Promotions, Labor Markets and Skill Acquisition∗

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February 4, 2010

Abstract

We study incentives for skill acquisition when skills cannot be contracted on. Our framework has two features. First, workers can acquire both firm specific and general skills at a cost, and second, job assignments signal skill acquisition to a less informed labor market. Our key insight is that each skill performs a different function. The firm specific skill helps in revealing information to a labor market whereas the general skill enforces wage promises from employers. As a consequence, both skills are acquired together in equilibrium, sometimes inefficiently. This endogenous complementarity between skills also yields implications for promotions and job design.

1 Introduction

In many jobs, workers acquire skills, firm specific or general, that are costly to themselves but that benefit their employers. Because of this externality, workers have to be given incentives to acquire skills. But a problem arises here. Skills, such as building relationships with clients and acquiring technical knowledge on the job, are often hard to verify by a third party and thus difficult to contract on. In particular, an employer who promises to pay the worker for acquiring skills, can renege on his promise. The worker anticipates this and does not acquire skills leading to an inefficiency.

How can incentives for skill acquisition be provided when skills cannot be contracted on? The key is to get employers to commit to pay workers when they acquire skills and the literature suggests three ways to do this. The first approach relies on repeated interaction where workers can threaten to terminate a relationship when employers renege on their promises (Baker, Gibbons, and Murphy (1994), Levin (2003)). The second approach (Prendergast (1993)) allows for contracts to be written on jobs (as opposed to skills). When higher level jobs are more sensitive

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to skills, employers have an incentive to promote workers with skills to increase output, and are thus forced by the contract to reward the worker. Finally, the third approach taken by Scoones and Bernhardt (1998) draws on the existence of an outside labor market and heterogeneity (in managerial ability and match quality) of workers. Worker heterogeneity bids wages up to reflect, at least partially, the worker’s investment in skills.

We suggest an alternative way in which employers can commit to repay workers and our framework has two main features. First, workers can invest in both firm specific and general skills at a cost. Second, current employers have better information about a worker’s skills which they can signal through job assignments to an outside labor market (Waldman (1984)). The key insight from this framework is that each type of skill performs a different function. The firm specific skill gives a share of the surplus to current employers and thus induces them to reveal their private information about a worker’s skills to the labor market. The general skill, on the other hand, forces the current employer to honor wage promises because of competition in the labor market. Using this insight, we show how skills can be acquired by workers even without legally enforceable contracts, and we derive implications for skill acquisition, promotion patterns and job design.

Our model is based on the principal-agent framework of Prendergast (1993) with a worker, a firm (where the worker is currently hired) and an outside labor market, all of whom are risk neutral. Like, Prendergast (1993), we assume that skills cannot be contracted on and that higher level jobs are more sensitive to skills. But we add two new features. First, we assume two types of skills, a firm specific and a general skill. Each type of skill has two outcomes, success or failure, and the worker can influence the probability of success at a cost. Second, we assume that the firm observes skill acquisition whereas the outside labor market only observes job assignments (Waldman (1984)).

Our main result is that firm specific and general skills are complements from an incentive viewpoint: when one type of skill is acquired in equilibrium then so is the other type. Interestingly, this result holds even if the firm specific and general skills are not technological complements. Underlying our main result is the insight that each skill performs a distinct function. The firm specific skill helps information to be revealed to a labor market whereas the general skill enables the market to enforce wage promises by an employer. Because both skills are always acquired together there could be situations in which skills are over accumulated relative to their efficient levels.

We then draw on this endogenous complementarity between skills to suggest implications for promotions and job design. Consider promotions first. As both

1Unlike Prendergast (1993), we assume that contracts written on jobs cannot be enforced by courts of law. Thus in our framework, the labor market is the only enforcement mechanism. This assumption applies well to settings where enforcement of contracts by a court of law is costly (especially to the worker) or involves significant delay.
types of skills must be acquired in equilibrium, promotions should get workers to view the skills as complements. When firm specific and general skills are natural substitutes for a worker (through a linear cost function), the only way to induce skill complementarity is to promote the worker if and only if he acquires both types of skills. This result suggests that a generalist in an organization who acquires both types of skills should be more likely to be promoted than a specialist who acquires only one type of skill.

Next, consider job design. Firms would like to design jobs that allow them to extract a larger share of the surplus. The only way to do this is to increase the value of the firm specific skill relative to the general skill. We find that by making skills technological complements, firms can extract a larger share of the surplus. The intuition is the following. Workers are rewarded if and only if they successfully acquire both skills. Technological complementarities thus increase this contingent reward, giving the worker stronger incentives to collect both types of skills. Firms then can exploit these stronger incentives and extract a larger share of the surplus without worrying about workers shirking.

Starting with Becker (1962) and Becker (1964), a large literature studies incentives for investing in specific and general skills, especially within the context of who pays for training. A number of papers also draw on the framework of Waldman (1984), where job assignments signal worker productivity to an outside labor market. Scoones and Bernhardt (1998) combine both of these features in a framework where heterogeneous workers (in managerial ability and match quality) can invest in firm specific or general skills (not both) and where current employers have better information about managerial ability. They find that workers can invest in firm specific human capital even though this does not directly increase wages. There are two reasons for this. First, heterogeneity in the labor market bids up wages to reflect at least part of the firm specific investment and second, firms are less likely to under-promote workers for strategic reasons when the firm specific skill is acquired. The main difference between their paper and ours is the source of commitment; in their paper it is worker heterogeneity whereas in ours it is the presence of a general skill. Yet another way for firms to commit to repay workers is the use of up or out contracts (Kahn and Huberman (1988)). If firms can commit to fire workers when they do not acquire skills then they have less of an incentive to claim that workers have not acquired skills. Finally, Chang and Wang (1995) and Chang and Wang (1996) study incentives for skill acquisition when current employers privately observe levels of training provided to their employees. Their focus, however, is on incentives for a firm to invest in training rather than incentives for a worker.

