Does Competition Make Firms More Efficient?

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Abstract

A large literature considers the impact of product market competition on the internal efficiency of firms–what Leibenstein (1966) called "X-Efficiency." Formal models tend to produce ambiguous results which depend crucially on the strategy space in the product market. By utilizing monotone methods, we are able to take a substantially more general approach. We provide a necessary and sufficient condition for increased product market competition to cause an agent in a principal-agent setting to take a harder action. We do so without imposing any structure on the product market. We further show that it can be the case that product market competition causes a harder action but actually *increases* the welfare loss. This is because the first-best action can be more responsive to product market competition than the second-best action.

Keywords: Managerial effort, principal-agent problem, monotone comparative statics, X-efficiency.

JEL Classification Codes: L13, L22

1   Introduction

The most celebrated result in economics–the First Welfare Theorem–tells us that product market competition increases *allocative* efficiency. Beginning in earnest with Leibenstein (1966), though with distant roots[1], economists have investigated whether competition increases the *internal* efficiency of firms.

A natural way to do so is to consider how product market competition affects the principal-agent problem–since much economic activity takes place in environments where there is a separation of ownership and control. A large literature finds that whether competition leads to a harder action from the agent depends crucially on the structure of product market competition. For instance, with Cournot competition Martin (1993) finds that a larger number of firms reduces managerial effort, while Raith (2003) reaches the opposite conclusion in a differentiated products Bertrand setting. Schmidt (1997) presents a number of different models in which competition can increase or decrease effort[2].

Some of these models take the classic approach to the principal-agent problem: where the agent is risk-averse and the principal must satisfy incentive compatibility and individual rationality constraints. Others, following Innes (1992), assume that the agent has limited liability and thus the principal must ensure a lower bound on the payment to the agent *ex post*.

In this paper we attempt to synthesize this literature. Using a technique for analyzing the principal-agent problem in a general setting we provide a condition which is necessary and sufficient for the direct effect of increased product market competition to cause the agent to exert more effort in both the classic model and the limited liability model. When the limited liability constraint does not bind, or where it does bind but there are two output levels the conditional takes a particularly simple form and is independent of the cost to the principal of inducing the optimal action. We go on to show that the major papers in this literature on competition and agency costs can be nested as special cases of this model. This helps to clarify the role of special assumptions–often

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[1]For instance: Smith (1776,1910): "Monopoly...is a great enemy to good management" and Hicks (1935):"The best of all monopoly profits is a quiet life."

[2]There is also a sizeable empirical literature. See for intance Nickell (1996), Blundell et al. (1999), Galdon-Sanchez and Jr. (2002) and Cunat and Guadalupe (2005)

on the structure of the product market–which such papers make.

The condition we identify is quite stringent. Only in certain cases will increased competition lead to a harder action being taken by the agent. Moreover, we show that it is quite possible for competition to lead to a harder action, but for the welfare loss (the difference between the principal's payoff in the first-best and the second-best) actually to *increase*. Even when competition increases the second-best action, it may increase the first-best action more. We conclude, therefore, that any claim that product market competition reduces agency costs, and therefore increases the internal efficiency of firms, is not generally true.

Much of the literature has focused solely on the direct effect of competition on incentives. But a change in the incentives at one firm has an indirect effect on the optimal contract at another. In principle, it is quite possible for this indirect effect to go in the opposite direction to the direct effect. Using results from the theory of supermodular games, we are able to provide a sufficient condition for the indirect effect to reinforce the direct effect. This approach has the virtue of not requiring the equilibrium to be solved for explicitly–and thus makes general settings tractable.

The remainder of the paper is organized as follows. Section 2 provides the model and approach. Section 3 uses this to analyze the effect of product market competition on the agent's *action*. Section 4 considers the effect of product market competition on the *welfare loss* from the principal-agent problem. Section 5 turns to equilibrium in contracts–thus analyzing the indirect effect of competition as well. Finally, Section 6 contains some concluding remarks.

2   The Model

2.1   Statement of the Problem

There are two players, a risk-neutral principal and a risk-averse agent[3]. The principal hires the agent to perform an action. She does not observe the action the agent chooses.

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[3]The analysis can be generalized to the case where the principal is risk-averse.

Rather she observes profits, which are a noisy signal of the action.

Let $\phi \in \mathbb{R}$ be a measure of product market competition which affects the profits which accrue to the principal. A higher value of ϕ means that, all else equal, profits are lower. Suppose that there are a finite number of possible gross profit levels for the firm. Denote these $q_1(\phi) < ... < q_n(\phi)$. These are profits before any payments to the agent. We assume that the principal is concerned only with net profit–i.e. gross profit less payments to the agent.

Definition 1. *A set X is a **product set** if $\exists$ sets $X_1, ..., X_n$ such that $X = X_1 \times ... \times X_n$. X is a **product set in** $\mathbb{R}^n$ if $X_i \subseteq \mathbb{R}, i = 1, ..., n$.*

The set of actions available to the agent, A, is assumed to be a product set in $\mathbb{R}^n$ which is closed, bounded and non-empty[4]. Let S be the standard probability simplex, i.e. $S = \{y \in \mathbb{R}^n | y \geq 0, \sum_{i=1}^{n} y_i = 1\}$ and assume that there is a twice continuously differentiable function $\pi : A \to S$. The probabilities of outcomes $q_1(\phi), ..., q_n(\phi)$ are therefore $\pi_1(a), ..., \pi_n(a)$.

Let the agent's von Neumann-Morgenstern utility function be of the following form:

$$U(a, I) = G(a) + K(a)V(I)$$

where I is a payment from the principal to the agent, and $a \in A$ is the action taken by the agent.

