

What's Old and What's New in Contract Theory

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Overview

- Emerged as a field from the late 1970s
- Basically: game theory where the rules of the game are endogenous
- Some early contributions: Mirrlees (1974), Spence (1976), Holmstrom (1979, 1982a,b,c), Grossman-Hart (1986), Laffont-Tirole (1990)
- Basic divide b/w “complete contracting” and “incomplete contracting”
- This talk: provide an overview of existing literature and highlight some recent trends and developments in the last few years
- Selective, not encyclopedic

Talk Outline

- Complete Contracting
 - Moral Hazard
 - Classic model and first-order approach
 - Linear Contracts and Dynamics
 - *Robust Contracting*
 - *Beyond the first-order approach*
 - Career Concerns
 - Multi-tasking
 - Relational Contracting
 - *Building Routines and PPDs in SSEs*
 - Adverse Selection and Screening

Talk Outline

- Incomplete Contracting
 - Grossman-Hart-Moore and Boundaries of the Firm
 - Financial Contracts and Capital Structure
 - Internal Organization of Firms
 - Foundations
 - *The “lightly behavioral” response*
 - *The “getting inside people’s heads” response*
- Empirical Work and Laboratory Experiments

The Classic Moral Hazard Problem

- $a \in A$ (Action Set)
- This leads to q (verifiable revenue)
- Stochastic relationship $F(q; a)$
- Incentive scheme $I(q)$
- The Principal solves the following problem:

$$\begin{aligned} & \max_{\hat{I}(\cdot), \hat{a}} \left\{ \int (q - \hat{I}(q)) dF(q; \hat{a}) \right\} \\ \text{s.t. (i)} & \hat{a} \text{ solves } \max_{a \in A} \left\{ \int u(a, \hat{I}(q)) dF(q; a) \right\} & \text{(IC)} \\ & \text{(ii) } \int u(\hat{a}, \hat{I}(a)) dF(q; \hat{a}) \geq \bar{U} & \text{(IR)} \end{aligned}$$

Classic Moral Hazard

- Use the deterministic problem of the Principal inducing the Agent to choose the action because there may be multiple actions which are equivalent for the Agent but the Principal might prefer one of them
- The Principal is really just a risk-sharing device

The First-Order Approach

- Suppose $A \subseteq \mathbb{R}$
- The problem is now

$$\max_{a, I(\cdot)} \left\{ \int_{\underline{q}}^{\bar{q}} (q - I(q)) f(q|a) dq \right\}$$

subject to

$$a \in \arg \max_{\hat{a} \in A} \left\{ \int_{\underline{q}}^{\bar{q}} u(I(q)) f(q|a) dq - G(a) \right\} \quad (\text{IC})$$

$$\int_{\underline{q}}^{\bar{q}} u(I(q)) f(q|a) dq - G(a) > \bar{U} \quad (\text{IR})$$

The First-Order Approach

- IC looks like a tricky object
- Maybe we can just use the FOC of the agent's problem
- That's what Spence-Zeckhauser, Ross, Harris-Raviv did

- FOC is

$$\int_{\underline{q}}^{\bar{q}} u(I(q)) f_a(q|a) dq = G'(a)$$

- SOC is

$$\int_{\underline{q}}^{\bar{q}} u(I(q)) f_{aa}(q|a) dq = G''(a)$$

The First-Order Approach

- If we use the first-order condition approach:

$$\frac{1}{u'(I(q))} = \lambda + \mu \frac{f_a(q; a)}{f(q; a)}$$

- f_a/f is the likelihood ratio
- $I \uparrow q \Leftrightarrow \frac{f_a}{f} \uparrow q$
- But the FOC approach is not always valid – you are throwing away all the global constraints
- The $I(q)$ in the agent's problem is endogenous!
- Valid under MLRP + CDFC, but...
- Equally importantly: **everything depends on the likelihood ratio**

