Welfare in the Eaton-Kortum Model of International Trade

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Abstract

I show that the welfare effects of changes in technologies or trade costs in the workhorse Ricardian model of international trade are identical under a wide range of preferences. Specifically, as long as products can be grouped into a finite number of sets within which they enter the utility function symmetrically, the model's welfare predictions are independent of the form of the utility function and depend only the domestic trade share and trade cost elasticity.

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1 Introduction

The Eaton-Kortum model (Eaton and Kortum, 2002) has become one of the primary workhorse models of the quantitative international trade literature. Among the model's appealing features are that it tractably relates micro-level Ricardian trade forces with observable aggregate trade flows and that it allows welfare implications to be related to changes in thse trade flows. In addition, since Arkolakis et al. (2012) other commonly employed quantitative trade models make identical predictions regarding aggregate trade flows and welfare, its tractability apparently does not come at the cost of ignoring non-Ricardian motivations for trade.

One feature shared by the Eaton-Kortum model and the other models delineated by Arkolakis et al. (2012) are their reliance on constant elasticity of substitution (CES) preferences to maintain a high degree of tractability. However, in this paper, I show that the predictions of the Eaton-Kortum model for bilateral trade flows, income, and welfare are entirely independent of assumptions regarding the demand side of the economy, given that preferences satisfy some very weak conditions. Specifically, I show that assuming that households have common preferences that treat goods symmetrically is sufficient to derive the model's aggregate quantitative predictions. Thus, as in the trade models based on CES demand, the welfare effects of a foreign shock can be inferred from only two variables, the domestic trade share and the trade cost elasticity.

I derive this result in three parts. First, I show that, when all goods enter the utility function symmetrically, the share of country n expenditure which is devoted to products from country i does not depend on the form of the utility function. Second, I show that the level of expenditure required to achieve a given level of utility is proportional to the scale parameter of the price distribution. Because changes in this parameter can be inferred from changes in the domestic trade share, given the trade cost elasticity, the welfare effect of any foreign shock can be inferred from these two variables. Finally, I show that this result can be extended to a more general set of arbitrary preferences over a finite number of sets of products as long as goods within each set enter preferences symmetrically.

2 Model

The world is composed of $n = 1, ..., N$ countries, each of which contains a measure L_n of identical households. Households each supply one unit of labor inelastically and maximize a utility function, $U(\mathbf{q}_n)$, where $\mathbf{q}_n = \{q_n(\omega)\}$ is the consumption of consumers in n over a continuum of goods indexed by $\omega \in [0, 1]$. I assume that $U(\cdot)$ is increasing, thrice continuously differentiable and strictly quasiconcave. I also assume that preferences are symmetric in the sense that

$$\frac{\partial U(\mathbf{q}_n)}{\partial q_n(\omega)} = v(q_n(\omega), \mathbf{q}_n)$$

and that $v(q_n(\omega), \mathbf{q}_n)$ is an invertible function of $q_n(\omega)$. This implies that, for any consumption bundle, the marginal utility of a particular good is independent of the good's identity, ω.

As the Eaton-Kortum, I assume a single factor of production (labor), constant returns to scale in production, perfect competition, and iceberg trade costs of delivering a good to a particular destination. Thus, the price of a unit of a good in n that was produced in i is given by

$$p_{ni}(\omega) = \frac{w_i d_{ni}}{Z_i(\omega)},$$

where w_i is the wage in i, $d_{ni} \geq 1$ is the iceberg trade cost of delivering a good from i to n, and $Z_i(\omega)$ is the productivity with which ω is produced in i. Also as in the Eaton-Kortum model, I assume that $Z_i(\omega)$ is the realization of a random variable drawn from a Fréchet distribution, given by

$$F(z; T_i) = e^{-T_i z^{-\theta}}.$$

3 The Price Distribution and Trade Flows

The form of $U(\cdot)$ implies that identical goods produced in different countries are perfect substitutes.[1] Thus, households will purchase each good only from the lowest-cost source, and the effective price of ω in n is $p_n(\omega) = \min_i\{p_{ni}(\omega)\}$. Given the distribution of productivity in each source, the distribution of prices in n is given by

$$G(p; \Phi_n) = 1 - e^{-\Phi_n p^{\theta}},$$

where $\Phi_n = \sum_i T_i(w_i d_{ni})^{-\theta}$. Because this distribution does not depend on the identity of a particular good, the measure of goods with a price equal to p in n is $g(p; \Phi_n) = \partial G(p; \Phi_n)/\partial p$.

