$