Linear Contracts

- Everything depending on the likelihood ratio is troubling
- Many real-world contracts similar in wildly different economic environments (i.e. different likelihood ratios)
- Holmstrom-Milgrom (ECMA, 1987)
- Say $w = t + vq$
- Assume normally distributed performance and CARA (exponential) utility
- Let $q = a + \varepsilon$ with $\varepsilon \sim N(0, \sigma^2)$
- Principal is risk-neutral
- The Agent is risk-averse with:

$$U(w, a) = -e^{-r(w - \psi(a))}$$

- Let $\psi(a) = \frac{ca^2}{2}$
- Note that r is the coefficient of absolute risk-aversion $-u''/u'$

Linear Contracts

- The CARA-Normal formulation allows for a neat trick
- Let $x \sim N(0, \sigma_x^2)$
- $E[e^{\gamma x}] = e^{\gamma^2 \sigma_x^2 / 2}$ (this is essentially the calculation done to yield the moment generating function of the normal distribution)
- With this certainty-equivalent formulation one can show the following

$$v = \frac{1}{1 + rc\sigma^2}$$

- Which is a nice, simple, closed form solution
- Justified by dynamic P-A problem with stationarity delivered through CARA-Normal (no wealth effects) plus no intermediate consumption assumptions

New: Robust Contracting

- Gabriel Carroll (Stanford)
- Principal is uncertain about what actions the agent can take
- P knows some actions, but unknown actions may exist
- Robustness: P evaluates contracts by their worst-case performance
- Risk-neutrality (P and A) and limited liability
- No other functional form assumptions
- Optimal contract is linear
- Rationalizes widespread use of linear contracts in practice
- Tractable modeling approach for moral hazard under non-quantifiable uncertainty
- See also: “Robust Incentives for Information Acquisition”

New: Beyond the First-Order Approach

- Grossman-Hart (ECMA, 1983) formulate a very general P-A problem without using the FOA
- Everything finite: $q_1, \dots, q_n, a \in A, I_1, \dots, I_n$
- Two steps: (i) find the cost-minimizing way to implement any action $C(a)$; (ii) choose the action that maximizes $B(a) - C(a)$
- But step (ii) is a non-convex programming problem—tough to say much
- Holden observation: but can do comparative statics using monotone methods (Topkis, Milgrom-Shannon)
- Can show that all of the comparative statics from the linear model generalize except the one on risk-aversion
- Also provides a method for applied work without function form assumptions

Career Concerns

- Holmstrom (1982, 1999)
- Non pecuniary incentives can be very important as well
- Principal and Agent—both risk neutral and neither know agent's ability
- But agent's ability revealed over time in REE from observed output

$$w_2 = \bar{\theta} \left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \right) + (y_1 - a_1^*) \left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \right)$$

- Agent disciplined w.r.t. out of equilibrium beliefs
- Agent may work too hard early in career
- Should see more formal contracts later in career (Gibbons-Murphy evidence)

Multitasking

- Grossman-Hart allows for multiple tasks—but maybe *too general*
- Let there be two tasks $i = 1, 2$.
- Suppose that the output that accrues to the principal on task i is $q_i = a_i + \varepsilon_i$.
- Joint distribution to the shocks to output on the two tasks is $(\varepsilon_1, \varepsilon_2) \sim N(0, \Sigma)$ where the variance-covariance matrix is

$$\Sigma = \begin{pmatrix} \sigma_1^2 & R \\ R & \sigma_2^2 \end{pmatrix}$$

- P's benefit function b is given by $b = a_1 + a_2$

$$U(a_1, a_2, w) = -e^{-r(w - \psi(a_1, a_2))},$$

where $\psi(a_1, a_2) = \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) + \delta a_1 a_2$.

