In the 2016 general election, sixty-five percent of the candidates running for an office in the Illinois General Assembly ran unopposed. Many did not even face opponents in their party primaries. While legislators might like to think the lack of competition is the result of their stellar records in Springfield, it has everything to do with the strange geography of the legislative districts. Politicians draw the maps to choose their voters, instead of the other way around. This often leads to unfair results, such as diminished minority representation.

These practices do not just affect the General Assembly. The earmuff-shaped 4th Congressional District, for example, is designed to consolidate two separate heavily Hispanic-populated areas of Chicago into a single district instead of two separate districts that would allow each their own representative. Current redistricting practices have not gone unopposed, however. There was a strong push recently for redistricting reform. The petition to place an Independent Map Amendment to the Illinois Constitution on the ballot has been signed by more than five hundred thousand Illinois voters intent on changing the way things work in Springfield. In order to ensure a fair, democratic process, Illinois must change the way electoral maps are drawn.

— Pat Hughes, President, Liberty Justice Center

CHAPTER 5

A Process for Non-Partisan Redistricting

Richard Holden

I. Introduction

At the heart of the democratic process is the ability of citizens to choose their elected representatives. Yet in the United States, politicians typically draw electoral boundaries, leading to the perverse result that politicians choose their voters rather than voters choosing their politicians.

Because populations change over time, political districts need to change as well. Indeed, Article I, Section 2, Clause 3 of the United States Constitution provides that "Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers." This means that the number of Congressional representatives, and hence districts in a state, can change as relative populations of the states change. In addition, shifts in population within states require changes to legislative districts in order to maintain equal numbers of voters in districts, as required under various United States Supreme Court precedents.[1] Together, these requirements mean that legislative districts in a given state (both

Congressional and for the state legislature) will periodically need to be redrawn.

Article I, Section 2 of the federal Constitution requires that redistricting occur "within every subsequent Term of ten Years, in such Manner as [the states] shall by Law direct". Thus, state electoral law governs the redistricting process. In most states — indeed, until relatively recently, in all states — this has meant that state legislatures have drawn legislative districts.

II. The Current Illinois Redistricting Process

The Illinois Constitution provides that, "[i]n the year following each Federal decennial census year, the General Assembly by law shall redistrict the Legislative Districts and Representative Districts."[2] This understates the complexity of the process involved. The following is a timeline of the most recent redistricting round.

Figure 1: Redistricting Timeline[3]

December 21, 2010

State Populations and [C]ongressional apportionment delivered to President [Barack] Obama. Illinois loses one of its 19 Congressional Districts.

January 12, 2011

Inauguration of 97th Illinois General Assembly.

June 30, 2011

If no redistricting plan becomes effective by this date, a Legislative Redistricting Commission shall be constituted.

July 10, 2011

Deadline for formation of Redistricting Commission. The Commission shall consist of eight members, no more than four of whom shall be members of the same political party.

The Speaker and Minority Leader of the House of Representatives shall each appoint to the Commission one Representative and one person who is not a member of the General Assembly. The President and Minority Leader of the Senate shall each appoint to the Commission one Senator and one person who is not a member of the General Assembly.

The members shall be certified to the Secretary of State by the appointing authorities. A vacancy on the Commission shall be filled within five days by the authority that made the original appointment. A Chairman and Vice Chairman shall be chosen by a majority of all members of the Commission.

August 10, 2011

Deadline for Redistricting Commission to file an approved plan with the Secretary of State approved by at least five members.

September 1, 2011

If the Redistricting Commission fails to file an approved plan, the Illinois Supreme Court shall submit the names of two persons, not of the same political party, to the Secretary of State no later than this date.

September 5, 2011

No later than this date the Secretary of State shall draw by random selection the name of one of the two persons to serve as the ninth member of the Commission.

October 5, 2011

Deadline for Redistricting Commission to file a redistricting plan with the Secretary of State approved by at least five members.

The Supreme Court shall have original and exclusive jurisdiction over actions concerning redistricting the House and Senate, which shall be initiated in the name of the People of the State by the Attorney General.