Assumption A1. *V is a continuous, strictly increasing, real-valued, concave function on an open ray of the real line $\mathcal{I} = (\underline{I}, \infty)$. Let $\lim_{I \to \underline{I}} V(I) = -\infty$ and assume that G and K are continuous, real-valued functions, and that K is strictly positive. Finally assume that for all $a_1, a_2 \in A$ and $I, \widehat{I} \in \mathcal{I}$ the following holds*

$$G(a_1) + K(a_1)V(I) \geq G(a_2) + K(a_2)V(I)$$
$$\Rightarrow G(a_1) + K(a_1)V(\widehat{I}) \geq G(a_2) + K(a_2)V(\widehat{I})$$

As Grossman and Hart (1983) note, this assumption implies the agent's preferences

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[4]Thus, by the Heine-Borel Theorem, it is compact. This fact is important for existence of an optimal second-best action.

over income lotteries are independent of actions[5], and that the agent's ranking over (only) perfectly certain actions is independent of income. This assumption is clearly not innocuous. It is worth noting, however, that if the agent's utility function is additively or multiplicatively separable in action and reward then the assumption is satisfied.[6]

Let the agent's reservation utility be $\overline{U}$ and let

$$\mathcal{U} = V(\mathcal{I}) = \{v | v = V(I) \text{ for some } I \in \mathcal{I}\}.$$

Assumption A2. $\left(\overline{U} - G(a)\right)/K(a) \in \mathcal{U}, \forall a \in A.$

A2 says that for every action a, there exists a payment such that the agents' reservation utility is achieved.

A third assumption ensures that $\pi_i(a)$ is bounded away from zero and hence rules out the scheme proposed by Mirrlees (1975) through which the principal can approximate the first-best by imposing ever larger penalties for actions which occur with ever smaller probabilities if the desired action is not taken[7].

Assumption A3. $\pi_i(a) > 0, \forall a \in A$ *and* $i = 1, ..., n.$

The principal is assumed throughout to know the agent's utility function $U(a, I)$, the action set A, and the function π.

Definition 2. *An **incentive scheme** is an n-dimensional vector* $\mathbf{I} = (I_1, ..., I_n) \in \mathcal{I}^n.$

Given an incentive scheme the agent chooses $a \in A$ to maximize her expected utility $\sum_{i=1}^n \pi_i(a) U(a, I_i).$

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[5]A result of Keeney (1973) implies the converse - that if the agent's preferences over income lotteries are independent of action the utility function has the form $G(a) + K(a)V(I)$.

[6]In fact, in either of the separable cases, preferences over income lotteries are independent of actions *and* preferences over action lotteries are independent of income. This is stronger than A1.

[7]This ensures that $\pi_i(a)$ is bounded away from zero because of the compactness of A, in conjunction with the fact that there are a finite number of states.

2.2 Solution

The problem which the principal faces is to choose an action and a payment schedule to maximize expected output net of payments, subject to that action being optimal for the agent (Incentive Compatibility), subject to the agent receiving her reservation utility in expectation (Individual Rationality) and subject to the agent not receiving a negative payment in any state of the world (Limited Liability). Let $\hat{I}$ be the realized payment to the agent. Now, denoting F to be the set of pairs of incentive schemes I^* and actions a^* such that, under I^* the agent will be willing to chose a^* the problem is:

$$\max_{(a,I)\in F} \left\{ \sum_{i=1}^{n} \gtrsim_i(a) (q_i - I_i) \right\} \tag{1}$$

subject to

$$a^* \in \arg\max_{a} \left\{ \sum_{i=1}^{n} \gtrsim_i(a) U(a, I_i) \right\} \tag{IC}$$

$$\sum_{i=1}^{n} \gtrsim_i(a^*) U(a^*, I_i) \geq \overline{U} \tag{IR}$$

$$\hat{I} \geq 0, \forall i. \tag{LL}$$

The model is solved in two stages. In stage 1 the principal determines the lowest cost way to implement a given action. In stage 2 she chooses the action which maximizes the difference between the expected benefits and costs.

Definition 3. *A vector* $\mathbf{I} = (I_1, ..., I_n)$ *which satisfies the constraints in (1) is said to **implement** action* a^*.

Definition 4. *Let:*

$$C(a^*, \phi) = \inf \left\{ \sum_{i=1}^{n} \pi_i(a^*) I_i \text{ such that } \mathbf{I} = (I_1, ..., I_n) \text{ implements } a^* \right\}$$

which implements a^* *if the constraint set in (1) is non-empty. If the constraint set is empty then let* $C(a^* \phi) = \infty$.

Stage 2 involves

$$\max_{a \in A} \{B(a, \phi) - C(a, \phi)\} \tag{2}$$

where $B(a, \phi) = \sum_{i=1}^{n} \succsim_i(a) q_i(\phi)$. **?** point out that this may well be a non-convex problem, for $C(a, \phi)$ will not generally be a convex function. Let the solution to this problem be a^{**}, which hence may be set valued. The following result applies to the setting where the limited liability constraint does not bind (or is not part of the program).

Proposition 1. *Suppose that LL does not bind, then the following condition is necessary and sufficient for a^{**} to be non-decreasing in ϕ*

$$\sum_{i=1}^{n} q_i'(\phi) \pi_i'(a) \geq 0, \forall a, \phi. \tag{3}$$

Proof. Since LL does not bind $C(a^*, \phi)$ is independent of ϕ. The conclusion is then a straightforward consequence of the Monotonicity Theorem of Milgrom and Shannon (1994). For a^{**} to be non-decreasing in ϕ in the Strong Set Order[8] requires that a^{**} have increasing differences in (a, θ). When q and π are differentiable this is simply (3). □

Since nothing hinges on differentiability, we maintain that assumption throughout.

Note that when LL does not bind the condition for a harder action to be induced is independent of the cost of implementation. When LL does bind matters are more complicated, as the following result highlights.