- The term $\delta(a_1, a_2)$ captures the interaction between the two tasks in the agent's cost of effort function

Multi-tasking

$$v_1 = \frac{1 + r\sigma_2^2(c_2 - \delta)}{1 + r\sigma_1^2c_1 + r\sigma_2^2c_2 + r^2\sigma_1^2\sigma_2^2(c_1c_2 - \delta^2)},$$

- Recover the single-task case if $\delta = 0$: highlights that it is interactions between the tasks that generate the interesting insights
- As we go from $\delta = 1$ to $\delta = -1$ (i.e. from tasks being substitutes to complements in the agent's cost of effort function) both v_1 and v_2 increase.
- When task stronger complements doing more of one task *lowers* the cost of effort on other tasks

Multi-tasking

- What happens as one task becomes easier to measure relative to the other?
- Suppose $\sigma_2^2 \rightarrow \infty$ (task 2 is really hard to measure) then note that:

$$v_2 \rightarrow 0$$
$$v_1 \rightarrow \frac{r(c_2 - \delta)}{rc_2 + r^2\sigma_1^2(c_1c_2 - \delta^2)}$$

- Put *all* the incentive on task 1.
- When the tasks are perfect substitutes in the agent's cost of effort function and task 2 becomes unmeasurable then incentive on *both* tasks goes to zero!
- Incentive compatibility means that the agent must equate the marginal return to each task, and the only way this can happen is for there to be zero incentive on both

Relational Contracts

- Bull (QJE, 1987), MacLeod-Malcolmson (ECMA, 1989), Baker-Gibbons-Murphy (QJE, 1994), Levin (AER, 2003)
- What can be achieved if *no* formal contract can be written?
- In a one-shot interaction, nothing
- But in repeated interactions, value of future relationship can discipline current temptation for bad behavior
- Bonuses a classic example
- Folk Theorem logic pretty straightforward w/ symmetric information
- More complex with hidden actions (MM, Levin)
- Mix of relational and formal interesting (BGM)

New: Building Routines

- Sylvain Chassang (AER, 2008)
- Can agents with conflicting interests learn to cooperate when the details of cooperation are not common knowledge?
- Repeated game with one-sided asymmetric information about how benefits accrue
- Start out w/ imperfect monitoring so need inefficient punishments on the equilm path (a la Green-Porter)
- As common history (relationship) grows monitoring becomes possible for the uninformed player
- More efficient “routines” are built
- Information revelation is costly so don’t want to reveal all
- Path dependent equilibria (PPDs in SSEs)–see also Ellison-Holden (JLEO, 2014)

Adverse Selection and Screening

- Begins with Mirrlees (ReStud, 1976) celebrated paper on optimal income taxation
- Remarkable for a number of reasons
 - Economics: (i) “information rents” required for players to reveal their type; (ii) no distortion at the top
 - Technical: (i) Single crossing property; (ii) integrate up from the bottom type/change of measure; (iii) downward IC constraints binding and IR of lowest type binding
- Baron-Myerson (ECMA, 1982): regulating a monopolist with unknown costs
- Laffont-Tirole (“A Theory of Incentives in Procurement Regulation”, MIT Press—numerous papers)
- Dynamic screening and Coase Conjecture (Coase 1972, Hart-Tirole 1988, Bulow, Gul,...)—revelation principle fails and information comes out slowly

Incomplete Contracts: Grossman-Hart-Moore

- Coase (1937): if markets are so good, why do firms exist?
- Fact: 50 percent of all economic activity takes place in firms
- Coase answer: transaction costs—developed by Williamson
- But fails to analyze the costs and benefits of ownership
- Grossman-Hart (JPE, 1986): asset ownership in a world of incomplete contracts
- Not all future contingencies can be contracted on (bounded rationality, hard to describe,...)
- Ex post hold-up gives rise to suboptimal ex ante investments
- Optimal ownership structure minimizes loss from this friction

Incomplete Contracts: Grossman-Hart-Moore

- Consider the relationship between a B (uyer) and a S (eller) of an intermediate good (a “widget”).
- B can use the widget to produce a final good which can be sold to a consumer.
- The consumer values the final good at v .
- S can make a privately costly investment which makes the widget cheaper to produce.
- If S makes the investment, which costs \$5, then the widget can be produced for \$10, otherwise it costs \$16 to produce.
- B can make a privately costly investment which makes the final good more valuable to the consumer.
- This investment also costs \$5.
- If B makes the investment $v = \$40$, otherwise $v = \$32$.