This process in 2011 led to the current Illinois Congressional districts, which look like this:

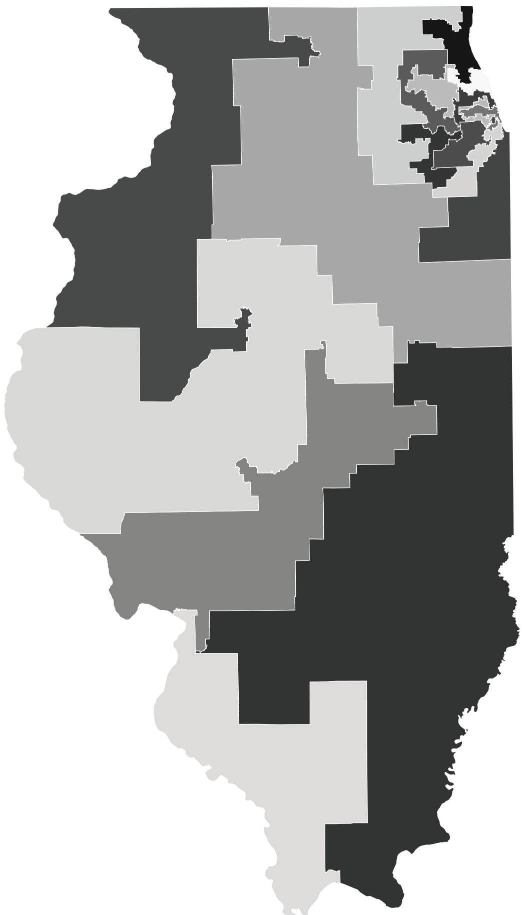

Figure 2

Legislative districts, which look like this:

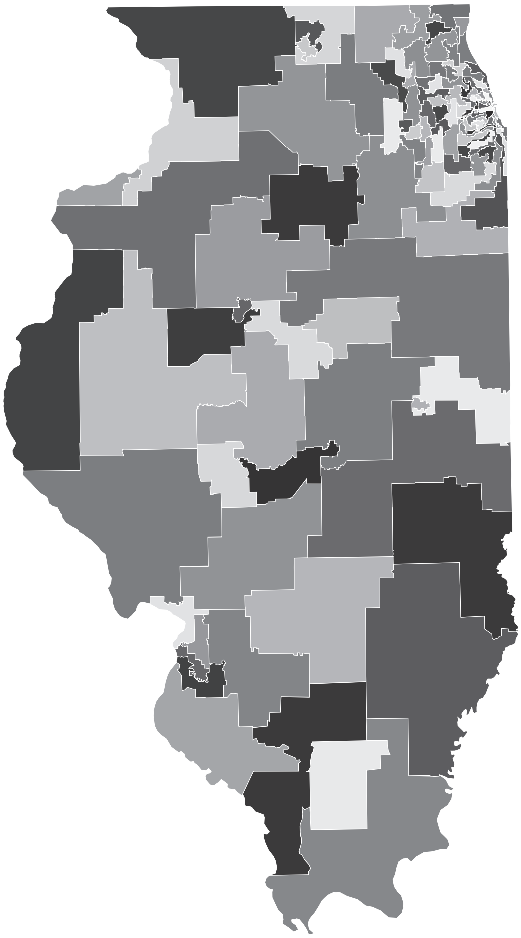

Figure 3

It is important to note that the constitutional process in place for redistricting is inherently political in nature. This could lead to a partisan outcome if one political party has effective control of the process, or it might lead to a compromise outcome favored by incumbents of both parties. The political nature of the redistricting process might lead to partisan or incumbent-protecting gerrymandering. Further, the timeline for the redistricting process is relatively short, meaning that, given the complexity of redistricting, existing districting plans are a natural starting point. This creates an inertia in how districts are drawn, which allows “bad” redistricting plans to persist for decades. Finally, the process in place for dealing with failure to produce an acceptable plan involves the Illinois Supreme Court providing two names from different political parties to the secretary of state from which the secretary of state randomly selects the ninth member of the Redistricting Commission. This means if the Redistricting Commission of eight members, consisting of an even number of members from the two main political parties, is unable to agree on a redistricting plan, a fifth member from one of the two main political parties will be randomly selected, giving one party a clear majority on which to redistrict in a manner favorable to that party.

III. A Proposal to Address the Partisan, Political Nature of Redistricting

It is hardly surprising that the redistricting process has been used for partisan political purposes. After all, politicians are self-interested actors. From Governor Elbridge Gerry of Massachusetts in 1812 — after whom the term “gerrymander” was coined,[4] to the present day, politicians draw political districts for partisan gain.