Our paper is also related to work on multi-tasking and multi-skilling. MacDon-

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2Examples of applications where employers signal worker traits to an outside market are up or out contracts (Waldman (1990)), layoffs (Gibbons and Katz (1991)) and discrimination (Milgrom and Oster (1987) and De Varo, Ghosh, and Zoghi (2008)).

3Similarly, a tournament with fixed prizes as in Carmichael (1983) and Malcomson (1984) acts as a commitment device.
ald and Marx (2001) consider a multi-task principal agent model where the agent views tasks as substitutes whereas the principal views tasks as complements. They show that incentive contracts which reward overall success more than other outcomes make the tasks complements for the agent as well. Our result where workers are promoted if and only if they are successful at both skills has a similar flavor. However, the complementarity of skills in our framework is endogenous. Another context in which multi-skilling serves as a commitment device is given by Carmichael and MacLeod (1993). They show how multi-skilling allows firms to commit to retain workers after workers suggest a labor saving innovation.

2 Model

We build on a model by Prendergast (1993) with a risk neutral firm, a risk neutral worker and a risk neutral labor market. There are two skills that the worker can acquire, a firm specific skill and a general skill, with each skill having two outcomes. The outcome for the firm specific skill is denoted by \( i \) with \( i \in \{0, f\} \) where 0 denotes failure and \( f \) denotes success at the firm specific skill. Similarly, the outcome for the general skill is denoted by \( j \) with \( j \in \{0, g\} \) where 0 denotes failure and \( g \) denotes success for the general skill. Thus there are four possible outcomes, failure at both skills \((0, 0)\), success only at the firm specific skill \((f, 0)\), success only at the general skill \((0, g)\) and success at both skills \((f, g)\). The worker can exert effort that is privately observable to influence the probability of success on each skill. To simplify notation, we assume that the worker chooses these probabilities directly. The probability of acquiring the firm specific skill is \( p_f \in [p, \bar{p}] \) and the probability of acquiring the general skill is \( p_g \in [p, \bar{p}] \) and these probabilities are independent of one another. To ensure that beliefs of the labor market can be determined by Baye’s rule, we assume that \( p = \frac{1}{2} - \epsilon \) and \( \bar{p} = \frac{1}{2} + \epsilon \) where \( 0 < \epsilon < \frac{1}{2} \). In the analysis that follows we refer to \( p_f \) as the level of the firm specific skill and \( p_g \) as the level of the general skill. We assume that the worker has a linear cost function given by \( C(p_f, p_g) = c_fp_f + c_gp_g \) where \( c_f \) and \( c_g \) are strictly positive constants. Having a more general cost function that is strictly increasing in \( p_f \) and \( p_g \) does not affect our main result on the complementarity of skills though it does affect equilibrium promotion patterns. We discuss the role of the linear cost function in greater detail following Proposition 2.

The relationship between skills and output depends on the job assignment. There are two jobs, a lower level job, \( L \), and a higher level job, \( H \), and these jobs vary in their sensitivity to skills. Output for the four possible outcomes is as follows. When the outcome is \((0, 0)\), output is 0. When the worker is only successful at the firm specific skill, output is \( \lambda y_L \) for the low job and \( \lambda y_H \) for the high job with \( 0 < y_L < y_H \) and where \( \lambda \in [0, 1] \) measures the relative value of firm specific skills.
to general skills. When the worker is only successful at the general skill, output is $(1 - \lambda)y_L$ for the low job and $(1 - \lambda)y_H$ for the high job. Finally for the outcome $(f, g)$, output is given by $y_L + \Delta$ for the low job and $y_H + \Delta$ for the high job with $\Delta > 0$. The notation $\Delta$ measures the extent to which both the skills are complementary. In other words, when we fix a job level, $\Delta$ is the cross partial of expected output with respect to $p_f$ and $p_g$. The table below summarizes the relationship between outcomes and output.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>(0,0)</th>
<th>(f,0)</th>
<th>(0,g)</th>
<th>(f,g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output from Job L</td>
<td>0</td>
<td>$\lambda y_L$</td>
<td>$(1 - \lambda)y_L$</td>
<td>$y_L + \Delta$</td>
</tr>
<tr>
<td>Output from Job H</td>
<td>0</td>
<td>$\lambda y_H$</td>
<td>$(1 - \lambda)y_H$</td>
<td>$y_H + \Delta$</td>
</tr>
</tbody>
</table>

The timing in the model is as follows. The worker chooses the probabilities of success for both skills which induces a probability distribution over outcomes. The outcomes for both of these skills are then realized. The firm observes these outcomes and decides on the job assignment of the worker. Finally, the labor market observes the job assignment, updates beliefs and offers a wage that is contingent on the assignment. In this setting a strategy for a worker is given by the vector $(p_f, p_g)$. The firm’s strategy is denoted by $(x_0, x_0, x_f, 0, x_0, g, x_f, g)$ with $x_{i,j} \in \{L, H\}$. The labor market observes job assignments and for each job assigns beliefs to each of the four outcomes. The beliefs associated with each of the outcomes for job $L$ are given by the vector of probabilities $\mu^L = (\mu^L_{0,0}, \mu^L_{f,0}, \mu^L_{0,g}, \mu^L_{f,g})$ and the beliefs associated with each outcome for job $H$ are given by the vector of probabilities $\mu^H = (\mu^H_{0,0}, \mu^H_{f,0}, \mu^H_{0,g}, \mu^H_{f,g})$. Wages offered by the labor market are denoted by $w_L$ when job $L$ is observed and $w_H$ when job $H$ is observed.

