Managerial Contracts at Regulated Firms

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Abstract

There is substantial empirical evidence that top managers at regulated firms have systematically different incentive contracts than those at unregulated ones. At regulated firms, managerial compensation is both lower and less performance sensitive. Less well understood, however, is what shape managerial contracts at regulated firms *should* take. We analyze an integrated model of shareholder-manager and regulator-firm agency relationships for a monopolist. We show that it is optimal for regulated firms to have less intense incentives for managers–and to provide less compensation in expectation. These results imply that regulated firms are less X-efficient than unregulated ones. *(JEL L51)*

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1 Introduction

A long tradition in economics argues that market pressure disciplines managers (Smith, 1776; Hicks, 1935; Leibenstein, 1966; Machlup, 1967). This was an important argument used by Prime Minister Thatcher and President Reagan in favor of deregulation. According to this argument, regulated firms are less subject to market pressure and hence waste resources, are less likely to innovate, and serve customers less well than unregulated ones. In the terminology of Leibenstein (1966), they are less X-efficient. The welfare consequences of this are potentially large.

Like unregulated firms, regulated firms are generally run by managers rather than by the owners of the firm. Thus, any X-inefficiency at regulated firms comes at the hands of non-owner managers whose payoffs are linked to the firm's, if at all, by a managerial incentive scheme. In fact, a significant literature suggests the systematic difference between compensation of top managers at regulated and unregulated firms. Empirical studies consistently find that top managers' pay at regulated firms is lower and less performance-sensitive than top managers' pay at unregulated firms (Hendricks, 1977; Caroll and Ciscel, 1982; Joskow, Rose and Shepard, 1993; Crawford, Ezzell and Miles, 1995; Hubbard and Palia, 1995; Wolfram, 1998; Palia, 2000). For instance, in Palia's (2000) sample of regulated electric and gas utilities and unregulated manufacturing firms, the level of salary, bonus and option grants was three times higher in the unregulated environment, while the estimated increase in CEO pay with a $1000 increase in shareholder value was nearly ten times greater in the unregulated environment. Likewise, Hubbard and Palia (1995) compare 55 banking CEOs' pay before and after banking deregulation and find both significantly greater salary, bonus and option grants and significantly greater performance-sensitivity of pay after deregulation than before.

While the empirical differences in the level and structure of managerial pay in regulated versus unregulated environments are clear, less well understood is what shape managerial contracts at regulated firms *should* take under the principal-agent paradigm that has been widely applied to the firm-manager relationship (Jensen and Meckling, 1976; Jensen and Murphy, 1990; Hayes and Schaefer, 2000; Prendergast, 2002). Under the principal-agent paradigm, the firm (the principal) puts the manager (the agent) on a profit-sharing incentive contract to induce the manager to maximize the firm's profits. In this paper we introduce regulation into the standard principal-agent model of firm-manager contracting and compare the resulting optimal managerial contract to the optimal managerial contract in the unregulated environment.

Our model contains three players: a regulator, a firm, and a manager. The firm–which is a monopolist–operates in one of two product market environments. The first is an unregulated or "market environment," in which output prices are determined in a market equilibrium. This environment is the standard principal-agent environment with no regulation. The second environment introduces regulation. Here a regulator sets the prices charged in the product market. When the regulator sets the prices, the firm's objective function and, thus, the managerial contract that is optimal differ from those in the unregulated setting. We show that, in the regulated environment, it is optimal for the firm to offer a less intense incentive contract than in the market environment. This is because regulation makes low cost states less valuable for the firm relative to high cost states. Regulation also leads to the manager receiving a lower expected payment, corresponding to the lower level of risk in the regulated environment.

In some sense this is not all that surprising. Consider, for example, a simple model where the firm offers a linear incentive scheme and regulation takes place through a linear tax. Well known results from Holmström and Milgrom (1987) then imply that the slope of the incentive scheme is:

$$\beta = \frac{B'(e)}{1 + rC''(e)\,\sigma^2},$$

where B is the benefit of effort function, r is the coefficient of absolute risk-aversion, C is the cost of effort function, and σ^2 is the variance of the shock. Since $B(e) = (e + \varepsilon)(1 - \tau)$, where τ is the tax rate, it is easy to see that $d\beta/d\tau < 0$. However, there are two things to note about this: a linear contract is probably not optimal in general, and the form of regulation is probably not optimal either. Our results can be thought of as providing conditions under which the conclusions from this simple model generalize to one with optimal managerial contracts and optimal regulation.

While this paper emphasizes differences in managerial pay resulting from the change in the firm's objective function as a result of regulation, other factors may also play a role in explaining the variation in the level and structure of pay in the market and regulated environments. Political constraints on managerial pay may lead to systematic departures from optimal contracting (Jensen and Murphy, 1990), and such constraints are likely to be stronger for regulated firms, whose behavior is regularly subjected to public scrutiny, than for unregulated ones (Joskow, Rose and Shepard, 1993; Joskow, Rose and Wolfram, 1996).[1] Whereas distributional and other political considerations that shape the regulatory process make themselves felt through changes in the objective function of the regulated firm (see section 2 below), political constraints act as direct barriers to optimizing behavior by firms. Regulated entities in particular may be driven to adopt suboptimal compensation arrangements out of fear of political retaliation operating through the regulatory process.