This brings with it a number of ills. First, it means that significant time and resources are devoted to a socially wasteful activity — partisan gerrymandering. Second, it has the potential to reduce electoral competition and affect the ideological position of the politicians who are eventually elected. Third, it damages the legitimacy of the political process and the political system.

This chapter proposes a manner of redistricting that addresses these problems by taking redistricting out of the hands of politicians by utilizing an algorithmic approach. Combined with a nonpartisan commission as an oversight body, algorithmic redistricting takes into account the important factors of contiguity, compactness, and equal population size, while removing the process from partisan political actors.[5] These factors aren't just important, they are constitutionally required. Article IV, Section 3(a) states that legislative districts and representative districts must "be compact, contiguous and substantially equal in population." The benefit of the proposed algorithmic approach is that it can more effectively take into consideration contiguity, compactness, and equal population size while effectively eliminating rent-seeking, increasing electoral competition, and restoring legitimacy to the redistricting process.

This proposal should be delineated from so-called "independent districting commissions" that have been both proposed and established in certain states — California being a large and notable example. Once there is any discretion involved in a process, there is the potential for politics to enter the equation. The advantage of an algorithmic approach is that it lays down precise districting principles that can be stated mathematically and then derives potential districting plans from those principles. The underlying principles that this chapter proposes are constitutionally required: equal population, contiguity and compactness. Thus, to the extent that those principles can be established mathematically and without human intervention, the less likely that the district can be drawn in a partisan or political manner.

Indeed, under the process proposed, once the algorithmic process is established, the only place that human intervention can occur is if there is some sort of unexpected mechanical error that can be established in court. In this regard, it takes politics out of districting.

The remainder of this chapter discusses the technical details of the algorithmic redistricting proposal advocated. The reader should be aware that it is sometimes necessary to use technical language in discussing these issues, but that the concepts and proposal advanced in this chapter should still be accessible to a reader without such specific technical knowledge.

IV. Compactness

An important, and constitutionally required, feature of districting plans is their "compactness" — that is, grouping people close together geographically into the same districts. In order to understand the algorithmic model proposed in this chapter for redistricting, it is necessary to understand the importance of "compactness." A specific requirement that districting plans be compact was contained in the apportionment acts of 1842, 1901, and 1911. Nineteen state constitutions currently contain a compactness requirement. Compactness has played a fundamental role in how courts assess gerrymandering. Since *Gomillion v. Lightfoot,*[6] the United States Supreme Court has recognized compactness as a relevant factor in considering racial gerrymandering claims. In that case the court referred to the proposed district as "an uncouth 28-sided figure".[7] As Justice Sandra Day O'Connor put it in one of the most important and still relevant redistricting cases: "We believe that reapportionment is one area in which appearances do matter."[8] In *Davis v. Bandemer,*[9] Justices Lewis Powell and John Paul Stevens suggested that compactness, or more precisely lack thereof, was a means of determining the existence of partisan gerrymandering. In the same case, Justices Byron White, William J. Brennan Jr., Harry Blackmun, and Thurgood Marshall suggested it was a useful criterion in making such determinations.

Given this backdrop, it is unsurprising that scholars have developed a large literature of various measures of compactness.[10] As Roland G. Fryer and Richard T. Holden point out, most of these measures suffer from one or more of the following significant failings — they only apply to individual districts rather than districting plans as a whole, they are sensitive to population density or topographical features, or they are *ad hoc* in nature. In response to this, Fryer and Holden develop an axiomatic measure, demonstrating how this can be efficiently calculated and the resulting districting plans ranked and mapped. Finally, they estimate counterfactual seats-votes curves in several states based on the maximally compact districts.

Fryer and Holden propose three axioms that they claim any reasonable districting plan should satisfy.[11]

> 1. *Anonymity:* The index does not depend on the identity of any given voter.
>
> 2. *Invariance:* The index does not depend on a state's population density, physical size, or number of districts.
>
> 3. *Clustering:* If two states with the same number of voters, the same number of voting districts, and the same value for the minimum-partitioning problem have different total intra-district distances, then the state with the larger value is less compact.

The first axiom ensures that all voters are weighted equally. The second axiom makes it possible to sensibly compare indices that satisfy it across states — a highly desirable property that many previous measures fail to satisfy. The third axiom is the heart of what compactness means — putting voters who are close together in the same district and voters that are far apart in different districts.