### 3 Efficiency

We start our analysis by characterizing efficient levels of skill acquisition. This characterization serves as a benchmark. The following assumption says that if the worker could get the outcome $(f, g)$ for sure, then the benefits to the firm excluding complementarities outweigh the costs to the worker. This assumption ensures that the efficient solution always sets at least one of the skill levels at its maximum level.

**Assumption 1.** $y_H > c_f + c_g$

Let $(p_f^{eff}, p_g^{eff})$ denote the efficient level of skill acquisition. Then $(p_f^{eff}, p_g^{eff})$ is the optimal solution to the following problem.

$$Max \quad p_f(1 - p_f) \lambda y_H + (1 - p_f)p_g(1 - \lambda)y_H + p_f p_g(y_H + \Delta) - C(p_f, p_g)$$
The following proposition characterizes the efficient skill acquisition levels. The proofs of all of the propositions and lemmas are in the Appendix.

**Proposition 1.** The efficient levels of skill acquisition are given by

\[
p^\text{eff}_f = \begin{cases} 
  p & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda \in \left[0, \frac{c_f - \bar{p}\Delta}{y_H}\right) \\
  [p, \bar{p}] & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda = \frac{c_f - \bar{p}\Delta}{y_H} \\
  \bar{p} & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda \in \left(\frac{c_f - \bar{p}\Delta}{y_H}, 1\right] \\
  \bar{p} & \text{if } \Delta > \frac{c_f}{p}
\end{cases}
\]

and

\[
p^\text{eff}_g = \begin{cases} 
  \bar{p} & \text{if } \Delta > \frac{c_g}{p} \\
  \bar{p} & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda \in \left[0, 1 - \frac{c_g - \bar{p}\Delta}{y_H}\right) \\
  [p, \bar{p}] & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda = 1 - \frac{c_g - \bar{p}\Delta}{y_H} \\
  p & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda \in \left(1 - \frac{c_g - \bar{p}\Delta}{y_H}, 1\right]
\end{cases}
\]

Proposition 1 highlights two factors that determine efficient skill levels: technological complementarities and the relative importance of both skills. When technological complementarities are high, both skills are acquired at their maximum level. When technological complementarities are low, the relative value of each skill plays an important role. If both skills are important (for intermediate values of \(\lambda\)), efficiency requires a worker to choose the highest skill level for both skills. If one skill is more important relative to another (\(\lambda\) close to 0 or 1), efficiency requires the highest skill level for the more valuable skill and the lowest skill level for the less valuable skill. Notice that these corner solutions arise because of the linear specification for the cost function. Figure 1 depicts the efficient level of skill acquisition with \(\lambda\) on the horizontal axis and \(p^\text{eff}_f\) and \(p^\text{eff}_g\) on the vertical axis when technological complementarities are not too high.
4 Equilibrium

The objective in this section is to compare equilibrium skill levels with the efficient skill levels above. The equilibrium concept we use is Weak Perfect Bayesian Equilibrium and we restrict our attention to pure strategies.

Definition 1. A Weak Perfect Bayesian Equilibrium with skill acquisition consists of a strategy profile \((p_f, p_g, (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))\), a belief system for the labor market \((\mu_L, \mu_H)\), and a pair of wages, one for each job, \((w_L, w_H)\) such that the following conditions hold:

1. The strategy profile \((p_f, p_g, (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))\) is sequentially rational given the wages offered by the market, \((w_L, w_H)\).

2. The belief system \((\mu_L, \mu_H)\) is derived, using Baye’s rule wherever possible, from the strategy profile \((p_f, p_g, (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))\).

3. Competition between the firm and the labor market ensures that the worker is paid his expected output in the labor market. That is \(w_L = (1 - \lambda)(\mu_{0,g}^L y_H + \mu_{f,g}^L (y_H + \Delta))\) and \(w_H = (1 - \lambda)(\mu_{0,0}^H y_H + \mu_{f,g}^H (y_H + \Delta))\).

4. Either \(p_f > p\) or \(p_g > p\) or both.

Henceforth, we refer to a Weak Perfect Bayesian Equilibrium as an equilibrium. The following lemma places restrictions on wages in equilibrium.

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Figure 1: Efficient Levels of Skill Acquisition.
Lemma 1. Suppose the worker is assigned to job $L$ for any of the outcomes $(f,0)$, $(0,g)$ or $(f,g)$ in equilibrium. Then $w_H > w_L$.

This lemma says that if a worker is assigned to a low job in equilibrium when he is successful at acquiring any skill, it must be the case that wages for the higher level job exceed wages for the lower level job. The reason for this is that higher level jobs are more sensitive to skills. If $w_L$ was at least as high as $w_H$, then the firm could always promote the worker, get a higher output and pay a lower wage and do strictly better.

The next proposition states the main result of the paper, that firm specific and general skills are complements from an incentive viewpoint.

Proposition 2. Let $\epsilon$ be sufficiently close to $\frac{1}{2}$. Also let

$$c_f + c_g \leq y_H - y_L \leq \frac{y_H + (1 + \tilde{p})\Delta}{(1 + \tilde{p})}$$

and suppose

$$\lambda \in \left[1 - \frac{(1 + \tilde{p})(y_H - y_L)}{y_H + (1 + \tilde{p})\Delta}, \min\{1 - \frac{(1 + \tilde{p})(c_f + c_g)}{y_H + (1 + \tilde{p})\Delta}, \frac{y_H + (1 + \tilde{p})\Delta}{y_H + (1 + \tilde{p})\Delta + (1 + \tilde{p})(y_H - y_L)}\}\right]$$

Then there exists a unique equilibrium with skill acquisition where $(p_f, p_g) = (\tilde{p}, \tilde{p})$ and where the firm promotes the worker if and only if the outcome $(f,g)$ is realized. For $\lambda$ outside of the interval specified above, there are no skill acquisition equilibria.