GHM

- Note that B and S have different human capital characteristics.
- B and S would like to write a contract which specifies that each party should make its respective investment, because that leads the total surplus in the relationship to be $40 - 10 - 5 - 5 = 20$.
- But suppose that contracts are incomplete—*observable* but not *verifiable*.
- Now B and S will have to bargain about the price that B pays to S for the widget *after* the investment stage.
- Suppose that B and S are non-integrated so that at the bargaining stage they split whatever surplus is generated 50:50.
- This split arises in a situation of Nash bargaining because B cannot produce the final good without the widget from S , and S has no use for the widget if it is not sold to B .

- Therefore, both B and S have zero outside options.
- In this situation B will not invest—If B does invest, she will bear a private cost of 5, but gets half of the increase in surplus of $40 - 32 = 8$, or 4.
- Similarly, S bears a cost of 5 by investing, but gets an increased payoff of $(16 - 10)/2 = 3$ in the bargaining. So S won't invest either.
- Thus neither B nor S invests, and total surplus is thus $32 - 16 = 16$.

GHM

- Now suppose B and S are vertically integrated, with S owning B 's machine that produces a final good.
- S no longer needs to bargain with B because S owns the machine.
- So S gets *all* of the increased surplus from investing in cost reduction (that is $16-10-5$), and thus will be prepared to invest.
- However B will not invest as she will get none of the benefit of making the final good more valuable.
- S cannot compel B to invest, nor contract on B making the investment.
- Total surplus is thus $32 - 10 - 5 = 17$.
- This is larger than under non-integration, so forward vertical integration is desirable.

GHM

- B ownership (backward integration), does better still.
- Now B invests, but S does not, yielding total surplus of $40 - 16 - 5 = 19$.
- This is not as good as if contracting was possible (that would yield a surplus of 20), but it is better than the other possible ownership structures.
- What makes B ownership preferable to S ownership is that B 's investment is relatively more important (at the margin) than S 's.
- Both cost 5, but B 's has a benefit of $40 - 32 = 8$, whereas S 's has a benefit of $16 - 10 = 6$.
- Since asset ownership presumably *can* be contracted upon, we would expect B ownership to emerge as the equilibrium ownership structure as it maximizes joint economic surplus.

Financial Contracting

- Choice of financial structure matters a lot in practice
- But Modigliani–Miller...
- Tax and other explanations
- Aghion-Bolton (ReStud, 1992) use the incomplete contracts approach: debt financing provides a contingent and thus more flexible governance structure for firms.
- More responsive to whether good or bad states of the world materialize in the future.
- Consider an entrepreneur who has ideas but no wealth
- E gets private benefits (e.g. empire building)
- Investor just cares about money

Financial Contracting

- 3 governance structures/3 different types of financial contracts
 - ① full entrepreneur control, e.g. when the outside investor only holds non-voting shares
 - ② full investor control, I holds all the voting equity in the firm
 - ③ contingent control: E has control if revenues are high (good state of nature), I has control if revenues are low (bad state of nature); this control allocation in turn can be achieved in each state.
- Timing
 - at the contracting stage, parties write a financial contract which allocates control rights.
 - State of nature is realized and is verifiable by a third party
 - An action must be chosen (e.g. expand/continue or shut down business)
- If good E wants to continue but I wants to get paid back
- In bad E will still want to continue, whereas I may want to liquidate

Financial Contracting

- Entrepreneurs will often place a higher value on the continuation of the firm, while investors are more likely than the entrepreneur to prefer pulling money out of the firm in a good state succeeds or closing it down in a bad state.
- Under E control, I risks incurring large losses if the bad state occurs—and anticipating this, the investor may refuse to finance the firm's investment through non-voting shares.
- Under I control, she is protected against the risk of large losses if the bad state occurs (can always decide to liquidate the firm in the bad state), but can also impose a suboptimal course of action in the good state (stop E expanding)
- Contingent control through debt financing allows E to maintain control over the business in the good state, while in the bad state control will be transferred to the I who can liquidate.