The Fryer-Holden (hereafter FH) measure of compactness has two components. Component A sums the squared distance between all pairwise combinations of voters in a given district, then sums that over all districts in a state. Component B is the Component A calculation, but for the districting plan that minimizes that sum. What FH refer to as the "relative proximity index" is Component A divided by Component B, for a given districting plan in a state.

Given the motivation and approach, it is not surprising that the FH index satisfies the three axioms they propose. What is more surprising is the following theorem that they establish — any districting plan satisfying the three axioms ranks districting plans identically to the relative proximity index (RPI). In other words, given the axioms, the RPI is essentially unique.

The difficulty FH face in calculating the RPI is that calculating component B (the denominator of the index) is computationally difficult. By developing an algorithm based on so-called "power diagrams" (used in tropical geometry and string theory), FH show how to calculate their index in a computationally feasible way and go on to do so, for the 106th Congress, with data from the U.S. census.

It turns out that the five states with the most compact districting plans are: Idaho, Washington, Arkansas, Mississippi, and New Hampshire, while the five least compact states are Tennessee, Texas, New York, Massachusetts, and New Jersey. It is also interesting to note that the RPI ranks districting plans quite differently to popularly-used alternative measures. For instance, the rank correlations between the RPI and the "dispersion" and "perimeter" measures are -0.37 and -0.29, respectively.

Given both the natural attraction of compact districts, and the fact that they play an important role in the way courts assess districting plans, it is very desirable that any algorithmic district process take account of compactness. The algorithmic approach to districting proposed utilizes the FH relative proximity index as part of its requirement for districts. Put differently, the districting that the algorithm arrives at are within a prescribed distance of maximally FH compact districts, *inter alia*.

V. Algorithmic Redistricting

In light of the aforementioned problems with partisan redistricting, it seems highly desirable to develop some kind of automated procedure that can take the politics out of redistricting to the greatest extent possible. In fact, this is a goal with a long intellectual history, dating to at least William Vickrey (1961).[12]

There are two steps to arriving at a working redistricting algorithm: (1) deciding the set of criteria to use to determine district boundaries and (2) developing an algorithm capable of mapping census and other data into districting plans in a computationally feasible manner. The latter is no small task, since the number of feasible districting plans satisfying equal population, even for a small state, is larger than the number of atoms in the observable universe.[13]

The approach advocated in this chapter is drawn from recent work by Benjamin Fifield, Michael Higgins, Kosuke Imai and Alexander Tarr (2015) — henceforth FHIT.[14] The FHIT approach has three steps. First, it develops an algorithm that samples from the set of all contiguous districting plans given a number of voters and a number of districts to be drawn. Second, it imposes a requirement that each district have an equal population. Third, it overlays the FH compactness measure, resulting in a set of contiguous, equipopulous, compact districting plans. It is important to note that this does not result in a unique districting plan, but rather a set of districting plans satisfying the criteria.

A. The FHIT Districting Algorithm

To be more precise, the primitives of the redistricting problem are a set of m *nodes* to be divided into n *districts*. We will impose two binding constraints on the redistricting problem.

Constraint 1 (C1): Each district must contain the same number of voters.

Constraint 2 (C2): Districts must be contiguous.

We will call such a plan *feasible* if it satisfies C1 and C2.

A key insight that FHIT make is that the districting problem is equivalent to a graph cutting problem. The second key insight that FHIT make is that when sampling for contiguous districting plans it is very useful to start with a plan that is contiguous — such as one that already has been adopted by a state.

Their algorithm to sample contiguous districting plans has five steps.[15]

Step 1: TURN ON EDGES. From the last partition, pick each edge with (independent) probability q, of the existing edge set.

Step 2: GATHER CONNECTED COMPONENTS. Find all components that are connected within the previous edge set and that are adjacent to another block in the partition.

Step 3: SELECT COMPONENTS. Randomly select a set of non-adjacent connected components, with uniform probability, such that a feasible partition is preserved.

Step 4: PROPOSE SWAPS. Propose voter swaps by reassigning connected components to adjacent districts to create a candidate partition (and return to Step 3 if the partition is empty).

Step 5: ACCEPT OR REJECT. Finally, accept or reject the plan according to a probability based on "turned-on" and "turned-off" edges that are connected to adjacent districts.