Before examining Proposition 2 in greater detail consider the condition in (1). This inequality places lower and upper bounds on the extent to which job $H$ is more sensitive to skills relative to job $L$. The lower bound ensures that there are a range of $\lambda$’s for which firms have an incentive to promote workers when the outcome $(f,g)$ is realized and for which workers have an incentive to acquire the maximum level of skills. On the other hand, $y_H - y_L$ should be low enough to deter firms from promoting workers when the outcome $(0,g)$ is realized. From (1), there always exist a range of $\lambda$’s for which there is a unique skill acquisition equilibrium.

Next, it is useful to split Proposition 2 into three parts to highlight the role of different assumptions. The first part says that firm specific skills and general skills are always acquired together in equilibrium ($p_f > p$ and $p_g > \tilde{p}$). The interesting feature of this result is that it holds even if the skills are not technological complements ($\Delta = 0$). The logic underlying this result is that both skills perform different functions for the firm. If only general skills were acquired in equilibrium, firms could reassign workers to the low job whenever they should have been promoted and take a larger part of the surplus. If only firm specific skills were acquired, the labor market
would not bid wages up and firms would not be able to commit to pay wages. The crucial assumption here is that $p$ is sufficiently close to 0. This drives the workers wage down to 0 and thus firms cannot commit to pay a wage that covers costs.

The second part of Proposition 2 follows from the first part. Given that both skills have to be acquired together in equilibrium, this part deals with how incentives can be provided through promotions. Proposition 2 says that to get workers to acquire both skills, firms must promote workers if and only if the outcome realized is $(f,g)$. Two additional assumptions play a role here. The first is that the upper limit for probability of success, $\bar{p}$, is sufficiently close to 1 and the second assumption is the linearity of the cost function. When the cost function is linear the worker naturally views the skills as substitutes. Promoting the worker only when the outcome $(f,g)$ is realized, induces the worker to view the skills as complements. This result holds even if we had cost functions of the form $h(\alpha p_f + \beta p_g)$ where $\alpha$ and $\beta$ are positive constants and where $h$ is a strictly increasing function. Relaxing the assumptions made on the upper limit for probabilities and the functional form of the cost function makes it difficult to rule out the strategies $(L,H,H,L)$ and $(L,H,H,H)$ as equilibrium strategies. Also, notice that because costs are linear, we get corner solutions where the maximum level of skills is acquired in any equilibrium with skill acquisition.

Finally, we can use Proposition 2 to relate equilibrium skill levels with their efficient counterparts for any given $\lambda$. Because the worker bears all the costs of skill acquisition and the firm gets all of the benefits we would typically expect an under accumulation of skills relative to the efficient level in equilibrium. As Proposition 2 indicates, this is true when the relative value of firm specific skills is high ($\lambda$ is sufficiently close to 1). The reason for this under accumulation is that wages are not bid up enough by the market to offset costs of the worker. There is also an under accumulation of skills when the relative value of the general skill is high ($1-\lambda$ is high) and jobs do not vary much in their sensitivity to skills (when $y_H - y_L$ is low). The reason here is that firms have little incentive to promote workers because the share of the surplus that the firm can extract is small relative to the output gain.

What is unusual about Proposition 2 is that there can be an over accumulation of skills. When jobs vary substantially in their sensitivity to skills (when $y_H - y_L$ is high) it is possible to have both skills being acquired in equilibrium even though efficiency requires only the general skill. The reason for this over accumulation is that both skills have to be acquired together in equilibrium. Figure's 2 and 3 depict equilibria where $y_H - y_L$ is low and high respectively. In the following section we build on Proposition 2 to see how firms can design jobs using technological

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5Note that for any $\lambda \in [0,1]$ there exist beliefs, such that the pooling strategy where firms promote the worker for all of the outcomes is an equilibrium. Thus when $\lambda \in [\max\{0,1 - (1 + \tilde{p})(y_H - y_L)\}, \min\{1 - \frac{(1 + \tilde{p})(c_f + c_g)}{y_H + (1 + \tilde{p})\Delta}, \frac{y_H + (1 + \tilde{p})\Delta}{y_H + (1 + \tilde{p})\Delta + (1 + \tilde{p})(y_H - y_L)}\}]$ there are multiple equilibria. Figure 2 only focusses on the skill acquisition equilibria within this interval.
complementarities.

5 Complementarities and Job Design

Two parameters that are of interest to the firm in our model are $\lambda$, which denotes the relative importance of the firm specific to the general skill, and $\Delta$, which specifies the extent to which the skills are technological complements. So far, we have assumed that $\lambda$ and $\Delta$ are exogenous. However, we can think of a setting where firms can design jobs by choosing skills with different $\lambda$’s and $\Delta$’s prior to the worker choosing skill levels.

We know that if the assumptions in Proposition 2 hold and if

$$\lambda \in \left[ 1 - \frac{(1 + \bar{p})(y_H - y_L)}{y_H + (1 + \bar{p})\Delta}, \min\{1 - \frac{(1 + \bar{p})(c_f + c_g)}{y_H + (1 + \bar{p})\Delta}, \frac{y_H + (1 + \bar{p})\Delta}{y_H + (1 + \bar{p})\Delta + (1 + \bar{p})(y_H - y_L)}\} \right]$$

then there is unique equilibrium with skill acquisition where the worker chooses the maximum level of both skills. The expected payoff to the firm from the equilibrium is given by

$$\bar{p}(1 - \bar{p})(y_L - (1 - \lambda)y_H) + \bar{p}^2 \lambda(y_H + \Delta)$$

Notice that this expression is strictly increasing in $\lambda$. Thus the firm would like to set $\lambda$ as large as possible to extract as much surplus as it can. Let $\lambda^*(\Delta)$ be a function that specifies the largest possible $\lambda$ that is consistent with a skill acquisition equilibrium. From Proposition 2, we know that $\lambda^*(\Delta) = \min\{1 -
Figure 3: Equilibrium Levels of Skill Acquisition: $y_H - y_L$ high.

$$\min\left\{1 - \frac{(1 + p)(c_f + c_g)}{y_H + (1 + p)\Delta}, \frac{u_H + (1 + p)\Delta}{y_H + (1 + p)\Delta + (1 + p)(y_H - y_L)}\right\}$$

The following proposition states that technological complementarities allow a firm to extract a larger share of the surplus.