Proposition 2. *Suppose that LL binds, then the following condition is necessary and sufficient for a^{**} to be non-decreasing in ϕ*

$$\sum_{i=1}^{n} q_i'(\phi) \pi_i'(a) - \frac{d^2 C(a, \phi)}{da d\phi} \geq 0, \forall a, \phi. \tag{4}$$

Proof. Since LL binds $d^2 C(a, \phi)/da d\phi$ does not equal zero in general, and result follows from the proof of Proposition 1. □

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[8]The Strong Set Order is defined as follows.

Let $A, B \subset \mathbb{R}^n$. Then A is **higher** than B **in the Strong Set Order** $(A \geq_s B)$ iff for any $\mathbf{a} \in A$ and $\mathbf{b} \in B$, $\max\{\mathbf{a}, \mathbf{b}\} \in A$ and $\min\{\mathbf{a}, \mathbf{b}\} \in B$.

In $\mathbb{R}^1$ the following is analogous. A set $S \subseteq \mathbb{R}$ is said to be as "High" as another set $T \subseteq \mathbb{R}$ $(S \geq_S T)$, if and only if (i) each $x \in S \backslash T$ is greater than each $y \in T$, and (ii) each $x' \in T \backslash S$ is less than each $y' \in S$.

In general the term $d^2C(a, \phi)/dad\phi$ is a complicated object. We will see in section 3.2, however, that in various models in the literature simplifying assumptions are made (e.g. risk-neutrality of the agent or two states of the world) which lead to a simple form for the term.

Definition 5. *Let $C_{FB} : A \to \mathbb{R}$ be the first-best cost of implementing action a, i.e. $C_{FB} = V^{-1}\left(\left[\overline{U} - G(a)\right]/K(a)\right)$.*

Remark 1. *C_{FB} induces a complete ordering over A. When we say that an action a is "harder" than another a' we mean $C_{FB}(a) > C_{FB}(a')$.*

The *(Strict) Monotone Likelihood Ratio Property* ("MLRP") MLRP holds if, given $a, a' \in A$, $C_{FB}(a') < C_{FB}(a) \Rightarrow \pi_i(a')/\pi_i(a)$ is decreasing in i.

When MLRP holds (3) can be written as:

$$\sum_{i=j+1}^{n} \pi_i'(a) q_i'(\phi) \geq \sum_{i=1}^{j} |\pi_i'(a)| \, q_i'(\phi). \tag{5}$$

We will assume throughout that MLRP holds. Recall also that MLRP implies First-Order Stochastic Dominance. We will sometimes make use of FOSD below.

3   Product Market Competition

The purpose of this section is to show that models in this literature can be better understood within the framework of this paper. Much like all asset pricing models are essentially models of the stochastic discount factor, models of competition and agency are, at their core, models of the relative impact which competition has on the different cost states. Models which find that more competition leads to a harder action do so because the assumptions mean that the states of nature which harder actions make more likely are more attractive. Those which find the converse do so because the assumptions make such states relatively less attractive.

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3.1 When Does Competition Increase Effort?

We now turn attention to what characteristics high profit states of nature have. To that end write profit in state i as:

$$q_i = p_i(x_i)x_i - \psi_i(x_i), \tag{6}$$

where x is quantity and ψ is the cost function. There are a number of ways to think about what it means to be to in a high profit state. One is that costs could be low. If "hard" actions by the agent make low cost states more likely then this is a natural interpretation. A second is that prices might be higher in high profit states. This might be the case if "hard" actions affect demand, or if they aid collusion among firms.

First, suppose that agent effort lowers costs and that product market competition affects revenues, with more competition lowering revenues. Hence, equation (6) becomes:

$$q_i = p_i(x_i, \phi)x_i - \psi_i(x_i),$$

with $\psi_1 > \psi_2 > ... > \psi_n$.

A "high profit" state is one in which, all else constant, there are low costs–and *vice versa*. Consider the version of the model without a limited liability constraint. From (3) competition increases agent effort iff:

$$\sum_{i=1}^n \pi_i'(a)x_i\frac{\partial p(x_i, \phi)}{\partial \phi} > 0 \tag{7}$$

It is easy to see that this condition need not hold in general. Consider, for example, the case where there are just two possible outcomes. Noting that $\pi_H'(a) = -\pi_L'(a)$, the above condition then becomes:

$$\pi_H'(a)\left(x_H\frac{\partial p(x_H, \phi)}{\partial \phi} - x_L\frac{\partial p(x_L, \phi)}{\partial \phi}\right) > 0.$$

By FOSD $\pi_H'(a) > 0$ and hence we require:

$$x_H\frac{\partial p(x_H, \phi)}{\partial \phi} > x_L\frac{\partial p(x_L, \phi)}{\partial \phi}.$$

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Under reasonable assumptions about product market competition the quantity produced in the low cost state will be higher than in the high cost state, so that $x_L > x_H$. But, without further assumptions, it is unclear that $\partial p(x_H, \phi)/\partial\phi > \partial p(x_L, \phi)/\partial\phi$.

More generally, write $q_i(a, \phi)$. The required condition is now:

$$\frac{\partial^2 B}{\partial a \partial \phi} = \sum_{i=1}^{n} \left[\pi_i'(a) \frac{\partial q_i(a, \phi)}{\partial \phi} + \pi_i(a) \frac{\partial^2 q_i(a, \phi)}{\partial \phi \partial a} \right] > 0.$$

Again consider the two outcome case and this becomes:

$$\pi_L'(a) \left(\frac{\partial q_L(a, \phi)}{\partial \phi} - \frac{\partial q_H(a, \phi)}{\partial \phi} \right) + \pi_L(a) \frac{\partial^2 q_L(a, \phi)}{\partial \phi \partial a} + \pi_H(a) \frac{\partial^2 q_H(a, \phi)}{\partial \phi \partial a} > 0. \qquad (8)$$

Clearly, substantial structure must be placed on a model in order to ensure that this condition is always met.