The effects of changes in the regulated firm's objective function and changes in the background political constraints are difficult to distinguish empirically; both sorts of changes predict managerial pay that is lower and less performance-sensitive than in the unregulated sector. Even the more nuanced empirical findings in the literature on managerial compensation at regulated firms are often consistent with both changes in the firm's objective function and changes in the background political constraints. Thus, for example, Joskow, Rose and Shepard (1993) find that differences in the level and structure of managerial pay at regulated firms are greater for firms subject to firm-level regulation (electric and gas utilities, interstate natural gas pipelines, and telephone companies) than for firms subject

[1] The belief that political constraints limit optimal contracting by firms and managers grew out of empirical findings suggesting that managerial pay is relatively insensitive to firm performance (Jensen and Murphy, 1990). However, later work (such as Haubrich, 1994; and Hall and Liebman, 1998) suggests a much greater sensitivity of managerial pay to firm performance.

to industry-level regulation (railroads, trucking, and airlines). If political constraints are stronger under firm-level regulation–because of direct regulatory oversight of the firm's compensation arrangements–than under industry-level regulation, then the finding of greater effects on pay under firm-level regulation may, as Joskow, Rose and Shepard suggest, point to a role for political constraints. We show below, however, that greater compensation effects under firm-level regulation are also to be expected if regulation changes the objective function of the regulated firm. The observed effects of regulation on the level and structure of managerial pay thus may often reflect some combination of changes in the regulated firm's objective function and changes in the governing political constraints.

Related to the analysis below is a literature exploring how the general nature of product market competition affects managerial compensation and incentives. Indeed, this literature was motivated by the claims of Leibenstein (1966), Machlup (1967) and others. Formal models exploring this began with Hart (1983) and Nalebuff and Stiglitz (1983). A large literature has followed and found that product market competition may lead to more or less intense incentives (and hence managerial effort)–depending on the specifics of the model. For instance, Martin (1993) shows that under Cournot competition an increase in the number of firms leads to *less* intense incentives because the marginal value of a cost reduction decreases. Raith (2003) obtains the opposite result in a model where firms compete in product variety and there is free entry and exit. Schmidt (1997), in a model with bankruptcy costs, obtains ambiguous results. Horn, Lang and Lundgren (1994) compare Bertrand competition, Cournot competition and an output cartel and find a non-monotonic relationship between competitiveness and the intensity of incentives under an optimal managerial contract. Holden (2005b) presents a unified treatment of this literature and identifies necessary and sufficient conditions–which prove to be fairly stringent–for product market competition to lead to increased intensity of managerial incentives.

In this paper we hold constant the degree of product market competition across the market and regulated environments. Specifically, we assume there is a single firm in both environments. Thus, our results are driven by the way in which regulation changes the objective function of the firm. In contrast to the case of increased product market competition–which, as just noted, can plausibly either increase or decrease the level and performance-sensitivity of managerial pay–movement from the regulated to the market environment will generally increase both the level and the performance sensitivity of managerial pay.

That said, the assumption of a single firm is not innocuous. Unfortunately we have not obtained our unambiguous results for a general number of firms $m \geq 2$. We discuss this further in the final section.

The remainder of the paper is organized as follows. Section 2 presents our model and main results under a general characterization of the regulatory process. Section 3 introduces optimal regulation into the model and shows that our main results continue to hold in this setting. Section 4 offers a simple example with two possible cost realizations which we are able to solve explicitly in order to illustrate the main results. Finally, Section 5 contains a discussion and some concluding remarks.

2 The Model

2.1 Preliminaries

We consider a general version of the familiar principal-agent relationship between a firm and its manager. In this model a risk averse manager takes an action which is observable only to the manager but affects the firm's outcome. We make the standard assumption that the firm is risk-neutral. There are n possible profit levels $q_1, ..., q_n$ for the firm, corresponding to n possible marginal cost realizations $c_1, ..., c_n$. We will interpret low cs as "bad" (high cost) states and high cs as "good" (low cost) states. The probability of each state i is influenced by the manager's action choice. If the manager chooses a "harder" action, then it is more likely that lower cost states will occur.

We now specify the manager's action choice more formally.

Definition 1. *A set* X *is a Product Set if* $\exists$ *sets* $X_1, ..., X_N$ *such that* $X = X_1 \times ... \times X_N$. X *is a Product Set in* $\mathbb{R}^N$ *if* $X_n \subseteq \mathbb{R}, n = 1, ..., N$.

We write the manager's action as $a \in A$ where A is a product set which is dense in $\mathbb{R}^N$, closed, bounded and non-empty. Hence, by the Heine-Borel theorem it is compact. Define the ordinary probability simplex $S = \{y \in \mathbb{R}^N | y \geq 0, \sum_{i=1}^N y_i = 1\}$ and assume that there is a once continuously differentiable function $\pi : A \rightarrow S$. The probabilities of outcomes $c_1, ..., c_n$ are therefore $\pi_1(a), ..., \pi_n(a)$. The payment to the agent in state i is I_i. The agent has reservation utility $\overline{U}$ and von Neumann-Morgenstern utility function:

$$U(a, I) = G(a) + K(a)V(I)$$

We make use of assumptions A1-A3 from Grossman and Hart (1983). These are largely technical in nature and ensure the existence of a second-best optimal incentive scheme and rule out schemes involving huge out-of-equilibrium punishments through which the first-best can be approximately arbitrarily closely (Mirrlees, 1975). For completeness these are provided in the appendix; we refer to them below as A1-A3.

A final background step is necessary because our model involves a multi-dimensional action choice which, in equilibrium, may be set valued. This is because the general moral hazard problem is not a convex programming problem (Grossman and Hart, 1983). We therefore require a way of comparing different multi-dimensional sets of actions. The following definition introduces the multi-dimensional Strong Set Order.

Definition 2. *Let* $X, Y \subset \mathbb{R}^n$. *Then* X *is higher than* Y *in the Strong Set Order* ($X \geq_s Y$) *iff for any* $\mathbf{x} \in X$ *and* $\mathbf{y} \in Y$, $\max\{\mathbf{x}, \mathbf{y}\} \in X$ *and* $\min\{\mathbf{x}, \mathbf{y}\} \in X$.