Eaton and Kortum (2002) detail two very useful properties of this distribution. The first is that the probability that i is the lowest-cost source for good ω for consumers in n is equal to

$$\pi_{ni} = \frac{T_i(w_i d_{ni})^{-\theta}}{\Phi_n}, \tag{1}$$

[1] Technically, $q_n(\omega) = \sum_i q_{ni}(\omega)$, where $q_{ni}(\omega)$ is the quantity of good ω that is produced in i and consumed in n.

which is independent of the identity of the good, ω. Second, conditional on buying a good from a particular source, the distribution of prices is also given by $G(p;\Phi_n)$. In other words, the set of products that i actually sells in n has the same price distribution as the set of goods supplied by any other source. Together with the properties of the utility function, these properties of $G(p;\Phi_n)$ lead to the following result.

Proposition 1. *The share of country n's total expenditure that is allocated to products purchased from i is equal to π_{ni}.*

Proof. The assumption of symmetric preferences and the invertibility of $v(\cdot)$ imply that households' (Walrasian) demand for a particular good can be expressed as $q_n(\omega) = q(p_n(\omega), \mathbf{p}_n, w_n)$, where $\mathbf{p}_n = \{p_n(\omega)\}$ is set of prices in n of all goods. The probability that i is lowest-cost source of a given good, given the price, is equal to

$$\Pr(p_{ni}(\omega) = p_n(\omega) | p_n(\omega) = p) = \prod_{i' \neq i} e^{-T'_i (w'_i d_{ni'})^{-\theta} p^{\theta}}.$$

Together, these two results imply that the share of n's expenditure devoted to products purchased from i is given by

$$\frac{x_{ni}}{x_n} = \frac{\int_0^\infty p q(p, \mathbf{p}_n, w_n) \theta T_i (w_i d_{ni})^{-\theta} p^{\theta-1} e^{-\Phi_n p^\theta} dp}{\int_0^\infty p q(p, \mathbf{p}_n, w_n) dG_n(p;\Phi_n)} = \pi_{ni}.$$

□

The key to this result is that the symmetry of preferences implies that goods anonymous in the sense that demand depends only the price of the good, the prices of the other goods, and income, not on the identity of the good, ω. This allows aggregate expenditure to be calculated by integrating over the price distribution without keeping track of which goods have a particular realized price. Given this, the result follows immediately from the properties of $G(p;\Phi_n)$.

To close the model, I assume that trade is balanced, which, along with the result from Proposition 1 and labor market clearing, implies that

$$w_i = \sum_n \pi_{ni} w_n \frac{L_i}{L_n}.$$

Alvarez and Lucas (2007) show that these conditions define a unique general equilibrium. This implies that equilibrium wages and trade flows are independent of the particular form of $\mathscr{U}(\cdot)$.

4 Welfare

I have shown that equilibrium prices, wages, and international trade flows do not depend on specific assumptions about preferences. It remains to be shown that the welfare predictions of the model are also independent of such assumptions. This is the subject of the following proposition. In what follows, any variable with a hat indicates a proportional change – i.e., $\hat{x} = x'/x$.

Proposition 2. *The relative equivalent variation of households in n for any foreign shock is given by*

$$\frac{EV_n}{w_n} = \hat{\pi}_{nn}^{-\frac{1}{\theta}} - 1. \tag{2}$$

Proof. Consider the expenditure minimization problem of households in n, given by

$$\begin{aligned} e(\mathbf{p}_n, u) = &\min_{\mathbf{q}_n} \int_0^1 p_n(\omega) q_n(\omega) d\omega, \\ &\text{s.t.} \quad U(\mathbf{q}_n) \geq u, \end{aligned} \tag{EMP 1}$$