To see how this works, consider the following example of a hypothetical state with three districts and ten geographic units. These units could be individual voters, or they could be a higher level of aggregation, such as census blocks or voting precincts, that for some reason (such as the state constitution) cannot be split.

The algorithm starts with an existing districting plan, so let us suppose that it is as depicted in the following figure.

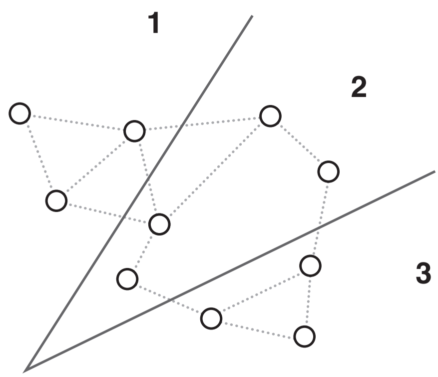

Figure 5: A Hypothetical Districting Plan

In this state there are three geographic units in district 1, four in district 2, and three in district 3. The lines (edges) connecting the units signify that those units are contiguous.

As stated above, the algorithm begins by randomly selecting, or "turning on," some of these edges, as depicted by the heavy dashed lines in the following figure.

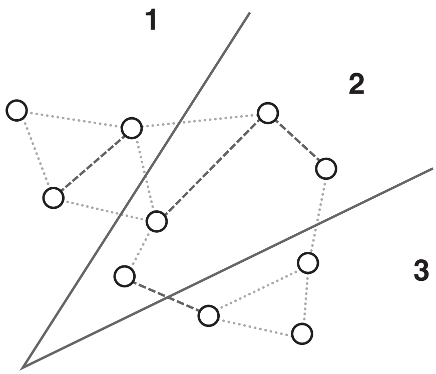

Figure 6: Step 1 of the FHIT Algorithm

The algorithm then selects connected components that are on district boundaries. These are captured by the gray irregular circles in the figure below.

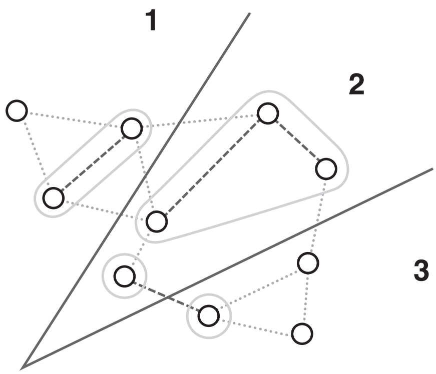

Figure 7: Algorithm Step 2

Some subsets of these are then selected as candidates to be moved to other districts, as depicted in the following figure.

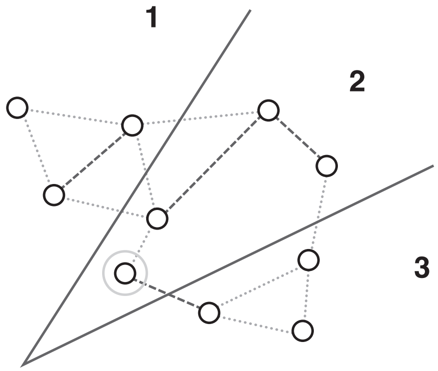

Figure 8: Algorithm Steps 3 and 4

Finally, the algorithm implements those if they lead to an improvement, leading to the new districting plan, represented in the following figure.

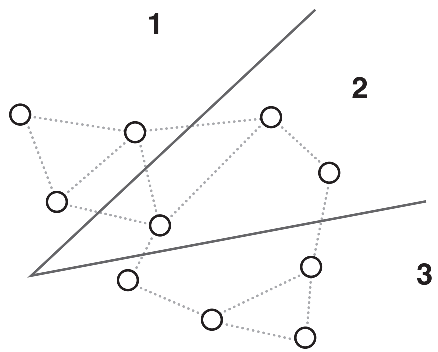

Figure 9: The Modified Districting Plan

This algorithm produces a sample of contiguous districts, but does not add the equal population constraints, nor does it impose a compactness tolerance. FHIT consider a number of possibilities for amending their algorithm to account for these additional constraints.

The first possibility they consider is to throw out any districting that does not satisfy the desired constraints by checking this after Step 4 of the algorithm. Unfortunately, this slows down the algorithm appreciably because of the number of plans that must be checked (and, of course, the complexity involved in checking the additional constraints). A second alternative is to run the algorithm to its conclusion and then throw out any plans that do not satisfy the constraints. This could throw out districting plans that one wants to be considered, by essentially taking too strict a view of the additional constraints.