**Proposition 3.** $\lambda^*$ is strictly increasing in $\Delta$.

There are two channels through which technological complementarities between skills increase profits for a firm. The first is the direct effect which increases output by $\Delta$ when both skills are acquired together. Much of the literature on job design focusses on this channel (see Gibbs and Levenson (2002) for a review of the job design literature). A second channel, which Proposition 3 emphasizes, is through the fraction of surplus a firm can extract. To see how this channel works, notice that there are two constraints on the value of the firm specific skill relative to the general skill. First, the more valuable the firm specific skill is relative to the general skill, the less wages get bid up. Thus a worker has greater incentive to deviate and acquire no skills. Second, the more valuable firm specific skills are relative to general skills, firms are more likely to deviate and promote workers when the outcome $(f, 0)$ is realized. Increasing the degree to which both skills are technological complements helps to relax both of these constraints. For the first constraint, notice that because workers are rewarded if and only if they acquire both skills, technological complementarities give very strong incentives for a worker to collect both skills. Firms then can use these strong incentives to extract a larger share of the surplus without worrying about workers shirking. For the second constraint, technological complementarities increase the wage for job $H$ and once again act as a deterrent to promote workers when $(f, 0)$ is realized.
6 Conclusion

We study incentives for skill acquisition when skills cannot be contracted on. The problem here is that firms find it difficult to commit to repay workers when they acquire costly skills. Our framework has two features which allows firms to commit. First, there are two types of skills, a firm specific skill and a general skill and workers can acquire both at a cost. Second, job assignments signal skill acquisition to a less informed labor market. In this framework each type of skill plays a distinct role; firm specific skills give a larger share of the surplus to current employers and thus reveal information to a labor market whereas general skills, via competition in the labor market, bid wages up, forcing current employers to honor their wage promises.

We find three main results. First, both types of skills are complements from an incentive viewpoint; if one skill is acquired in equilibrium then so is the other type. This leads to a potential over accumulation of skills. Second, when workers view skills as natural substitutes (they have linear cost functions) they are promoted if and only if they successfully acquire both skills. Finally, we study job design and show how technological complementarities allow firms to capture a larger share of the surplus.

To be sure, there are other ways to provide incentives for skills: legally enforceable contracts can be written on jobs (Prendergast (1993)), firms can strategically under promote workers based on their natural abilities (Scoones and Bernhardt (1998)) and firms can resort to up or out contracts (Kahn and Huberman (1988)). Nevertheless, we believe that our framework with contracts enforced by a labor market, multiple types of skills and signalling through job assignments is a realistic and rich framework. Furthermore, we derive implications for skill acquisition and promotion patterns that are distinct from the papers above and which can potentially be taken to the data. This is an area for future research.
Appendix

**Proof of Proposition 1:** The first order necessary conditions for the optimal solution are

\[
\left( \frac{\lambda y_H + p_g \Delta - c_f}{(1 - \lambda)y_H + p_f \Delta - c_g} \right) = \mu_f \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mu_f \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \mu_g \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mu_g \begin{pmatrix} 0 \\ -1 \end{pmatrix}
\]

where \(\mu_f\) is the non negative multiplier associated with the constraint \(p_f \leq \bar{p}\) and \(\mu_f\) is the non negative multiplier associated with the constraint \(p_f \geq \bar{p}\). Likewise, \(-\mu_g\) and \(\mu_g\) are non negative multipliers associated with the inequality constraints for \(p_g\).

In the following claim we establish that the efficient skill level has at least one of the skills at its maximum level.

**Claim** The efficient level of skills has either \(p_f = \bar{p}\) or \(p_g = \bar{p}\) or both.

**Proof** Suppose the optimal solution had \(p_f < \bar{p}\) and \(p_g < \bar{p}\). Then the first order conditions imply \(\lambda y_H + p_g \Delta - c_f \leq 0\) and \((1 - \lambda)y_H + p_f \Delta - c_g \leq 0\). Adding both inequalities and rearranging we get \(y_H + (p_g + p_f)\Delta \leq c_f + c_g\). But this contradicts Assumption 1.

Now suppose \(\Delta > \frac{c_f}{\bar{p}}\) and suppose to the contrary that \(p_f < \bar{p}\) at the optimum. From the claim above it follows that \(p_g = \bar{p}\) at the optimum. Since \(\Delta > \frac{c_f}{\bar{p}}\) the first order condition with respect to \(p_f\) can be written as \(\lambda y_H + \bar{p} \Delta - c_f = \mu_f - \mu_f > 0\).

From the complementary slackness conditions we have \(p_f = \bar{p}\) at the optimum which is a contradiction. Similarly when \(\Delta > \frac{c_g}{\bar{p}}\) we must have \(p_g = \bar{p}\) at the optimum.

Next suppose \(\Delta \leq \frac{c_f}{\bar{p}}\) and \(\Delta \leq \frac{c_g}{\bar{p}}\) and consider the following partition of the interval \([0, 1]\).