where $p_n(\omega)$ is the realization of a random variable distributed according to $G(p; \Phi_n)$. Because goods enter the utility function symmetrically, they can be relabelled without altering the household's expenditure minimization problem. In particular, consider sorting the goods in descending order of realized effective price in n, such that each good's label, ω, is matched one-to-one with a price $p(\omega)$. This implies that the measure of goods with a price greater than $p(\omega)$ is equal to ω. As a result, $\omega = 1 - G(p(\omega); \Phi_n)$, and thus

$$p(\omega) = -\Phi_n^{-\frac{1}{\theta}} \ln(\omega)^{\frac{1}{\theta}},$$

which further implies that EMP 1 can be rewritten as

$$\begin{aligned} e(\Phi_n, u) = &\Phi_n^{-\frac{1}{\theta}} \min_{\mathbf{q}_n} \int_0^1 - \ln(\omega)^{\frac{1}{\theta}} q_n(\omega) d\omega, \\ &\text{s.t.} \quad U(\mathbf{q}_n) \geq u. \end{aligned} \tag{EMP 2}$$

Without loss of generality, I choose labor in country n as the numeraire, so that w_n is constant. Then, using the form of EMP 2, the relative equivalent variation for a household in n of moving from an initial equilibrium to one characterized by Φ_n' is equal to

$$\frac{e(\Phi_n, u')}{e(\Phi_n', u')} - 1 = \hat{\Phi}_n^{\frac{1}{\theta}} - 1,$$

where u' is the maximized level of utility when household income is w_n and the price distribution is $G(p;\Phi'_n)$. The final result is obtained by totally differentiating (1) to show that $\hat{\Phi}_n = \hat{\pi}_{nn}^{-1}$, as long as T_n and d_{nn} are held constant.[2] $\square$

The intuition behind this result is rather straightforward. Because of the convenient properties of the Fréchet distribution employed in the Eaton-Kortum model and the assumptions of constant returns and perfect competition, the distribution of prices in n is entirely summarized by the value of Φ_n, given the value of θ.[3] Because any expenditure function is linearly homogeneous in prices, the welfare effect (measured by the equivalent variation) of a proportional change in all prices is independent of households' underlying preferences.[4] When goods enter the utility function symmetrically, a shift in the scale of the price distribution is equivalent to a common proportional change in all prices.[5] Because $G(p;\alpha\Phi_n) = G(\alpha^{\frac{1}{\theta}}p,\Phi_n)$, this implies that a change in Φ_n has the same welfare effect as a proportional increase in wealth equal to $\hat{\Phi}_n^{\frac{1}{\theta}}$. Finally, because trade flows follow a gravity equation (1), unobservable changes in Φ_n can be inferred from observable changes in π_{nn}.

Proposition 2 implies that, as in the trade models based on CES demand that are delineated in Arkolakis et al. (2012), the welfare effects of a foreign shock can be inferred from only two variables, the domestic trade share and the trade cost elasticity, θ. In particular, the welfare effects of moving from the baseline equilibrium to autarky is equal to $\pi_{nn}^{-\frac{1}{\theta}}$. In addition, following the methodology of Dekle et al. (2008), changes in π_{ni} resulting from any changes in trade costs can be computed as the solution to the following equations:

$$\hat{\pi}_{nn} = \left(\sum_i \pi_{ni}(\hat{w}_i\hat{d}_{ni})^{-\theta}\right)^{-1},$$

where

$$\hat{w}_i = \sum_n \frac{w_n L_n}{w_i L_i} \cdot \frac{\pi_{ni}(\hat{w}_i\hat{d}_{ni})^{-\theta}}{\sum_i \pi_{ni}(\hat{w}_i\hat{d}_{ni})^{-\theta}}.$$

Thus, for any specification of symmetric preferences, the welfare effects of any change in trade costs in the Eaton-Kortum model of international trade are identical and can be calculated from data on bilateral trade flows and the trade cost elasticity.