The SSO allows comparisons of multi-dimensional actions. With this in hand we can compare the intensity of different incentive schemes.

We make one final assumption, which is purely for ease of exposition.[2] Using the real

[2]See Holden (2005a) for a demonstration that this assumption can easily be relaxed.

line as the action space means that we can present the main results using calculus, rather than a series of difference inequalities.

Assumption A4. $A \subseteq \mathbb{R}$.

2.2 Solving the Model

The timing of our model is as follows.

- $t = 0$ The regulator announces the pricing schedule (if applicable).
- $t = 1$ The firm and manager contract.
- $t = 2$ An action is chosen by the manager and cost schedule determined.
- $t = 3$ Production takes place and profits are realized.

As in Grossman and Hart (1983), the step-one problem for the principal is to find the lowest cost way to implement a given action a^* (i.e. the lowest cost way to satisfy the agent's incentive compatibility and participation constraints with action choice a^*). Denote that cost by the function $C(a^*)$.

The second step is to choose which action should be implemented. That is, choose the action which maximizes the expected benefits minus the costs of implementation:

$$\max_{a \in A} \{B(a, \phi) - C(a)\} \tag{1}$$

where $B(a, \phi) = \sum_{i=1}^{n} \pi_i(a) q_i(\phi)$ and ϕ is an indicator variable which takes the value 1 if the firm is in the regulated environment and 0 if it is in the market environment. In approaching this programming problem we shall make the familiar assumption that the likelihood ratio is monotonic. This condition says that higher outcomes are indeed a correct signal of harder actions It holds for a wide range of probability distributions (see Milgrom, 1981).

Definition 3. $C_{FB} : A \to \mathbb{R}$ *is the first-best cost of implementing action a given by:*

$$C_{FB}(a) = h\left(\frac{\bar{U} - G(a)}{K(a)}\right).$$

Condition 1 (Monotone Likelihood Ratio Property)**.** *MLRP holds if, given* $a, a' \in A$, $C_{FB}(a') < C_{FB}(a) \Rightarrow \pi_i(a')/\pi_i(a)$ *is decreasing in* i.

While we rely on MLRP, our results do not require us to make the strong set of assumptions associated with the traditional first-order approach to the principal-agent problem, such as the Convexity of the Distribution Function Condition ("CDFC") (see Mirrlees (1975), Grossman and Hart (1983), Rogerson (1985), Holden (2005a)).

2.3 Comparison of the Market and Regulated Environments

This section establishes our two central results about managerial contracts at regulated firms under a general characterization of the regulatory environment. Let $\Delta q_i = q_i(1) - q_i(0)$ be the difference in firm profits in state i in the regulated versus unregulated environment, and denote the set of optima corresponding to the problem in (1) as $a^{**}(\phi)$.

Theorem 1 (Holden, 2005a)**.** $a^{**}(\phi)$ *is non-increasing in the SSO as a function of* ϕ *iff:*

$$\sum_{i=1}^{n} \pi_i'(a)(\Delta q_i) \leq 0, \qquad \forall a, \phi. \tag{2}$$

Proof. From the Monotonicity Theorem of Milgrom and Shannon (1994) a necessary and sufficient condition for the stated conclusion is that $B(a, \phi)$ have decreasing differences in the pair (a, ϕ). Since π is differentiable this condition is simply $\sum_{i=1}^{n} \pi_i'(a)(\Delta q_i) \leq 0$. ■

This theorem allows us to relate the optimal action a^{**} in the regulatory environment to its counterpart in the market environment and, as a result, compare the optimal contracts across the two environments.

Our first main result is that it is optimal for the firm to offer a less intense incentive contract in the regulated environment. Our second main result is that the optimal contract gives the manager lower expected compensation in the regulated environment. We first make precise the notion of more or less intense incentive schemes in our model. In the special case of linear contracts, the slope of the incentive scheme–the derivative of managerial compensation with respect to the firm's profit–is a sufficient statistic for the intensity of incentives. In our more general setting, matters are slightly more complicated.

Definition 4. *Consider two incentive schemes* $I = (I_1, ..., I_n)$ *and* $\hat{I} = (\hat{I}_1, ..., \hat{I}_n)$ *with equilibrium actions from the agent* a_I^{**} *and* $a_{\hat{I}}^{**}$ *respectively. We say that* I *has "More Intense Incentives" than* $\hat{I}$ *if and only if* $a_I^{**} >_S a_{\hat{I}}^{**}$.

Note that this definition is consistent with the outcomes obtained in the linear contracts setting popularized by Holmström and Milgrom (1987). In their model the equilibrium action for the agent is $e = \beta/c$, where β is the slope of the contract and c is the second-derivative of the agent's cost of effort function. Since c is a constant, higher e corresponds to higher β.

The general characterization of regulation that we adopt in this section is that regulation compresses the spread of firm profits across the n cost states. In other words, profits are less variable across states under regulation. This general characterization, which we particularize in condition C2 below, fits with the empirical evidence on the effects of regulation; we derive its optimality in a model of regulation in section 3 below.