This illustrates the fact that general claims about the impact of product market on agent effort have little hope of success. Whether product market competition increases or decreases agent effort will generally depend on the specifics of product market interactions. Models which make specific assumptions about such interactions clearly make them more tractable–but do so at the expense of generality. We now turn to a number of such models.

3.2 Other Models of Product Market Competition

Although it has long been conjectured that product market competition may be a mitigant of agency costs, the first formal models of the issue did not emerge until the early 1980s. The early models considered how competition might provide a change to the information content of the Principal-Agent Problem (Nalebuff and Stiglitz (1983); Hart (1983); Scharfstein (1988b); Scharfstein (1988a)). In these models, the agent's choice of effort probabilistically impacts cost, and therefore profit for the firm. Within such a model, any shock or event which provides some information about the distribution which maps agent effort to the profit function is valuable, since it allows the principal to infer some information about the choice of effort, and therefore introduce a payment

scheme which is partially contingent on this. Competition does this, in such models, because it helps reveal the common shock faced by all firms. In Hart (1983) and Scharfstein (1988b), this takes place though the price mechanism.

Martin (1993) finds that in a Cournot model as the number of firms increases agency costs actually increase, since the principal has less incentive to provide a contract which induces the agent to reduce marginal cost[9]. Horn et al. (1994) compare the case of monopoly with three duopoly models: Bertrand, Cournot and an output cartel (in decreasing order of product market competitiveness). They find, however, that the Bertrand case has lower agency costs compared to the monopoly case but that the Cournot and output cartel cases exhibit the opposite trend. Schmidt (1997) introduces the possibility of bankruptcy, and finds that competition can increase or decrease effort. Finally, Raith (2003) presents a model with linear contracts where there is (importantly) free entry and exit in the product market and firms compete in product variety. He obtains a result where competition decreases managerial slack because a fall in profits induces exit by some firms, leading to a business creation effect and hence there is more marginal cost to save and there are increasing returns to cost reduction.

As we mentioned in the introduction, there is an important distinction between models which consider only the direct effects of product market competition–in the sense that they take the other firms' actions as fixed and consider only the incentive for a given firm to induce a harder action–and those models which consider the *equilibrium* incentive to induce a harder action. That is, they require a given firm to induce a harder action, and given that strategy, it be optimal for all other firms to also induce a harder action. For instance, the basic bankruptcy model in Schmidt (1997) does not consider equilibrium effects, but his n-firm Bertrand extension does. We return to this issue in Section 5.

The following four subsections present four models from the literature and demonstrate why our basic condition is satisfied–or not–in those models.

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[9]Bertoletti and Poletti (1996) argue that, due to special assumptions, asymmetric information plays no role in Martin's model and hence it cannot be applied to determining the impact of competition on agency costs.

3.2.1 Schmidt's Basic Model

Schmidt (1997) presents a model in which the firm goes bankrupt if realized profits are below a certain level. He first develops a basic model in which there is a reduced form measure of product market competition, ϕ. An increase in ϕ corresponds to a more competitive product market. He extends this model by imposing Bertrand competition on the product market and varying the number of competitors.

In Schmidt's model, the agent is risk-neutral and so to avoid the first-best being implementable he makes assumptions on primitives that rule this out. Thus the LL constraint must bind. He also assumes that effort by the agent affects costs. There are two possible states: high cost and low. Denote these L and H respectively. There is a loss to the agent of $\overline{L}$ if the firms goes bankrupt (e.g. a reputation cost), which occurs with positive probability in the high cost state and with zero probability in the low cost state. He assumes that the probability of this occurring is $l(\phi)$ with $l'(\phi) > 0$.

Schmidt's main result (his Proposition 3) states that the increase in agent effort is unambiguous if IR binds and is ambiguous otherwise. To see this in the context of our model note that if IR binds then

$$\pi_L(a) I_L + \pi_H(a) I_H = \overline{U}.$$

Since LL binds it must be that $I_H = 0$, and hence $\pi_L(a) I_L = \overline{U}$.

$$
\begin{aligned}
C(a^*, \phi) &= \pi_L(a^*) I_L + \pi_H(a^*) I_H \\
&= \pi_L(a^*) I_L \\
&= \overline{U}.
\end{aligned}
$$

Hence $d^2 C(a^*, \phi)/dad\phi = 0$.

Given MLRP, (4) becomes:

$$\pi'_L(a) \left[q'_L(\phi) - q'_H(\phi) \right] > 0 \tag{9}$$

By FOSD $\pi'_L(a) > 0$ (a harder action makes the low cost state more likely). Schmidt's

result requires $q'_H(\phi) < q'_L(\phi)$. Since the agent suffer a loss of $\overline{L}$ with positive probability in the high cost state and with zero probability in the low cost state the principal's profits are lower in the high cost state since it affects the agent's utility and hence the payment that the Principal must make if the participation constraint binds. In effect, then $q_H(\phi) \equiv \overline{q}_H(\phi) - l(\phi)\overline{L}$. Clearly $q'_L(\phi) > q'_H(\phi)$, since the expected loss of $l(\phi)\overline{L}$ occurs only in state H.

If the PC is slack at the optimum then the effect of competition is ambiguous because the loss of L is only equivalent to profits being lower if L is sufficiently large. Thus, for $\overline{L}$ sufficiently small we have $q'_L(\phi) = q'_H(\phi)$ and hence the condition is not satisfied.