[2]This result implicitly assumes that the value of $e(\Phi_n,u)$ exists – i.e., that preferences are such that an equilibrium exists. Formally, this requires that the integral $\int_0^1 -\ln(\omega)^{\frac{1}{\theta}} q_n(\omega,\Phi_n,u)d\omega$ be well-defined, where $q_n(\Phi_n,u)$ denotes the Hicksian (compensated) demand for $q_n(\omega)$ in the general equilibrium. For example, if $U(\cdot)$ were a CES utility function, it would require that the elasticity of substitution across goods, $\sigma < 1+\theta$.

[3]It is worth noting that the Fréchet distribution is not employed by Eaton and Kortum (2002) purely for convenience. Kortum (1997) shows that this distribution of productivity arises from an endogenous search process in which only best production technique discovered to date is used.

[4]See, e.g., Mas-Colell et al. (1995), Proposition 3.E.2.

[5]A shift in the scale of a probability distribution is defined as a change in a scale parameter, s, of a distribution function, $H(\cdot)$, where it is the case that $H(x;s,\beta) = H(x/s;1,\beta)$.

5 Asymmetric Preferences: Two-Stage Budgeting

While Proposition 2 requires that goods enter the utility function symmetrically, this result can be generalized to the case in which goods can be allocated to groups within which preferences over them are symmetric. An example of a utility function that fits this description is the "constant relative income elasticity" preferences used in Caron et al. (2014). It is also closely related to multi-sector trade models such as those of Caliendo and Parro (2015) and Levchenko and Zhang (2014).

Suppose that the continuum of goods from the previous setup can be partitioned into a finite number of sets, denoted Ω^k, where $k = 1, ..., K$ and $\bigcup_{k=1}^{K} \Omega^k = [0,1]$, and that households maximize a utility function that is weakly separable across these sets and symmetric across products within each set. Specifically, suppose that $U(\mathbf{q}_n)$ is given by $U\left(u_1(\mathbf{q}_n^1), ..., u_K(\mathbf{q}_n^K)\right)$, where $\mathbf{q}_n^k = \{q_n(\omega) : \omega \in \Omega^k\}$, and where each subutility function, $u_k(\cdot)$, satisfies the same set of assumptions as above. The following proposition shows that the main results are unaffected by generalizing preferences in this way.

Proposition 3. *Given preferences that are weekly separable across a finite number of sets of products and symmetric across products within each set, the relative equivalent variation of households in n for any foreign shock is given by*

$$\frac{EV_n}{w_n} = \hat{\pi}_{nn}^{-\frac{1}{\theta}} - 1. \tag{3}$$

A formal proof is given in the appendix and is similar to the proofs of Propositions 1 and 2. The intuition for this result is related to the composite commodity theorem of Hicks (1939) and Leontief (1936), which states that, if the prices of multiple goods move in parallel, they can be treated as a single good in the utility function.[6] As before, a change in Φ_n has the same effect as a proportional shift in all prices. Because the distribution of prices is the same for each set of products, the change in Φ_n has an effect equivalent to an identical proportional shift for every set of products. Thus, the problem reduces to the simpler one in which every good enters the utility function symmetrically.

6 Concluding Remarks

This paper has shown that the predictions of the Ricardian trade model of Eaton and Kortum (2002) – characterized by constant returns to scale in production, productivity levels drawn from country-specific Fréchet distributions, and international trade subject to iceberg trade

[6]See Deaton and Muellbauer (1980) for a modern treatment of the topic.

costs – are independent of the specific form of the utility function. This result requires only that it be possible to group the continuum of goods into a finite number of sets, within which they enter preferences symmetrically, which is satisfied by nearly every utility function over a continuum of goods that has been employed in the literature.

For this result to hold, some special conditions do need to be met, notably that the productivity distribution is identical for all sets of products and that prices move proportionally with changes in production and trade costs.[7] However, this paper makes clear that different assumptions regarding preferences do not alter the predictions of this workhorse quantitative trade model and, thus, that any deviation in predictions from the Eaton-Kortum model with CES preferences arises due to the interactions among deviations from the baseline model on both the supply and demand sides.