First, let \(0 < \lambda < \frac{c_f - \bar{p} \Delta}{y_H}\). Rearranging the second inequality we get \(\lambda y_H + \bar{p} \Delta - c_f < 0\). Combining this inequality with the constraint \(p_g \leq \bar{p}\), we can write the first order condition with respect to \(p_f\) as \(\lambda y_H + p_g \Delta - c_f = \mu_f - \mu_f < 0\). Since the multipliers are non-negative, it follows that \(\mu_f > 0\). The complementary slackness
conditions imply that \( p_f = p \) at the optimum. It follows from the claim above that \( p_g = \bar{p} \) at the optimum.

Second, let \( \lambda = \frac{c_f - \bar{p} \Delta}{y_H} \). Rearranging, we get \( -\lambda y_H = \bar{p} \Delta - c_f \). Adding \( y_H \) to both sides of the equality we get

\[
(1 - \lambda)y_H = y_H + \bar{p} \Delta - c_f \quad (2)
\]

Using equation (2) and Assumption 1 the first order condition with respect to \( p_g \) is \( (1 - \lambda) y_H + p_f \Delta - c_g = \bar{\mu}_g - \mu_g > 0 \). Since the multipliers are non-negative it follows that \( \bar{\mu}_g > 0 \). Thus \( p_g = \bar{p} \) at the optimum. Likewise the first order conditions with respect to \( p_f \) yield

\[
\lambda y_H + p_g \Delta - c_f = \bar{\mu}_f - \mu_f \quad (3)
\]

Substituting \( \lambda y_H = c_f - \bar{p} \Delta \) and \( p_g = \bar{p} \) into (3) we get

\[
\lambda y_H + p_g \Delta - c_f = c_f - \bar{p} \Delta + \bar{p} \Delta - c_f = 0 \quad (4)
\]

Since the objective function is linear with respect to \( p_f \) when \( p_g \) is held fixed it follows from (4) that any \( p_f \) in the interval \([\underline{p}, \bar{p}]\) is an optimal solution.

Third, let \( \frac{c_f - \bar{p} \Delta}{y_H} < \lambda < 1 - \frac{c_g - \bar{p} \Delta}{y_H} \). Rearranging the first inequality we get \( \lambda y_H > c_f - \bar{p} \Delta \). Substituting this inequality into the first order condition with respect to \( p_f \) we get \( \lambda y_H + p_g \Delta - c_f = \bar{\mu}_f - \mu_f > 0 \). Since the multipliers are non-negative, it follows that \( \bar{\mu}_f > 0 \). The complementary slackness conditions imply that \( p_f = \bar{p} \) at the optimum. Similar reasoning can be used to show that \( p_g = \bar{p} \) at the optimum.

Fourth, let \( \lambda = 1 - \frac{c_g - \bar{p} \Delta}{y_H} \). Using reasoning similar to the second case we can show that \( p_f = \bar{p} \) at the optimum along with any \( p_g \) from the interval \([\underline{p}, \bar{p}]\).

Finally for the fifth case suppose \( 1 - \frac{c_g - \bar{p} \Delta}{y_H} < \lambda \leq 1 \). Using reasoning similar to the first case we can show that \( p_f = \bar{p} \) and \( p_g = p \) at the optimum. [\[14\]]

**Proof of Lemma 1:** Suppose \( w_H \leq w_L \) in equilibrium. Then the firm can change the workers assignment from \( L \) to \( H \) whenever any one of the outcomes
\((f,0),(0,g)\) or \((f,g)\) is realized. Since \(y_H > y_L\), the firm makes a strictly higher profit leading to a contradiction. 

**Proof of Proposition 2:** The proof is divided into two parts. The first part consists of a set of ten claims that rule out all of the strategies besides \((L,L,L,H)\) as skill acquisition equilibrium strategies. In the second part of the proof we establish conditions under which \((L,L,L,H)\) is part of a skill acquisition equilibrium.

**Claim 1** The strategies \((L,L,L,L)\) and \((H,H,H,H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** Suppose \((L,L,L,L)\) or \((H,H,H,H)\) was part of a skill acquisition equilibrium. Then, no matter what the beliefs of the market are, the workers could do strictly better by choosing \((p,p)\) and saving on costs which is a contradiction.


**Proof** Suppose \((H,H,L,L)\) or \((H,H,H,L)\) were equilibrium strategies. Since beliefs of the market have to be consistent with the firm’s and workers strategies it follows that \(w_L \geq (1-\lambda)y_H \geq w_H\). But this contradicts Lemma 1.

Next, suppose \((H,L,L,L)\) or \((L,H,L,L)\) were equilibrium strategies. Since beliefs of the market have to be consistent with the firm’s and workers strategies it follows that \(w_L \geq w_H = 0\) for each of the strategies above. But this contradicts Lemma 1.

Finally consider the strategy \((H,H,L,H)\) and suppose it was an equilibrium strategy. The worker’s expected payoff is

\[
w_H - (1 - p_f)p_g(w_H - w_L) - c_fp_f - c_gp_g
\]

Given this expected payoff and Lemma 1, the worker’s optimal choice must be \(p_g = p\). Since beliefs of the market have to be consistent with the firm’s and workers strategies it follows that \(w_L = (1 - \lambda)y_H\) and \(w_H = \frac{pp_f(1 - \lambda)(y_H + \Delta)}{1 - p(1 - p_f)}\). Taking limits we have

\[
\lim_{\epsilon \to 0} w_L - w_H = (1 - \lambda)y_H > 0
\]

But this contradicts Lemma 1.

**Claim 3** The strategy \((H,L,H,H)\) cannot be part of an equilibrium with skill acquisition.
Proof The worker’s expected payoff given the firm’s strategy \((H, L, H, H)\) is

\[ w_H - p_f(1 - p_g)(w_H - w_L) - c_fp_f - c_gp_g \]

Suppose \(p_f > p\) in equilibrium. From Lemma 1, we know that \(w_H - w_L > 0\). Thus the worker can do strictly better by choosing \(p_f = p\) leading to a contradiction.