3.2.2 Schmidt's Price-Cap Model

An alternative model which Schmidt considers is price-cap regulation of a monopolist. In fact, bankruptcy plays no role in this model. He allows the firm to have constant marginal cost of either c^L or $c^H > c^L$. The regulator does not observe costs, but sets a price cap of $1/\phi$. Schmidt interprets a larger value of ϕ as a more competitive product market. Denoting demand at the cap (which is assumed to be binding regardless of the cost realization) as $D(1/\phi)$, profits are:

$$q(c^j, \phi) = D\left(\frac{1}{\phi}\right)\left(\frac{1}{\phi} - c^j\right)$$

Differentiating with respect to ϕ yields:

$$\frac{\partial q(c^j, \phi)}{\partial \phi} = -\frac{1}{\phi^2}\left[D\left(\frac{1}{\phi}\right) + D'\left(\frac{1}{\phi}\right)\left(\frac{1}{\phi} - c^j\right)\right]$$

Our condition for a harder action in this two outcome model is the same as in the previous subsection, i.e.

$$\pi'_L(a)\left[q'_L(\phi) - q'_H(\phi)\right] \geq 0$$

Since $\pi'_L(a)$ is positive, we require $q'_L(\phi) - q'_H(\phi) \geq 0$ - i.e. $q'_L(\phi) \geq q'_H(\phi)$. This

requires:

$$-\frac{1}{\phi^2}\left[D\left(\frac{1}{\phi}\right) + D'\left(\frac{1}{\phi}\right)\left(\frac{1}{\phi} - c^L\right)\right] \geq$$
$$-\frac{1}{\phi^2}\left[D\left(\frac{1}{\phi}\right) + D'\left(\frac{1}{\phi}\right)\left(\frac{1}{\phi} - c^H\right)\right]$$

which reduces to requiring:

$$\frac{(c^L - c^H)D'\left(\frac{1}{\phi}\right)}{\phi^2} \geq 0$$

Obviously $D'\left(\frac{1}{\phi}\right) < 0$, and, by construction, $c^H > c^L$.

Therefore a tighter price cap leads to a harder action by the agent. The intuition is that the price cap reduces profits less in the low cost state than in the high cost state. This makes the low cost state relatively more attractive to the principal - and they induce the agent to put more probability weight on that state by providing more incentives. This induces a harder action.

3.2.3 Schmidt's Bertrand Model

Another model which Schmidt provides, under the bankruptcy umbrella, is Bertrand competition with an increasing number of competitors. He finds that agent effort is higher for a two-firm oligopoly than a monopoly, but declines as the number of firms increases beyond two. In considering the difference between monopoly and duopoly he assumes: that there are two cost realizations, c^H and c^L; the difference between c^H and c^L is "drastic" in the sense that the monopoly price given marginal cost c^L is lower than if the firm faced marginal cost c^H; if one firm achieves c^L then it becomes a monopolist and the other firm is liquidated - if both firms have the same costs then they compete in prices and profits are zero.

Now, fix firm 2's strategy at a_2^* (which induces probability $p_L^2(a_2^*)$ of being low cost) and consider whether firm 1 wants to induce a harder action from their agent. For this

to be the case we require, as before:

$$\pi'_{1L}(a) \left[q'_{1L}(\phi) - q'_{1H}(\phi)\right] \geq 0 \qquad (10)$$

Since an increase in competition means the existence of a duopoly, we can denote $q'_{1L}(\phi)$ as $q_{1L}^M - q_{1L}^D \equiv \Delta q_{1L}$, and similarly for Δq_{1H}, and hence write (10) as:

$$\pi'_{1L}(a) \left[\Delta q_{1L} - \Delta q_{1H}\right] \geq 0 \qquad (11)$$

Since $\pi'_{1L}(a) > 0$, this requires $\Delta q_{1L} \geq \Delta q_{1H}$. Recall that, by assumption, if costs are c^H then if firm 2's costs are c^L, firm 1 goes bankrupt and earns negative profits (because of the liquidation cost l). But if costs are c^L, then firm 1 either earns the monopoly profit (if firm 2's costs are c^H), or zero profit (if firm 2's costs are c^L). Hence:

$$
\begin{aligned}
\Delta q_{1L} &= q_L^M - \left(p_L^2\left(a_2^*\right)0 + (1 - p_L^2\left(a_2^*\right))q_L^M\right) \\
&= p_L^2\left(a_2^*\right)q_L^M
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta q_{1H} &= q_H^M - \left(-L \cdot p_L^2\left(a_2^*\right) + (1 - p_L^2\left(a_2^*\right)) \cdot 0\right) \\
&= q_H^M + L \cdot p_L^2\left(a_2^*\right)
\end{aligned}
$$

Therefore the necessary and sufficient condition for firm 1 to induce a harder action is:

$$\Delta q_{1L} \geq \Delta q_{1H} \Leftrightarrow p_L^2\left(a_2^*\right)q_L^M \geq q_H^M + L \cdot p_L^2\left(a_2^*\right)$$

Now fix firm 1's strategy at a_1^* and perform the same exercise. The condition for firm 2 to induce a harder actions is $p_L^1\left(a_1^*\right)q_L^M \geq q_H^M + L \cdot p_L^1\left(a_1^*\right)$. Therefore, it is a Nash equilibrium for both firms to induce harder actions from their agents if and only if the following two conditions hold:

$$
\begin{aligned}
q_H^M &\geq p_L^2\left(a_2^*\right)\left[L - q_L^M\right] \\
q_H^M &\geq p_L^1\left(a_1^*\right)\left[L - q_L^M\right]
\end{aligned}
$$

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Hence, if q_H^M is sufficiently large, or if harder actions lead to a large increase in the probability of the low cost state (i.e. $p_L'^2(a_2^*)$ and $p_L'^1(a_1^*)$ are large which implies $p_L^2(a_2^*)$ and $p_L^1(a_1^*)$ small by concavity which Schmidt assumes), then both firms inducing a harder action is a Nash equilibrium. This is just as Schmidt finds.