Condition 2 (C2). $\Delta q_1 \geq \Delta q_2 \geq ... \geq \Delta q_n$, *with at least one inequality strict.*

Condition C2 states that profits in the regulated environment are relatively higher, by comparison with the market environment, in higher cost states. (Note that the manager's choice of an action a affects the probability of each state but does not affect either the cost level or the profit level associated with state i. Note also that profit levels may–or may not–be lower in all states under regulation; condition C2 is met as long as they are relatively lower in low cost states.) Under condition C2, regulation in effect acts as a cushion against large negative effects on profits in high-cost states. This characterization of regulation is broadly consistent with empirical findings on the effect of regulation on firm returns. Joskow, Rose and Shepard (1993), for instance, find that annual stock market return variance was about 25 percent less for regulated firms than for unregulated firms over the 1970 to 1990 period, and that annual accounting return variance was about 10 percent less in the regulated sector than in the unregulated sector over that period.[3] Likewise, Palia (2000) finds that the standard deviation of daily stock returns is only a third as high in his sample of regulated firms as in his sample of unregulated firms over the period from 1988 to 1993. And Murphy (1987) finds that five year stock market return variance was more than 50 percent less for regulated firms than for unregulated firms over the 1964 to 1983 period. It is plausible that these patterns result precisely from a regulatory cushion against high cost states–an idea that accords well with general intuitions about the effects of regulation.

Condition C2 is sufficient for the result that the managerial contract involves less intense incentives under regulation, as formalized in the following proposition:

Proposition 1. *Assume A1-A4 and that C1-C2 hold. Then under regulation the optimal managerial contract has less intense incentives than without regulation.*

Proof. See appendix. ■

The intuition for this result is straightforward. Starting from the benchmark of the market environment, regulation increases the profits of the firm less (or reduces them more) in low cost states than it does in high cost states (Condition C2). Thus, the presence of regulation makes low cost states relatively less attractive to the firm. This leads the firm to offer a contract to the manager which induces the manager to choose a lower action, and hence less probability weight on low cost states. Thus, the optimal behavior for the firm

[3] In the case of stock market return, the mean values in the regulated and unregulated sectors were almost identical, so direct comparison of the variance values seems clearly appropriate. In the case of accounting return, the regulated sector mean was lower than the unregulated sector mean, and the mean-scaled variance was actually slightly higher in the regulated sector than in the unregulated sector. However, most incentive pay in managerial compensation packages is stock-based, and, therefore, the key measure is stock market return variance as opposed to accounting return variance.

under regulation involves providing less intense incentives for the manager.

As noted in the introduction, less intense incentives in the regulatory environment are consistent with the existing empirical literature, including work by Murphy (1987), Joskow, Rose and Shepard (1993), Crawford, Ezzell and Miles (1995), Hubbard and Palia (1995), and Palia (2000). These studies all find that, controlling for a variety of firm and CEO characteristics, the pay of CEOs at regulated firms is significantly less sensitive to firm performance than is the pay of CEOs in unregulated firms.[4] Proposition 1 indicates that this pattern of differential performance-sensitivity is predicted by optimal contracting.

Our second major result is the following.

Proposition 2. *Assume A1-A4 and that C1-C2 hold. Then under regulation the expected payment to the manager is lower than without regulation.*

Proof. See appendix. ■

Intuitively, because regulation induces an incentive scheme with less intense incentives (Proposition 1), the manager, by definition, exerts less effort. Thus there is a lower effort cost in the regulated environment. But since the manager's reservation utility is a constant and the participation constraint binds by A1 (Grossman and Hart, 1983), the expected *utility* must be the same in the regulated and unregulated environments. The only way lower effort cost could coexist with higher expected compensation while keeping expected utility at the reservation level would be to impose more risk on the manager under regulation, but this would be suboptimal. Thus, the lower effort cost in the regulated environment implies that the expected payment must be lower.

As noted in the introduction, the conclusion that the level of compensation is lower in the regulatory environment as a consequence of optimal contracting between regulated firms and their managers is consistent with empirical findings on the level of managerial pay at

[4] Control factors used in these studies include firm size (measured by sales), CEO tenure, CEO age, whether the CEO founded the company, whether the CEO was an outside hire or an internal promotion, the number of employees at the firm, the value of the firm's assets, the firm's accounting or stock market return, and the percent of the firm's stock owned by the CEO. Specific controls used naturally vary across the studies.

regulated and unregulated firms. Wolfram (1998), for instance, finds that the average British electric utility CEO saw a pay increase of nearly 300% after industry privatization; likewise, Carroll and Ciscel (1982), Murphy (1987), Joskow, Rose and Shepard (1993), Hubbard and Palia (1995), and Palia (2000) find significantly higher pay in unregulated than in regulated environments.[5]

In addition to explaining the observed differences in the structure and level of managerial pay at regulated and unregulated firms, the optimal contracting account can also help explain some of the more nuanced empirical findings on regulation and managerial pay. For example, as noted in the introduction, firm-level regulation appears to produce greater changes in both the incentive intensity and the overall level of managerial compensation (relative to the unregulated setting) than does industry-level regulation (Joskow, Rose and Shepard, 1993). Applying Propositions 1 and 2, this result is predicted by optimal contracting if $\Delta q_1^R \geq \Delta q_2^R \geq ... \geq \Delta q_n^R$, with at least one inequality strict, for $\Delta q_i^R = q_i(F) - q_i(IN)$, where $q_i(F)$ is profit under firm-level regulation and $q_i(IN)$ is profit under industry-level regulation. The empirical evidence suggests that, in fact, regulation does act as more of a cushion in the case of firm-level regulation than in the case of industry-level regulation; Joskow, Rose and Shepard (1993) report that annual stock market return variance was over 40 percent less for firms subject to firm-level regulation than for firms subject to industry-level regulation over the 1970 to 1990 period.[6] The relationship between performance variance under firm-level regulation and performance variance under industry-level regulation suggests that optimal contracting by regulated firms and their managers may explain the pattern of compensation across regulated sectors that we observe in practice.

A important caveat to this conclusion is that results from the single-firm regulatory

[5] Again, these empirical results reflect controls for a variety of factors: firm size (measured by sales), CEO tenure, CEO age, whether the CEO founded the company, whether the CEO was an outside hire or an internal promotion, the number of employees at the firm, the value of the firm's assets, the firm's accounting or stock market return, and the percent of the firm's stock owned by the CEO. Again, specific controls used vary across the studies.