FHIT instead modify their algorithm by oversampling the redistricting plans that are likely to meet the constraints and then re-weight the remaining plans in a way that jointly approximates uniform sampling. This is achieved by using the Gibbs distribution from statistical physics and a modification known as *parallel tempering*.

This leads to the set of districting plans that are contiguous, equipopulous, and within an FH compactness tolerance. Importantly, FHIT provide open-source software that implements their algorithm[16] and show that the algorithm performs well in the cases of New Hampshire and Mississippi.

B. Implementing the FHIT Algorithm

A districting commission could easily be established to implement the FHIT algorithm. This could be a statutory authority, or even the type of redistricting commission foreshadowed by the existing Illinois process if the legislature fails to adopt a plan by a certain date (recall Section II, above). The technical requirements, in terms of both data and execution of the algorithm, are modest.

Of course, a fundamental practical consideration is the integrity of the districting plan(s) that the algorithm produces. Because the algorithm is designed to produce districting plans that adhere to three well-defined criteria (equal population, contiguity, and FH compactness), it is relatively straightforward for a court to adjudicate a claim that a given districting plan violates one of those criteria.[17]

It is important to note that the algorithm proposed here produces a *set* of districting plans satisfying the three key criteria, not a single or unique districting plan. Some of the plans within the set inevitably will be more favorable to some political actors than others. In the interests of keeping human factors and politics out of the process, one way to decide is to merely choose at random (say by assigning each plan in the set a lottery number and then picking the number in a publicly televised lottery). Another alternative would be to choose a plan from the algorithmically generated set that requires changing voters/districts the least from the existing districting plan. One can imagine other possibilities.

C. Political Economy and "Gaming" Considerations

Core to the success of any algorithmic process is ensuring that the rules specified cannot be "gamed" by political actors. Such considerations are very significant in the provision of financial incentives.[18]

An appealing aspect of the redistricting process suggested in this chapter is that political actors do not have control over the geographical variables that determine the criteria on which the algorithm is based. Even contiguity, as commonly and naturally defined, does not allow political actors to separate voters. This suggests, therefore, that gaming considerations are unlikely to plague the kind of algorithmic districting proposed in this chapter.

The algorithmic redistricting proposal does not explicitly consider the resulting composition of the legislature, either in partisan or racial terms. As Coate and Knight (2007) emphasize, if one takes a consequentialist view of legislative outcomes then one should care about who is elected to the legislature and how they vote. This has implications for how districts should be drawn — especially in terms of the resulting seats-vote curves (*see also* Shotts, 2002). Even if one does not take a consequentialist view, the partisan or racial composition of legislatures might be ends in themselves, and this approach does not explicitly factor those in.

However, once one starts to consider such factors as partisan affiliation or race, there are a host of auxiliary questions that arise, such as who such legislators are supposed to represent and how effective they are in so doing. By sticking to a small number of important, well-defined and precisely measurable criteria, this proposal makes it possible to essentially eliminate, or at least severely reduce, political decision-making in the redistricting process. No other redistricting process — including non-partisan commissions of the California variety — is as effective at doing so.

D. Legal Considerations

For algorithmic districting to be practically useful, the districting plans that the algorithm outputs must be lawful. An obvious requirement in this regard is that the algorithmic plans be consistent with the Illinois Constitution. The substantive provisions in this regard are:

> "Legislative Districts shall be compact, contiguous and substantially equal in population,"[19]
>
> and
>
> "Representative Districts shall be compact, contiguous, and substantially equal in population."[20]

The three core components of the algorithm proposed in this chapter are just those: compactness, contiguity, and equal population. There are, of course, broader legal considerations stemming from United States Supreme Court precedents. Many of those (including several mentioned above) concern issues of equal population and compactness. There are, however, two other notable issues worth touching on.

The first concerns the involvement, or lack thereof, of the state legislature in the redistricting process. After all, the algorithm proposed in this chapter is designed specifically to "take the politics" out of the districting process, and one might observe that the Elections Clause of the U.S. Constitution, which states: "The Times, Places and Manner of holding Elections for Senators and Representatives, shall be prescribed in each State by the Legislature thereof,"[22] and therefore ponder the constitutionality of algorithmic districting.