Now consider $m > 2$ firms. Schmidt assumes that $q_L^M > L$, that all firms choose interior values of $p_L^y(a_y^*)$, and considers symmetric equilibria. If a firm is the only one which succeeds in being low cost then it becomes the monopolist. A high cost firm gets liquidated provided that there is at least one low cost firm. Note that the probability that no firm other than y succeeds in being low cost is:

$$\prod_{y \neq z}(1 - p_L^y(a_y^*))$$

Fixing all other firm's strategies we have:

$$
\begin{aligned}
q_{1L}(\phi) &= \prod_{y \neq z}(1 - p_L^y(a_y^*))q_L^M \\
&= q_L^M(1 - p_L(a^*)^{m-1})
\end{aligned}
$$

and

$$
\begin{aligned}
q_{1H}(\phi) &= -L\left(1 - \prod_{y \neq z}(1 - p_L^y(a_y^*))\right) \\
&= -L\left(1 - (1 - p_L(a^*))^{m-1}\right) \\
&= L(1 - p_L(a^*))^{m-1} - L
\end{aligned}
$$

where the second line in each uses the symmetric equilibrium assumption, denoting $p_L^y(a_y^*) = p_L(a^*), \forall y$. Therefore the derivatives with respect to an increase in the number of firms are:

$$q'_{1L}(\phi) = -q_L^M(m-1)p_L(a^*)^{m-2}$$

and

$$q'_{1H}(\phi) = L(m-1)(1 - p_L(a^*))^{m-2}$$

Hence, our condition for firm 1 to induce a harder action is:

$$-q_L^M(m-1)p_L(a^*))^{m-2} \geq L(m-1)(1-p_L(a^*))^{m-2}$$

Upon rearrangement this becomes:

$$-\frac{q_L^M}{L} \geq \frac{(1-p_L(a^*))^{m-2}}{p_L(a^*)^{m-2}}$$

Since the RHS is strictly positive, this can never be the case. Again, this is just as Schmidt finds. An increased number of competitors reduces the probability that a given firm becomes the monopolist. This effect always outweighs the probability of liquidation increasing in this model. Consequently, the principal induces a weaker and weaker action as the number of firms increases beyond 2.

3.2.4 Cournot Competition

Consider an m-firm model of Cournot competition, where firms need not by symmetric. Let $X = \sum_{f=1}^m x^f$ be aggregate industry output. To simplify the analysis suppose that $P(X) = a - bX$ and assume constant marginal cost of c^f. Hence, profit for each firm is $q^f = P(X)x^f - c^f x^f$. The first-order condition for each firm is therefore:

$$P(X) - c^f - bx^f = 0 \tag{12}$$

Summing yields:

$$mP(X) - bX = \sum_{j=1}^m x_j.$$

Denoting the average marginal cost of the m firms as $\bar{c}$ and solving yields the following total output and equilibrium price:

$$X^* = \frac{(a-\bar{c})m}{b(m+1)} \tag{13}$$

$$P^* = \frac{(a+\bar{c})m}{m+1}. \tag{14}$$

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The profit of each firm is therefore:

$$q^* = \frac{(a - \bar{c})(a + \bar{c})m}{b(m+1)^2} - c$$

Recall that a necessary and sufficient condition for product market competition to increase agent effort is:

$$\sum_{i=j+1}^{n} \pi_i'(a)q_i'(\phi) > \sum_{i=1}^{j} |\pi_i'(a)| \, q_i'(\phi)$$

Now suppose, as in Hermalin (1992) and Martin (1993), that agent effort reduces marginal cost, c^f, and that costs are subject to a random shock. Consequently, high profit states of nature are those in which c^f is low and low profit states are those in which c^f is high. Effort makes the low cost, high profit states more likely. Increased competition corresponds to an increased number of firms. The timing of the game is as follows. Agents take an action and costs are realized. These costs are observed by all firms, who then set prices. Hence, firms who get a good shock and/or have an agent which takes a hard action, will have lower costs.

Now note that the derivative of profit for a given firm, f, with respect to competition is:

$$\frac{dq^*}{dm} = -\frac{(a - \bar{c})(a + \bar{c})(m-1)}{b(m+1)^3}$$

Therefore, for competition to increase agent effort we require:

$$\sum_{i=j+1}^{n} \pi_i'(a) \left[\frac{(a - \bar{c}_i)(a + \bar{c}_i)(m-1)}{b(m+1)^3} \right] \tag{15}$$

$$> \sum_{i=1}^{j} |\pi_i'(a)| \left[\frac{(a - \bar{c}_i)(a + \bar{c}_i)(m-1)}{b(m+1)^3} \right]$$

By construction, $\bar{c}_z > \bar{c}_y$ for all $z < y$. Hence $(a - \bar{c}_z)(a + \bar{c}_z)(m-1)/b(m+1)^3 > (a - \bar{c}_y)(a + \bar{c}_y)(m-1)/b(m+1)^3$ for all $z < y$. Since $\sum_{i=j+1}^{n} \pi_i'(a) = \sum_{i=1}^{j} |\pi_i'(a)|$ it follows that the inequality (15) can never be satisfied.

This is consistent with the statement of Hermalin (1992): "in the classic model, an increase in competition cannot lead the executive to choose a harder action." Martin

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(1993) generates a similar result to Hermalin in a Cournot model. In his model, as the number of firms increases marginal revenue falls and therefore there is less marginal cost to save. Since agent effort affects marginal cost there is less incentive to exert effort.

Martin considers the product market equilibrium, and shows that equilibrium effort decreases for all firms, as the number of firms increases. We will have more to say about this in Section 5.

4   Competition and Agency Costs

To this point we have considered situations in which an increase in product market competition leads to a harder action by the agent. The welfare loss from the principal-agent problem, however, is related to the difference between the first-best and second-best actions. Simply because an increase in competition may cause the agent to take a harder action does not mean that there has been a reduction in agency costs. In principle, it could be the case that the first-best action has increased by a larger amount.