Next, suppose \(p_f = p\) and \(p_g > p\) in equilibrium. It follows that \(p_g\) maximizes

\[ w_H - p(1 - p_g)(w_H - w_L) - c_fp - c_gp_g \]

By deviating and choosing \(p_g = p\) the gain for the worker from the deviation is

\[ c_g(p_g - p) \]

and the loss from the deviation is

\[ (p_g - p)p(w_H - w_L) \]

Since \(w_H - w_L\) is bounded above, for \(\epsilon\) sufficiently close to \(\frac{1}{2}\) the gain from the deviation outweighs the loss once again leading to a contradiction.

Claim 4 The strategy \((H, L, H, L)\) cannot be part of an equilibrium with skill acquisition.

Proof The worker’s expected payoff given the firm’s strategy \((H, L, H, L)\) is

\[ w_H - p_f(w_H - w_L) - c_fp_f - c_gp_g \]

Suppose \(p_f > 0\) or \(p_g > 0\) in equilibrium. From Lemma 1, we know \(w_H - w_L > 0\). Thus the worker can do strictly better by choosing \(p_f = p\) and \(p_g = p\) leading to a contradiction.

Claim 5 The strategy \((H, L, L, H)\) cannot be part of an equilibrium with skill acquisition.

Proof The worker’s expected payoff given the firm’s strategy \((H, L, L, H)\) is

\[ w_H - (p_f(1 - p_g) + p_g(1 - p_f))(w_H - w_L) - c_fp_f - c_gp_g \]
Suppose we have an equilibrium with skill acquisition with either $p_f > p$ or $p_g > p$. Consider a deviation where the worker chooses $(p, p)$. The difference in expected payoff from choosing $(p, p)$ is

\[
(p_f(1 - p_g) + p_g(1 - p_f) - 2p(1 - p))(w_H - w_L) + c_f(p_f - p) + c_g(p_g - p)
\]

From Lemma 1, we know that $w_H - w_L > 0$. Thus for $\epsilon$ sufficiently close to $\frac{1}{2}$ the expression above is strictly positive leading to a profitable deviation for the worker which is a contradiction.

**Claim 6** The strategy $(L, L, H, L)$ cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy $(L, L, H, L)$ is

\[
w_L + (1 - p_f)p_g(w_H - w_L) - c_f p_f - c_g p_g
\]

Suppose $p_f > p$ in equilibrium. From Lemma 1, we know that $w_H - w_L > 0$. Thus the worker can do strictly better by choosing $p_f = p$ leading to a contradiction.

Next, suppose $p_f = p$ and $p_g > p$ in equilibrium. Since beliefs are consistent in equilibrium it follows that $w_H = (1 - \lambda)y_H$ and $w_L = \frac{pp_g(1 - \lambda)(y_H + \Delta)}{1 + p_g(p - 1)}$. Now suppose the outcome $(0, g)$ is realized. The firm’s payoff is 0. Whereas if the firm deviates it gets a payoff of

\[
(1 - \lambda)y_L - \frac{pp_g(1 - \lambda)(y_H + \Delta)}{1 + p_g(p - 1)}
\]

Taking limits as $\epsilon$ tends to $\frac{1}{2}$ we see that the firm gets a strictly positive payoff from the deviation leading to a contradiction.

**Claim 7** The strategy $(L, L, H, H)$ cannot be part of an equilibrium with skill acquisition.

**Proof** Suppose $(L, L, H, H)$ was an equilibrium strategy. Since beliefs are consistent in equilibrium it follows that $w_H \geq (1 - \lambda)y_H$ and $w_L = 0$. Now suppose the outcome is $(0, g)$. The firm’s payoff is non positive. Whereas if the firm deviates
it gets a payoff of \((1 - \lambda)y_L > 0\). Thus the firm has a profitable deviation and \((L, L, H, H)\) cannot be an equilibrium strategy.\[\]

Claim 8 The strategy \((L, H, L, H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy \((L, H, L, H)\) is

\[
w_L + p_f(w_H - w_L) - c_fp_f - c_gp_g
\]

Suppose \(p_g > p\) in equilibrium. The worker can do strictly better by choosing \(p_g = p\) leading to a contradiction.

Next, suppose \(p_g = p\) and \(p_f > p\) in equilibrium. By deviating and choosing \(p_f = p\) the worker’s gain from the deviation is

\[
c_f(p_f - p)
\]

whereas the cost from deviating is

\[
(p_f - p)(w_H - w_L)
\]

Since beliefs are consistent in equilibrium it follows that \(w_H = p(1 - \lambda)(y_H + \Delta)\) and \(w_L = p(1 - \lambda)y_H\). Taking limits as \(\epsilon\) tends to \(\frac{1}{2}\) we have \(w_H - w_L\) tending to 0. Thus the gain from deviation strictly dominates the loss leading to a contradiction.\[\]

Claim 9 The strategy \((L, H, H, L)\) cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy \((L, H, H, L)\) is

\[
w_L + (p_f(1 - p_g) + p_g(1 - p_f))(w_H - w_L) - c_fp_f - c_gp_g
\]

Now suppose, \(p_f = \bar{p}\) and \(p_g \neq p\) in equilibrium. Consider a deviation where the worker chooses \((\bar{p}, p)\). The difference in expected payoff from choosing \((\bar{p}, p)\) is

\[
(\bar{p}(1 - p) + p(1 - \bar{p}) - \bar{p}(1 - p_g) - p_g(1 - \bar{p}))(w_H - w_L) + c_g(p_g - p)
\]

From Lemma 1, we know that \(w_H - w_L > 0\). Thus for \(\epsilon\) sufficiently close to \(\frac{1}{2}\) the expression above is strictly positive leading to a profitable deviation for the worker which is a contradiction.
Similar reasoning can be used to show that \( p_g = \tilde{p} \) and \( p_f \neq \tilde{p} \) cannot be an equilibrium strategy.