[6] Stock market return variance scalued by stock market return mean was 29 percent less for firms subject to firm-level regulation than for firms subject to industry-level regulation over the 1970 to 1990 period. The relationship between the variance of firm performance and condition C2 was described earlier in the paper.

model generalizes to multi-firm regulation (otherwise industry-level and firm-level regulation are synonymous). We discuss this further in the final section.

Optimal contracting by regulated firms and their managers therefore generates predictions about regulation and managerial pay that appear to be consistent with the available empirical evidence. Whenever regulation acts as a cushion against high cost states (a characterization of regulation that is consistent with the empirical evidence), it reduces the benefit associated with inducing high managerial action choices, and lower action choices correspond to both less intense incentives and lower levels of pay. Likewise, forms of regulation that increase the cushioning feature of regulation by relatively greater amounts have relatively larger effects on the benefit of inducing higher managerial actions and, therefore, generate relatively greater effects on managerial pay. In the next section we provide a model of optimal regulation under which regulation acts as a cushion against high cost states in the manner described above.

3 Optimal Regulation

We consider a symmetric information model in which the regulator is able to observe the (marginal) cost of the firm(s) and set a pricing schedule contingent on this cost. Since the pioneering work of Baron and Myerson (1982)[7] there has been a large literature exploring to consequences of regulation under asymmetric information. For the purposes at hand, however, such a framework introduces more analytical complexity than is warranted for the conclusions we draw. We return to this issue later in the paper.

Assume that there is a single firm in both the regulated and market environments.As is conventional in the literature, the regulator in our model seeks to maximize a social welfare function that puts some weight on consumer surplus and some on producer surplus. The control variables are state contingent prices $p_1, ..., p_n$.

By making the control variables for the regulator the state contingent prices we are ruling

[7] See also Laffont and Tirole (1986).

out lump sum transfers which could achieve the social optimum. Thus the form of regulation we consider is optimal with the class of instruments considered here.

We assume that the regulator is able to credibly commit to the pricing schedule announced at $t = 0$. It is natural to think that regulators have a particular ability to pre-commit in this way. We discuss the consequences of the regulator being unable to commit in this way in the final section of the paper.

The regulator solves the following problem, where $X(p_i)$ is market demand at price p_i, $S(X(p_i))$ is consumer surplus at that price, λ is the weight on producer surplus, and π_i is the probability that state i occurs (which depends on the actions chosen by the firm(s)).

$$\max_{\tilde{p}=(p_1,\ldots,p_n)} \left\{ W = \sum_{i=1}^{n} \pi_i \left(a^{**}(\tilde{p})\right) \left[S\left(X\left(p_i\right)\right) + \lambda q_i\right] \right\}. \tag{3}$$

This is a member of a standard class of objective functions in the study of regulation (Laffont and Tirole, 1993, chapter 1).

It is convenient to write this problem as:

$$\max_{\Delta\tilde{p}=(\Delta p_1,\ldots,\Delta p_n)} \{\Delta W\},$$

where ΔW is the difference between the regulator's objective function evaluated at the regulated and market prices, and $\Delta p_i = p_i^R - p_i^M$, where p_i^R and p_i^M denote the price in state i in the regulated and market environments respectively.

Since we will be interested in the difference between the market environment and the regulated environment, we first characterize the market environment, before proceeding to analyze optimal regulation, and how firm profits differ between the two.

3.1 The Market Environment

At each firm the manager chooses an action $a \in A$ which leads to (marginal) cost states $c_1 > c_2 > ... > c_n$ with probabilities $\pi_1(a), ..., \pi_n(a)$.

Condition 3 (C3)**.** *$0 \leq p_1^M - c_1 \leq p_2^M - c_2 \leq ... \leq p_n^M - c_n$, with at least one inequality strict.*

This condition requires that market prices are higher when firms' costs are higher. This is a mild condition which is obviously satisfied for a monopolist.

3.2 The Regulated Environment

To ensure that comparisons between the market and regulated environments are about changes in the objective function for the firm(s), rather than changes in the *number* of firms, we assume that there is also a single firm in the regulated environment.

It is clear from (3) that there is both a direct effect of a change in price (prices affect consumer and producer surplus), and an indirect effect (prices affect the managers' actions and hence firms' costs). Since we will show that the direct effect pushes the regulator to set $\Delta p_1 \geq \Delta p_2 \geq ... \geq \Delta p_n$, the indirect effect must be considered, because it works in the opposite direction. That is, with the price relative price changes across the different states $\Delta p_1 \geq \Delta p_2 \geq ... \geq \Delta p_n$ we will end up establishing, regulation has the effect of inducing the firm to offer a contract to the manager with less intense incentives. This effect, on its own reduces welfare. It is useful to decompose this indirect effect into its two component parts: the indirect effect on consumer surplus and that on producer surplus.

First, consider producer surplus. Recall that the step two problem that the firm solves is $\max_{a \in A} \{f = B(a, \tilde{p}) - C(a)\}$. Denote the value function of the firm as $V(a^{**}(\tilde{p}), \tilde{p})$ and then note that:

$$\frac{dV(a^{**}(\tilde{p}), \tilde{p})}{d\tilde{p}} = \frac{\partial f}{\partial \tilde{p}} + \frac{\partial f}{\partial a^{**}(\tilde{p})} \frac{\partial a^{**}(\tilde{p})}{\partial \tilde{p}}$$

By optimality of the contract between manager and firm we have $\partial f/\partial a^{**}(\tilde{p}) = 0$ and hence the first-order effect of a local change in $\tilde{p}$ is simply $\partial f/\partial \tilde{p}$. This demonstrates that the indirect effect of local changes in prices on producer surplus are second-order relative to the direct effect.