The meaning of the Elections Clause with regard to "independent" districting commissions was considered by the U.S. Supreme Court in *Arizona State Legislature v. Arizona Independent Redistricting Commission.*[22] The majority opinion ruled that the meaning of "Legislature" under the Election Clause may include institutions other than the state's representative body, and there is no reason to believe that similar reasoning would not apply to algorithmic districting of the type proposed here.

The second additional issue is race. Section 2 of the Voting Rights Act,[23] which prohibits districting plans that (whether by intent or not) improperly dilute the voting power of racial minorities. A vote dilution claim under Section 2 would be assessed according to the so-called "Gingles test."[24] The first limb of that test concerns the possibility of drawing compact majority-minority districts. Since compactness is a key part of the algorithm proposed here, any such hypothetical plan would be an element of the set of districting plans produced by the algorithm.

VI. Conclusion

The algorithmic districting proposal that has been laid out in this chapter seeks to take the politics out of redistricting. The algorithm takes into account factors such as equality of population, contiguity, and compactness — constitutionally required factors that are widely viewed as somewhere between desirable and essential in drawing districts. The algorithmic approach outlined here also has the virtue of not necessarily starting with the existing districting plan and incrementalizing from it. This "start fresh" feature of the process is helpful in avoiding path dependence of unappealing districting plans.

Public confidence is a crucial part of any electoral process. When districts look strange, are drawn through a partisan process, and result in partisan outcomes, it is hardly surprising that public confidence is shaken. This is clearly part of the rationale for so-called “independent” districting commissions. However, even with exceptionally well-defined criteria, such commissions always will involve a degree of discretion. As a result it always will be possible to question the districts that are drawn by them. Only through an algorithmic process, such as the one outlined in this chapter, can public confidence in the drawing of electoral boundaries be fully restored.

1 *See e.g.*, Wesberry v. Sanders, 376 U.S. 1 (1964) (addressing Congressional districts); Reynolds v. Sims, 377 U.S. 533 (1964) (addressing statewide legislative bodies); Avery v. Midland County, 390 U.S. 474 (1968) (addressing local governments).
2 Ill. Const., art. IV, § 3(b).
3 *Id.*; *see also Timeline*, Illinois Redistricting, www.ilhousedems.com/redistricting/?page_id=2 (last visited Aug. 16, 2016).
4 A political cartoon of the day depicted the contorted district that Gerry drew as a salamander, and the resulting district quickly became known as "Elbridge Gerry's salamander," giving rise to the portmanteau "Gerrymander."
5 It is also possible to factor in respecting political subdivisions.
6 364 U.S. 339 (1960).
7 *Id*. at 340
8 Shaw v. Reno, 509 U.S. 630, 647 (1993).
9 476 U.S. 173 (1986).
10 *See* Roland G. Fryer & Richard T. Holden, *Measuring the Compactness of Political Districting Plans*, 54 J. L. Econ. 493 (2011), http://research.economics.unsw.edu.au/richardholden/publications/fryer-holden-2011.pdf, for references.
11 In the following description I quote directly from Fryer and Holden, *supra* note 7 at 501.
12 The fact that Vickrey, later a Nobel Laureate for his pioneering work on auctions and mechanism design, was interested in the problem is indicative of its importance.
13 For California the number of feasible districting plans is $7.2 \times 10^{46,037}$ compared to 10^{80} atoms in the observable universe.
14 Benjamin Fifield, et al., A New Automated Redistricting Simulator Using Markov Chain Monte Carlo (Aug. 17, 2016) (mimeo.) (available at http://imai.princeton.edu/research/files/redist.pdf).
15 For precise details the interested reader is referred to FHIT.
16 Available at https://github.com/redistricting/redist.
17 Respect for political subdivisions could also be handled in this fashion.
18 *See* Florian Ederer, et al., Gaming and Strategic Opacity in Incentive Provision (2014) (mimeo.), and the references therein.
19 Ill. Const. art. IV, § 3(a).
20 *Id.*
21 U.S. Const. art. I, § 4, cl. 1 (emphasis added).
22 ___U.S.____ , ____, 135 S. Ct. 2652, 2662 (2015).
23 Voting Rights Act of 1965, Pub. L. No. 89-110, § 2.
24 Thornburg v. Gingles, 478 U.S. 30 (1986).