In this section, we address this issue. The following definition, due to Grossman and Hart (1983), makes precise what we mean by the amount of agency costs.

Definition 6. *Let $L = \max_{a \in A} \{B(a, \phi) - C_{FB}(a)\} - \sup_{a \in A} \{B(a, \phi) - C(a)\}$ be the difference between the principal's expected profit in the first-best and second-best.*

Let the value function be given by:

$$Q = B(a^{FB}(\phi), \phi) - C_{FB}(a^{FB}) - \left(B(a^{SB}(\phi), \phi) - C(a^{SB})\right), \qquad (16)$$

where a^{FB} and a^{SB} are the first-best and second-best optimal actions respectively.

The question is how Q changes as product market competition increase. Differentiating (16) with respect to ϕ and noting that the first-order conditions are $\partial B/\partial a -$

$\partial C_{FB}/\partial a = 0$ and $\partial B/\partial a - \partial C_{SB}/\partial a = 0$ yields:

$$\frac{\partial Q}{\partial \phi} = \frac{\partial B(a^{FB}(\phi), \phi)}{\partial \phi} - \frac{\partial B(a^{SB}(\phi), \phi)}{\partial \phi}$$

$$= \sum_{i=1}^{n} \pi_i(a^{FB}) q_i'(\phi) - \sum_{i=1}^{n} \pi_i(a^{SB}) q_i'(\phi)$$

where the last line utilizes the fact that $B = \sum_{i=1}^{n} \pi_i(a) q_i(\phi)$.

In general it is not possible to say whether $a^{FB} \geq_S a^{SB}$. But, to bias the case in favor of competition reducing the welfare loss, we will assume that $a^{FB} \geq_S a^{SB}$. The following proposition provides conditions under which an increase in product market competition increases, decreases, or leaves unchanged the welfare loss from the principal-agent problem.

Notation 1. *Let p_i be the set of states in which $\pi_i(a) > \pi_i(a')$ for $a > a'$ and p_d be the set of states in which $\pi_i(a) \leq \pi_i(a')$ for $a > a'$.*

Proposition 3. *Assume A1-A3 and that $a^{FB} \geq_S a^{SB}$. Then if $q_i'(\phi)$ is constant in i then $\partial Q/\partial \phi = 0$; if $\sum_i \left(\pi_i^{FB} - \pi_i^{SB} \right) q_i'(\phi) > 0$ then $\partial Q/\partial \phi > 0$; and if $\sum_i \left(\pi_i^{FB} - \pi_i^{SB} \right) q_i'(\phi) < 0$ then $\partial Q/\partial \phi < 0$.*

Proof. Since $a^{FB} \geq_S a^{SB}$ it follows that that π^{FB} first-order stochastically dominates π^{SB}. Note that if $q_i'(\phi)$ is constant in i then $\sum_{i=1}^{n} \pi_i(a^{FB}) q_i'(\phi) - \sum_{i=1}^{n} \pi_i(a^{SB}) q_i'(\phi)$ since the probabilities sum to one. If $\sum_i \left(\pi_i^{FB} - \pi_i^{SB} \right) q_i'(\phi) > 0$ then FOSD means more probability weight is place on states in p_i than p_d in the first-best than in the second-best. Hence $\sum_{i=1}^{n} \pi_i(a^{FB}) q_i'(\phi) - \sum_{i=1}^{n} \pi_i(a^{SB}) q_i'(\phi) > 0$. If $\sum_i \left(\pi_i^{FB} - \pi_i^{SB} \right) q_i'(\phi) < 0$ then less weight is put on states in p_i in the first-best and hence $\sum_{i=1}^{n} \pi_i(a^{FB}) q_i'(\phi) - \sum_{i=1}^{n} \pi_i(a^{SB}) q_i'(\phi) < 0$. $\qquad \square$

The following example illustrates this result.

Example 1. *Suppose there are two possible outcomes so that $n = 2$. Let $\pi_1(a^{FB}) = 0.2, \pi_2(a^{FB}) = 0.8, \pi_1(a^{SB}) = 0.4, \pi_2(a^{SB}) = 0.6, q_1' = -2$ and $q_2' = -1$. Note that π^{FB} FOSDs π^{SB}, reflect-*

ing the fact that the first-best action is harder than the second-best action. It then follows that:

$$\frac{\partial V}{\partial \phi} = \sum_{i=1}^{n} \pi_i(a^{FB})q_i'(\phi) - \sum_{i=1}^{n} \pi_i(a^{SB})q_i'(\phi)$$

$$= 0.2 \times (-2) + 0.8 \times (-1) - (0.4 \times -2 + 0.6 \times -1)$$

$$= 0.2$$

Thus an increase in product market competition actually increases the welfare loss L. Furthermore, recall that in the two outcome case, a necessary and sufficient condition for increased product market competition to lead to a harder action is:

$$\pi_1'(a) [q_1'(\phi) - q_2'(\phi)] \geq 0$$

Since a harder action makes the low profit state (state 1) less likely, this requires $q_1'(\phi) < q_2'(\phi)$. Indeed, in this example $q_1' = -2 < q_2' = -1$. Therefore, in this example, increased product market competition causes a harder action but leads to an increase in the welfare loss from the principal-agent problem.

Proposition 3 shows that there are circumstances in which competition can lead to a harder action, but still lead to an increase in agency costs. This is because the second-best action is less sensitive to ϕ than the first-best action. Therefore an increase in competition can cause the first-best action to increase by more than the second-best action and imply an increase in agency costs. On the other hand, where competition leads to an easier action by the agent, it can actually *reduce* agency costs because the first-best action can fall by a larger amount.