However, welfare is the (weighted) sum of consumer and producer surplus. In general, the indirect effect of a change in prices on consumer surplus need not be second-order relative to the direct effect. The effects of a change in price on consumer surplus can be seen from the following:

$$\frac{dCS}{dp_i} = \pi_i(\cdot)\, S'(X(p_i))X'(p_i) + \frac{\partial a^{**}}{\partial p_i} \sum_{i=1}^{n} \pi_i'(\cdot)\, S(X(p_i))$$

The indirect effect, the second term in the above, comes from the effect in a change in price on the equilibrium action taken by the manager. The indirect effect of a change in price of Δp_i on the change in consumer surplus is therefore:

$$\frac{\partial a^{**}}{\partial p_i} \sum_{i=1}^{n} \pi_i'(\cdot) \left[S(X(p_i^R)) - S(X(p_i^M)) \right]$$

Note that as demand becomes less elastic $S(X(p_i^R)) - S(X(p_i^M))$ approaches zero. We have thus proved:

Lemma 1. *Suppose that demand is sufficiently inelastic. Then for small values of* $\Delta p_1, ..., \Delta p_n$ *the effect on* ΔW *of the change in the contract offered by the firm is smaller than the direct effect on* ΔW.

We can now state the following result.

Proposition 3. *Suppose* λ *sufficiently large but finite, that C3 holds and Lemma 1 applies. Then in any solution to (3),* $\Delta p_1 \geq \Delta p_2 \geq ... \geq \Delta p_n$ *with at least one inequality strict.*

Proof. See appendix. ■

We can now state the major result of this section on optimal regulation—namely, that the change in profits between the regulated environment and the market environment is larger in higher (marginal) cost states. First, let $\Delta q_i = q_i^R - q_i^M$, where R and M index the market and regulated environments respectively.

Proposition 4. *Suppose λ sufficiently large but finite, that C3 holds and Lemma 1 applies. Then $\Delta q_1 \geq \Delta q_2 \geq ... \geq \Delta q_n$ with at least one inequality strict.*

Proof. See appendix. ■

Proposition 4 simply states that, relative to the market environment, regulation increases profits relatively more in high (marginal) cost states than in low (marginal) cost states. The intuition for this is straightforward. When the regulator cares sufficiently about producer surplus she must increase profits above the level of the market environment. Because any increase is distorting, she wants to do this in the least distorting manner, because she puts positive (though relatively less) weight on consumer surplus. Since the distortion from any price wedge is proportional to the *square* of the wedge, the least distorting states in which to increase price are those in which the market price is closer to marginal cost. These are high cost states.

4 An Example

In order to illustrate the results of the previous section we now offer a simple example.

The firm can have one of two possible (marginal) cost realizations c_1 or $c_2 < c_1$. The manager takes an action from the set $\{L, H\}$, which we interpret as low or high effort. This action affects the probability of c_1 or c_2 occurring. Let the probability distribution for each firm be as follows:

$$\begin{aligned}
\Pr(c &= c_1|L) = \eta \\
\Pr(c &= c_2|L) = 1 - \eta \\
\Pr(c &= c_1|H) = \zeta < \eta \\
\Pr(c &= c_2|H) = 1 - \zeta.
\end{aligned}$$

This implies that a harder action make low cost states more likely (i.e. FOSD). Suppose the inverse demand curve is $P(x) = 1 - \beta x$, where x is the quantity produced. The manager chooses price after observing the marginal cost realization and solves:

$$\max_{x} \left\{ (1 - \beta x - c)x \right\},$$

which yields equilibrium prices and quantities:

$$\begin{aligned} p^* &= \frac{1+c}{2} \\ x^* &= \frac{1-c}{2\beta}. \end{aligned}$$

This means that the equilibrium prices in states 1 and 2 are respectively $p_1^* = (1 + c_1)/2$ and $p_2^* = (1 + c_2)/2$. The associated demands are obviously $X_1^* = (1 - c_1)/2\beta$ and $X_2^* = (1 - c_2)/2\beta$.

Consumer surplus in the two states is therefore:

$$\begin{aligned} CS_1 &= \frac{c_1 (1 - c_1)}{4\beta}, \\ CS_2 &= \frac{c_2 (1 - c_2)}{4\beta}. \end{aligned}$$

From (3) then, welfare in the regulated environment is:

$$W^R = \sum_{i=1}^{2} \pi_i^R (a) \left[CS_i + \lambda q_i^R \right], \tag{4}$$

where Π_i^R is the firm's profit in state i, given by:

$$q_i^R = \sum_{i=1}^{2} \pi_i^R \left[x_i^R \left(1 - b x_i^R - c_i \right) \right].$$

The regulator's problem is:

$$\max_{p_1,p_2}\left\{W^R\right\}.$$

It is legitimate to use the quantities as control variables, since there is a one-to-one mapping between them and prices given the demand curve. This yields the first-order conditions:

$$\begin{aligned}
\pi_1^R(a)\left(\frac{c_1}{2}-\beta x_1^R\lambda+\left(1-c_1-\beta x_1^R\right)\lambda\right) &= 0,\\
\left(1-\pi_1^R(a)\right)\left(\frac{c_2}{2}-\beta x_2^R\lambda+\left(1-c_2-\beta x_1^R\right)\lambda\right) &= 0.
\end{aligned}$$

Solving simultaneously yields:

$$\begin{aligned}
x_1^{*R} &= \frac{c_1+2\lambda\left(1-c_1\right)}{4\beta\lambda},\\
x_2^{*R} &= \frac{c_2+2\lambda\left(1-c_2\right)}{4\beta\lambda}.
\end{aligned}$$

In terms of prices this is:

$$\begin{aligned}
p_1^{*R} &= 1-\frac{c_1+2\lambda\left(1-c_1\right)}{4\lambda},\\
p_2^{*R} &= 1-\frac{c_2+2\lambda\left(1-c_2\right)}{4\lambda}.
\end{aligned}$$

It is then straightforward to calculate the change in profit between the regulated and unregulated environment. They are:

$$\begin{aligned}
\Delta q_1 &= -\frac{\left(c_1\right)^2}{16\beta\lambda}\\
\Delta q_2 &= -\frac{\left(c_2\right)^2}{16\beta\lambda}.
\end{aligned}$$

Since $1>c_1>c_2$ it follows that $\Delta q_1>\Delta q_2$. For instance, when $c_1=1/4, c_2=1/2, \lambda=1$

and $\beta = 1$ the values are $\Delta q_1 = -1/256, \Delta q_2 = -1/64$.