Internal Organizational Structure

- Inside the firm asset ownership isn't possible, but allocation of authority is.
- Distinguish real from formal authority: taking decisions requires acquiring information, which takes effort. Design of the organization, together with the allocation of formal decision rights, will determine how real authority is distributed within the firm (Aghion-Tirole, JPE 1997)
- P(principal) and A(agent)
- *Formal authority* can be allocated contractually: e.g. shareholders allocate formal authority to a board of directors.
- *Real authority* is exerted either by the party which enjoys formal authority if that party also has the information, and by the other party if the only the other party has the information.

Internal Organizational Structure

- Contractual incompleteness key: any formal agreement between P and A cannot specify particular project choices, as these are not verifiable by third parties.
- Both P and A can invest in information acquisition: by investing effort $\frac{1}{2}E^2$, P acquires the relevant information to make a decision with probability E .
- Similarly, by investing effort $\frac{1}{2}e^2$, A acquires the relevant information to make a decision with probability e .
- Congruence b/w P and A: let α denote the probability that P's preferred project is also A's preferred project (congruence), and suppose that a party gets utility zero if the other party chooses her preferred project and preferences are not congruent.
- Finally, we assume that an uninformed party will never pick a project at random as this might be too risky.

Internal Organizational Structure

- Timing
 - First, the two parties sign a contract that allocates formal authority to one party, either P or A
 - Then, both parties make effort choices
 - Then, the party with no formal authority proposes a project to the party with formal authority. If the party with formal authority has acquired information, then the party with formal authority picks its own preferred project. If the party with formal authority has not acquired information, then it accepts the project submitted by the party without formal authority.
- In this latter case, *real authority* differs from *formal authority*, since the party with formal authority is uninformed and therefore can only rubberstamp the other party's project proposal.

Internal Organizational Structure

- P delegating formal authority to A involves a cost and a benefit.
- Cost: A may choose a project which P does not like. This is the *loss of control* effect.
- Benefit: encourages A to invest more effort in information acquisition. This is the *initiative* effect.
- Which effect dominates will depend on a key parameter: the extent of congruence between P and A
- Low congruence: P retains formal control; high congruence: P delegates authority.
- Span of control / overstretching as a commitment device.

Foundations

- The key premise of GHM is that there is information that is *observable* to the contracting parties, but not *verifiable* to a third party / court
- Maskin and Tirole (ReStud, 1999) argue that this distinction is without foundation
- If it is observable it can be *made* verifiable by using a suitable mechanism
- Use Moore-Repullo (ECMA, 1988) subgame perfect implementation–3 stage mechanism

Foundations

- 1 B announces either “high” or “low.” If “high” then B pays S a price equal to 14 and the game then stops.
- 2 If B announces “low” and S does not “challenge” B ’s announcement, then B pays a price equal to 10 and the game stops.
- 3 If S challenges B ’s announcement then:
 - 1 B pays a fine F to T (a third party)
 - 2 B is offered the good for 6
 - 3 If B accepts the good then S receives F from T (and also the 6 from B) and we stop.
 - 4 If B rejects at 3b then S pays F to T
 - 5 B and S Nash bargain 50:50 over the good.
- With common knowledge of v , this does the trick whether $v = 14$ or $v = 10$

New: The “Lightly Behavioral Response”

- Aghion-Fudenberg-Holden-Kunimoto-Tercieux (QJE, 2012) show that this depends critically on the common knowledge assumption.
- Suppose v is common- p belief for p arbitrarily close to 1 (1 is c.k.)
- Thm: Moore-Repullo mechanisms never admit a truthful equilibrium (and cannot be approximated as $p \rightarrow 1$ in mixed strategies)
- Thm: Consider *any* mechanism that implements a desired non-Maskin-monotonic social choice function under c.k. Then if $p \neq 1$ there always exists a “bad” Nash equilibrium of the game induced by the mechanism that can be sustained as a sequential equilibrium.
- Takeaway: These mechanisms are not robust.
- Then show that with asymmetric information, outside option allocation (e.g. assets) always beats the best mechanism (see also Segal-Whinston 2013, Baliga-Sjostrom 2013).