5 Equilibrium Effort Effects

Much of the received literature on product market competition consider only the direct effect of product market competition. That is, it ignores the possibility that increased product market competition causes other firms to induce their agents to take harder actions, and that this could have an effect on the optimal second-best action for a given

firm to implement. A complete understanding of this issue clearly depends on the particular structure of the product market game, and the equilibrium of it. In this section, however, we are able to reach some general conclusions. This is useful in two ways. First, calculating the equilibrium of a particular game can be very involved, and thus it is useful to be able to draw some conclusions without having to do so. Secondly–and related to this–attention is often restricted to symmetric equilibria for tractability. Here we can do better–we can make comparative statics statements for the greatest and least equilibria, be they symmetric or not.

Definition 7. *A noncooperative game* $(N, \mathbf{S}, \{\mathbf{f}_i \colon \mathbf{i} \in \mathbf{N}\})$, *is a Supermodular Game if the set* S *of feasible joint strategies is a sublattice of* $\mathbb{R}^m$, *the payoff function* $f_i(\mathbf{y}_i, \mathbf{x}_{-i})$ *is supermodular in* y_i *on* S_i *for each* x_{-i} *in* S_{-i} *and each player* i, *and* $f_i(\mathbf{y}_i, \mathbf{x}_{-i})$ *has increasing differences in* $(\mathbf{y}_i, \mathbf{x}_{-i})$ *on* $S_i \times S_{-i}$ *for each* i.

Theorem 4.2.3 of Topkis (1998) provides conditions under which the strategy of *each* player in the greatest equilibrium point, and the least equilibrium point, is increasing in the number of players in the game.

Furthermore, Topkis's also provides conditions under which the strategy of *each* player in the greatest equilibrium point, and the least equilibrium point, is increasing in a parameter, t. Although these two theorems apply to a finite number of players, analogous results have been proved for infinitely many players–and also for quasi-supermodular games (see Milgrom and Shannon (1994)).

We can now use these result to provide conditions under which the principal of *every* firm in the market induces a harder action from her agent in the greatest and least equilibrium of the game. To that end, we interpret a player as being a principal, and a strategy for her as being a feasible second-best action (correspondence), $a^{**} = \sup_{a \in A} \{B(a, \phi) - C(a)\}$, and a product market strategy $\mathbf{z}_i \in Z_i$, where Z_i is the set of product market strategies for player i. If this game is a supermodular game then the above Theorems imply that the actions implemented by all principals are increasing in the relevant measure of product market competition. We will refer to this game as the "Product Market with Agency Game". It is natural to ask under what conditions this game is a supermodular game. The first thing which is required is that the set of feasible joint strategies be compact. If the sets of product market strategies Z_i are non-empty

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and compact for all i then it follows trivially from Tychonoff's Theorem that the set S of feasible joint strategies in the Product Market with Agency Game is compact. For example, if a product market strategy is a price, quantity or supply function then S will be compact.

A second requirement is that the payoff function be supermodular in $\mathbf{y}_i \in S_i$. The key part of this requirement is that the agent's action and the product market strategy be complements. For instance, in a Cournot game where agent effort reduces cost this condition requires that lower costs make choosing higher quantities more desirable. Whether or not this condition is met clearly depends crucially on the nature of the product market and the effect of the agents' actions. The final important condition is that the payoff exhibit increasing differences in $(\mathbf{y}_i, \mathbf{x}_{-i})$ on $S_i \times S_{-i}$ for all i. Again, whether this is satisfied will depend on the particulars of the game. Using the Cournot example, this requires that a higher effort-quantity pair from one firm makes a higher effort-quantity pair from another firm more desirable.

This illustrates the fact that the above theorems can be used to analyze equilibrium effort effects *without the need to explicitly solve for the equilibrium of the game.* An added advantage is that the results hold for the greatest and least equilibria, not merely symmetric equilibria[10]. Most applied analysis restricts attention to symmetric equilibria for tractability (see for instance Schmidt (1997), Aghion et al. (2000)).

A drawback is that only a necessary condition can be provided, rather than a necessary and sufficient condition as in earlier sections.

6 Discussion and Conclusion

This paper has provided a condition which is necessary and sufficient for the direct effect of product market competition to lead to a harder action from the agent in a general principal-agent model: with incentive compatibility, individual rationality and limited liability constraints. When the limited liability constraint does not bind, or where it does bind but there are two output levels the conditional takes a particularly simple

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[10]Although in this example we have used it only to analyze symmetric equilibria.

form and is independent of the cost to the principal of inducing the optimal action. This condition basically requires that competition reduce profits *less* in states which are made more likely by harder actions from the agent. When this is satisfied, agents exert more effort. Various well-known models which impose particular market structures are special cases.

The above analysis also helps to clarify the intuition for why previous models have produced the results which they have. By laying bare the basic mechanism by which product market competition affects agent effort we suggest that the channels through which previous models have produced the results they do are more clearly understood.

We have also provided a sufficient condition for the indirect effect (the fact that other firms change their incentive schemes) to reinforce the direct effect. Moreover, this condition applies to the greatest and least equilibrium–even asymmetric equilibria.

Beginning with his 1966 paper, Leibenstein argued repeatedly that internal firm efficiency is an important consideration, not simply allocative efficiency. He proposed a number of essentially behavioral postulates as reasons why X-inefficiency may exist. Notwithstanding these reasons, the welfare loss from the principal-agent problem is a clear reason why firms may operate inside the production possibilities frontier. Leibenstein concluded, however, that competition was a powerful force which could eliminate X-inefficiency. He was certainly not alone in this view. According to this argument, competition not only leads to allocative efficiency–as the First Welfare Theorem so elegantly establishes–but it also leads to X-efficiency. This paper suggests that whilst competition is a powerful force for allocative efficiency, it does not necessarily perform a similar role with respect to X-efficiency. Moreover, even when competition leads to a harder action from the agent it may actually *increase* the welfare loss.

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