Note that the probabilities of being high or low cost conditional on action do not enter either the Δqs. This is because the indirect effect is second order relative to the direct effect, per Lemma 1.

It is then easy to show that the firm chooses a lower action for their manager in the regulated environment. From (2) a necessary and sufficient condition for this here is:

$$\sum_{i=1}^{2} \pi_i'(a)(\Delta q_i) \geq 0.$$

Noting that $\pi_1'(a) = -\pi_2'(a)$ (since probabilities sum to one) this becomes:

$$\pi_2'(a) \left[\Delta q_2 - \Delta q_1\right] \geq 0.$$

By FOSD (which is implied by MLRP) $\pi_2'(a) < 0$ (a higher action makes the high cost state 2 less likely). For the above inequality to hold, therefore, we require $\Delta q_2 - \Delta q_1 \leq 0$. This is indeed the case, since

$$\Delta q_2 - \Delta q_1 = \frac{(c_2)^2 - (c_2)^2}{16\beta\lambda^2},$$

which is negative since $1 > c_1 > c_2$. Thus optimal regulation makes low cost states less attractive and the firms offers contracts which induces their managers to put less probability weight on those states by taking a lower action.

5 Discussion and Conclusion

5.1 Multiple Firms

A major simplification we made in our model was to consider a single firm in both the regulated and unregulated environments. This is not innocuous, and it would certainly be desirable to obtain results for a general number of firms. This, however, raises a number of issues which we have not, at this juncture, been able to resolve satisfactorily. First, with multiple firms the optimal contract involves relative performance evaluation in order to filter out common-shocks to each agent (Holmstrom, 1982). Second, with multiple firms it is well known that the optimal contract differs in the sense that the strategic interactions between managers must be taken into account (Fershtman and Judd, 1987). Both of these factors substantially complicate the analysis of the optimal contract, and hence the optimal regulatory response. We conjecture that our two main results do not extend to this general case without qualification, though we do not have a counterexample.

5.2 Alternative Models of Regulation

We assumed in section 3 (and in the example in section 4) that the regulator is able to observe the realized (marginal) cost of the firm. There is a vast literature, beginning with Baron and Myerson (1982), which considers regulation when costs are unobservable. In this section we briefly explore the impact of asymmetric information in our model.

Since Mirrlees (1971), it has been understood that a fundamental property of any screening model, such as Baron-Myerson, is that agents who are "better" types from the perspective of the principal must be accorded an "information rent" in order for them to reveal themselves as being such a type. If this were not the case, then it would be optimal for them to pretend to be a "bad" type, violating the incentive compatibility constraints of the principal's problem.

Thus, in any asymmetric information model of regulation in which the regulator uses screening (which is in the regulator's interest), low cost types must be rewarded for revealing themselves as such. This runs exactly counter to the symmetric information based model of regulation we have presented. In our model, regulation is relatively more advantageous when the firm is in a high cost state than a low cost state. In fact, our Propositions 1 and 2 would be exactly reversed under any standard model of regulation with asymmetric information.

That is, managers at regulated firms would face more intense incentive contracts and would receive higher *ex post* compensation. This is in stark contrast with the empirical evidence outlined in Sections 1 and 2.

It could be the case that the appropriate model of regulation is one with asymmetric information, and that the other factors mentioned in Section 1, such as political constraints, are so strong as to swamp the contracting effect. We certainly cannot rule this out, and if it were these case, then we believe it would be of interest that such other factors are so strong.

5.3 Regulator Commitment

As discussed in section 3, it is natural to assume that the regulator is able to credibly commit to a price schedule. If, however, this were not the case, then our conclusions would likely be strengthened. With commitment the regulator essentially acts as a Stackelberg leader in determining the indirect effect of the price schedule on firm behavior. The case without commitment is analogous to a simultaneous move game since the firm would know that the regulator would always set the price which maximized the W function *ex post.* This conclusion, however, is merely a conjecture at this point, and warrants more detailed study.

5.4 Concluding Remarks

We have presented an integrated model of regulator-firm and firm-manager agency relationships in which it is optimal for the firm to offer managers at regulated firms less intense incentive contracts. The change in firm profits that such regulation induces also means that managers at regulated firms receive lower expected compensation, since they face less risk. Our findings are consistent with a growing body of empirical evidence that documents precisely these two facts in a variety of regulatory settings.

Our analysis implies that regulated firms are less X-efficient than their unregulated counterparts. Because the regulatory environment makes it optimal for them to offer less intense incentives to their managers, the agency problem within the firm is larger. This is a cost of regulation, which should be weighed against any benefits which such regulation may deliver.