New: Contracts as Reference Points

- Hart and Moore (2008)
- Contracting parties can agree on some aspects of performance, but not on others
- e.g. might be possible to agree on a contract to paint a house, but not on whether the painting of the house is done in a timely manner
- Performance could be “consummate” (I paint your house, and quickly), or “perfunctory” (I paint it, but I take my time)
- Only perfunctory performance is enforceable by a court, while consummate performance is non-contractible.
- Suppose consummate performance is slightly more costly for the party performing the service, but much more valuable to the party receiving the service.

New: Contracts as Reference Points

- HM assume that being treated well is equivalent to a party getting that to which the party feels *entitled*.
- This sense of entitlement creates a reference point, which in turn creates the possibility of one or another party feeling “aggrieved.”
- Show how shading induced by reference points can provide a rationale for simple ownership contracts.
- Immune to Maskin-Tirole critique
- Inefficiency is not ex ante underinvestment–ex post post aggrievement instead (c.f. Williamson)
- Can the reference point change from the time of contracting to the time of performance, perhaps if market conditions change?
- How applicable is this theory to sophisticated parties, including many firms?

New: Cognition and Endogenously Incomplete Contracts

- How does the level of contractual incompleteness depend upon cognitive ability? (Tirole, AER 2009)
- Two parties, B and S, who contract over the delivery of a good.
- Standard specification is denoted A .
- But it may turn out that A does not suit the buyer after the contract is signed—and that some alternative specification A' , which is a priori undescribable in the initial contract is preferable.
- Moving from A to A' costs a but is efficient.
- S has hold-up power
- B can invest in cognitive effort to learn whether alternative A' is better

New: Cognition and Endogenously Incomplete Contracts

- If B finds out that A' is better, then will disclose this to S at the contracting stage to avoid hold up.
- If B does not find out about A' and yet A' is better, renegotiation and the adjustment cost are unavoidable.
- Contractual incompleteness is measured by the equilibrium probability that B learns about A' .
- Even if the adjustment cost of actually changing the product was zero, B wants to invest to avoid hold up.
- Contractual completeness is increasing in the adjustment cost (greater scope for hold-up by S); in S's bargaining power; and decreasing in S's "patience."

Empirical Work

- Holmstrom: “A field with too much theory and not enough evidence.”
- Tough to get evidence
- Coase’s idea: a repository of contracts
- Some clever approaches
 - Baker-Gibbs-Holmstrom (QJE, 1994): org. structure/promotions
 - Elfenbein-Lerner (RAND, 2003): internet portals and alliances
 - Kaplan-Stromberg (ReStud, 2002): financial contracts b/w VCs and entrepreneurs
 - Sufi (JF, 2007): Syndicated loans and voting rules
 - Forbes-Lederman (AER, RAND): asset ownership/control in airlines
 - Acemoglu-Aghion-LeLarge-van Reenen (QJE, 2007): technology and decentralization
- But how many top job market candidates do “empirical contract theory” ?!

Empirical Work

- New(ish) frontier is laborator experiments
- If the data won't come to you...
- Pro: control the environment very precisely (e.g. i'll tell you what your cost of effort is)
- Con: external validity concern is especially important (Columbia undergrads versus CEOs...)
- Some selected works
 - Fehr-Schmist (EER, 2000): principals want less complete contracts than theory predicts
 - Fehr-Schmidt (AER, 2007): carrots & sticks (interaction of bonuses and fines)
 - Fehr-Klein-Schmidt (ECMA, 2007): fairness and contracting–bonus contracts good
 - Aghion-Fehr-Holden-Wilkening (2011): subgame perfect implementation w/ lack of common knowledge

Concluding Musings

- Contract theory is definitely now a field—e.g. 22/25 MIT PhD students typically take 14.281 (2nd year Contract Theory course)
- Influence in: trade, macro, **finance**,...
- “Who’s your contract theorist?”
- Behavioral contract theory is “a thing” and will remain so
- Open question: “lightly” behavioral, “get inside people’s heads” or a mix of the two...
- Empirical: impact of text recognition approaches? Coase meets PERL?