[7]The latter follows trivially from the assumption of perfect competition. The results of this paper would continue to hold if firms charged a constant percentage markup over marginal cost or if the markup distribution were invariant to foreign shocks. However, with monopolistic competition, for example, additional restrictions must be placed on preferences for such conditions to arise. In particular, CES utility is required for markups to be constant, and Arkolakis et al. (2015) delineate a set of preferences for which the latter holds with Pareto distributed productivity.

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A Proof of Propsition 3

First, it must be shown that, as in Proposition 1, π_{ni} is the share of n's expenditure devoted to all goods purchased from i. Because the utility function is separable across sets of products, and because goods within each set enter preferences symmetrically, households' Walrasian demand for a given good, $\omega \in \Omega^k$, can be expressed as

$$q_n(\omega) = q^k(p_n(\omega), \mathbf{p}_n, w_n).$$

Given this, expenditure by a household in n on all goods in set k is given by

$$x_n^k = \int_{\Omega^k} p_n(\omega) q^k(p_n(\omega), \mathbf{p}_n, w_n) d\omega.$$

Because $G_n(p; \Phi_n)$ is independent of the set to which a good belongs, this expression can be rewritten in terms of the price distribution as

$$x_n^k = \mu^k \int_0^\infty p q^k(p, \mathbf{p}_n, w_n) dG_n(p; \Phi_n),$$

where $\mu^k = \int_{\Omega^k} d\omega$ is the measure of goods in set k.

The same argument applies to the set of goods that n buys from i. Thus, following the proof of Proposition 1, the share of n's expenditure devoted to products purchased from i is given by

$$\frac{x_{ni}}{x_n} = \frac{\sum_{k=1}^K x_{ni}^k}{\sum_{k=1}^K x_n^k} = \frac{\sum_{k=1}^K \mu^k \int_0^\infty p q^k(p, \mathbf{p}_n, w_n) \theta T_i (w_i d_{ni})^{-\theta} p^{\theta-1} e^{-\Phi_n p^\theta} dp}{\sum_{k=1}^K \mu^k \int_0^\infty p q^k(p, \mathbf{p}_n, w_n) dG_n(p; \Phi_n)} = \pi_{ni}.$$

What remains is to show that $EV_n/w_n = \hat{\Phi}_n^{1/\theta} - 1$. To this end, and without loss of generality, I label the goods sequentially, so that each set is made up of products with contiguous values of ω. Formally, I label the goods such that $\Omega^k = \{\omega : \omega \in [m^{k-1}, m^k)\}$, where the set boundaries, m, are defined inductively by $m^0 = 0$ and $m^k = m^{k-1} + \mu^k$. Given this notation, the households' expenditure minimization problem can be expressed as

$$\begin{aligned} e(\mathbf{p}_n, u) = & \min_{\mathbf{q}_n} \sum_{k=1}^K \int_{m^{k-1}}^{m^k} p_n(\omega) q_n(\omega) d\omega, \\ & \text{s.t.} \quad U(\mathbf{q}_n^1, ..., \mathbf{q}_n^k) \geq u. \end{aligned} \tag{EMP 1'}$$

Because goods in a given set enter the subutility function symmetrically, as in the proof of Proposition 2, the household's expenditure minimization problem is unaffected by relabelling goods *within each set*. In particular, the goods can be sorted, within each set, in descending order of realized effective price in n, such that each label, $\omega \in \Omega^k$, is matched one-for-one with a price $p(\omega)$. This implies that the measure of goods in set k with price greater than $p(\omega)$ is equal to $\omega - m^{k-1}$, and thus $\omega - m^{k-1} = \mu^k(1 - G_n(p; \Phi_n))$. Using this labelling of goods, EMP 1′ can be rewritten as

$$\begin{aligned} e(\Phi_n, u) = &\Phi_n \min_{\mathbf{q}_n} \sum_{k=1}^{K} \int_{m^{k-1}}^{m^k} - \ln \left(\frac{\omega - m^{k-1}}{\mu^k} \right)^{\frac{1}{\theta}} q_n(\omega) d\omega, \\ &\text{s.t.} \quad U(\mathbf{q}_n^1, ..., \mathbf{q}_n^k) \geq u. \end{aligned} \tag{EMP 2′}$$

The remainder of the proof is identical to that for Proposition 2. □