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6 Appendix

6.1 Assumptions A1-A3 from Grossman and Hart (1983).

Assumption A1. *In the managerial utility function*

$$U(a, I) = G(a) + K(a)V(I)$$

V is a continuous, strictly increasing, real-valued, concave function on an open ray of the real line $\mathcal{I} = (\underline{I}, \infty)$. Let $\lim_{I \to \underline{I}} V(I) = -\infty$ and assume that G and K are continuous, real-valued functions and that K is strictly positive. Assume that for all $a_1, a_2 \in A$ and $I, \widehat{I} \in \mathcal{I}$ the following holds

$$G(a_1) + K(a_1)V(I) \geq G(a_2) + K(a_2)V(I)$$
$$\Rightarrow G(a_1) + K(a_1)V(\widehat{I}) \geq G(a_2) + K(a_2)V(\widehat{I}).$$

Finally, assume that either $G = 0$ or K is constant on A.

Notation 1. $\mathcal{U} = V(\mathcal{I}) = \{v | v = V(I) \text{ for some } I \in \mathcal{I}\}$.

Assumption A2. $\left(\overline{U} - G(a)\right) / K(a) \in \mathcal{U}, \forall a \in A$.

Assumption A3. $\pi_i(a) > 0$, $\forall a \in A$ *and* $i = 1, ..., n$.

6.2 Proofs

Proof of Proposition 1. Recall that a necessary and sufficient for regulation to cause an easier action is:

$$\sum_{i=1}^{n} \pi_i'(a)(\Delta q_i) \leq 0. \tag{5}$$

Condition 2 implies that $\Delta q_1 \geq \Delta q_2 \geq ... \geq \Delta q_n$, with at least one equality. Differentiating the identity $\sum_{i=1}^{n} \pi_i(a) = 1$ (probabilities sum to one) yields $\sum_{i=1}^{n} \pi_i'(a) = 0$. Holden (2005a) Lemma 2 shows that MLRP implies that $\exists x$ such that $\pi_i'(a) < 0, \forall i \leq x$ and $\pi_i'(a) > 0, \forall i > x$. Then it must be that $\sum_{i=1}^{n} \pi_i'(a)(\Delta q_i) < 0$. ■

Proof of Proposition 2. Recall that the manager's participation constraint is:

$$\sum_{i=1}^{n} \pi_i(a^*)U(a^*, I_i) \geq \overline{U}.$$

From Proposition 1 $a^{**}(1) <_S a^*(0)$. Then by A1

$$\sum_{i=1}^{n} \pi_i(a^*(1))U(a^*(1), I_i(1)) > \sum_{i=1}^{n} \pi_i(a^*(0))U(a^*(0), I_i(0)).$$

Since $\overline{U}$ is a constant and by A1 the participation constraint is binding (see Grossman and Hart, 1983 for a proof) it must be that

$$\sum_{i=1}^{n} \pi_i(a^*(1))U(a^*(1), I_i(1)) = \sum_{i=1}^{n} \pi_i(a^*(0))U(a^*(0), I_i(0)).$$

Therefore a sufficient condition for

$$\sum_{i=1}^{n} \pi_i(a^*(1))I_i(1) > \sum_{i=1}^{n} \pi_i(a^*(0))I_i(0)$$

is that the conditional distribution which gives rise to $\pi_1(a^*(1)), ..., \pi_n(a^*(1))$ is not second-order stochastically dominated by that which gives rise to $\pi_1(a^*(0)), ..., \pi_n(a^*(0))$. Suppose this is not the case. Then the firm could implement $a^{**}(1)$ in the regulated environment and receive a higher payoff, in contradiction of optimality. ■

Proof of Proposition 3. From Lemma 1 we can focus only on the direct effect of $\Delta\tilde{p}$. Note that $\lambda = 1$ is equivalent to the regulator maximizing the sum of consumer and producer surplus. For $\lambda = 1$, $p_i^R = c_i \ \forall i$ and $q_i^R = 0$. As $\lambda \to \infty$, $p_i^R \to p_i^m \ \forall i$ and hence $q_i^R \to q_i^m$, where p_i^m and q_i^m are the monopoly price and profit respectively. Therefore, by the continuity of W in λ, the Intermediate Value Theorem implies that $\exists \hat{\lambda} \ s.t. \ q^R = q$. Hence for $\lambda > \hat{\lambda}$ it must be that $\Delta p_i > 0$ for some i. Let $W^*(\lambda)$ be the maximized value of the welfare function, given λ. Further, let $\theta(\lambda)$ be the associated weight on the term $\sum_{i=1}^n \pi_i (a^{**}(\tilde{p})) S\left(X\left(p_i^R\right)\right)$. Since λ is bounded from above, $\theta(\lambda)$ is strictly bounded away from zero, otherwise $W^*(\lambda)$ could be increased, in contradiction of being at an optimum. Denote $D(\lambda, \tilde{p})$ the deadweight loss given the pair $(\lambda, \tilde{p})$. Note that, by duality, $\tilde{p}^* = \arg\max_{\tilde{p}} \{W\} = \arg\min_{\tilde{p}} \{D\}$. By C3, $p_1 - c_1 \leq p_2 - c_2 \leq ... \leq p_n - c_n$. Then from a basic result of optimal commodity taxation theory, it follows that the welfare loss from an increase in the price of a commodity is smaller when the increase is from a price closer to the optimum (see, for instance, Mas-Colell, Whinston and Green, 1995, pp.332). Therefore $\Delta p_y \geq \Delta p_z$ for all $z > y$, and $\Delta p_y > \Delta p_z$ for some $z > y$. ■

Proof of Proposition 4. From Proposition 3 we have $\Delta p_1 \geq \Delta p_2 \geq ... \geq \Delta p_n$, with at least one inequality strict. Since $c_i^R = c_i$ for all i by construction, and for λ finite $p_i < p_i^m, \forall i$, it follows that $\Delta q_y \geq \Delta q_z$ for all $z > y$, and $\Delta q_y > \Delta q_z$ for some $z > y$. ■