Now suppose the equilibrium strategy \((p_f, p_g)\) was interior. Since the expected payoff of the worker is linear in \( p_f \) holding \( p_g \) fixed it follows that by choosing \( p_f = \tilde{p} \) and \( p_g \) the worker gets exactly the same payoff. But by the reasoning above the worker can do strictly better by deviating and choosing \((\tilde{p}, \tilde{p})\). Thus an interior solution cannot be an equilibrium as well.

This leaves us with two possible cases to check. First, suppose \( p_f = \tilde{p} \) and \( p_g \neq \tilde{p} \) in equilibrium. Then since beliefs are consistent it follows that

\[
 w_L = \frac{pp_g(1 - \lambda)(y_H + \Delta)}{1 + 2pp_g - \tilde{p} - \tilde{p}_g}
\]

and

\[
 w_H = \frac{p_g(1 - \tilde{p})(1 - \lambda)y_H}{\tilde{p} + \tilde{p}_g - 2pp_g}
\]

Now suppose the outcome is \((0, g)\). Notice that as \( \epsilon \) tends to \( \frac{1}{2} \), the payoff to assigning the worker to \( H \) tends to 0 whereas the payoff to assigning the worker to \( L \) tends to \((1 - \lambda)y_L > 0\). Thus the firm has a profitable deviation leading to a contradiction.

Finally, consider the last case with \( p_g = \tilde{p} \) and \( p_f \neq \tilde{p} \) in equilibrium. Then since beliefs are consistent it follows that

\[
 w_L = \frac{p_f p(1 - \lambda)(y_H + \Delta)}{1 + 2p_f p - p_f - \tilde{p}}
\]

and

\[
 w_H = \frac{(1 - p_f)p(1 - \lambda)y_H}{p_f + \tilde{p} - 2p_f p}
\]

Now suppose the outcome is \((f, g)\). Notice that as \( \epsilon \) tends to \( \frac{1}{2} \), the payoff to assigning the worker to \( H \) tends to \( y_H + \Delta \) whereas the payoff to assigning the worker to \( L \) tends to \( y_L + \Delta \). Thus the firm has a profitable deviation leading to a contradiction. ■
Claim 10 The strategy \((L, H, H, H)\) cannot be part of an equilibrium with skill acquisition.

Proof The worker’s expected payoff given the firm’s strategy \((L, H, H, H)\) is

\[
w_L + (pf + pg - pfpg)(w_H - w_L) - c_fpf - c_gpg
\]

Now suppose, \(pf = \bar{p}\) and \(pg \neq p\) in equilibrium. Consider a deviation where the worker chooses \((\bar{p}, p)\). The difference in expected payoff from choosing \((\bar{p}, p)\) is

\[
(\bar{p}(1-p) + p - \bar{p}(1-pg) - pg)(w_H - w_L) + c_g(pg - p)
\]

From Lemma 1, we know that \(w_H - w_L > 0\). Thus for \(\epsilon\) sufficiently close to \(\frac{1}{2}\), the expression above is strictly positive leading to a profitable deviation for the worker which is a contradiction.

Similar reasoning can be used to show that \(pg = \bar{p}\) and \(pf \neq p\) cannot be an equilibrium strategy.

Now suppose the equilibrium strategy \((pf, pg)\) was interior. Since the expected payoff of the worker is linear in \(pf\) holding \(pg\) fixed it follows that by choosing \(pf = \bar{p}\) and \(pg\) the worker gets exactly the same payoff. But by the reasoning above the worker can do strictly better by deviating and choosing \((\bar{p}, p)\). Thus an interior solution cannot be an equilibrium as well.

This leaves us with two possible cases to check. First, suppose \(pf = \bar{p}\) and \(pg \neq p\) in equilibrium. Then since beliefs are consistent it follows that

\[
w_L = 0
\]

and

\[
w_H = \frac{pg(1-\lambda)(p\Delta + y_H)}{p + pg - pgpg}
\]

Suppose the outcome \((0, g)\) is realized. Then taking limits as \(\epsilon\) tends to \(\frac{1}{2}\), the firm’s expected payoff is 0. By deviating and choosing the strategy \(L\), the firm’s payoff is \((1-\lambda)y_L > 0\). Thus the firm has a profitable deviation.

Now consider the last case with \(pg = \bar{p}\) and \(pf \neq p\) in equilibrium. Then since beliefs are consistent it follows that

\[
w_L = 0
\]
and
\[ w_H = \frac{p(1 - \lambda)(p_f \Delta + y_H)}{p_f + p - p_f p} \]

Notice that
\[ \lim_{\epsilon \to 1/2} w_H - w_L = 0 \]

Also, the worker’s expected payoff from this strategy is
\[ w_L + (p_f + p - p_f p)(w_H - w_L) - c_f p - c_g p \]

Suppose the worker deviates and chooses \((p, p)\) then his expected payoff is
\[ w_L + (2p - p^2)(w_H - w_L) - c_f p - c_g p \]

Taking limits as \(\epsilon\) tends to \(1/2\) we see that the worker gains from a deviation leading to a contradiction. ■

The ten claims above have ruled out all strategies besides \((L, L, L, H)\) as equilibrium strategies with skill acquisition. We now consider the strategy \((L, L, L, H)\).

The worker’s expected payoff given the firm’s strategy \((L, L, L, H)\) is
\[ p_f p_g (w_H - w_L) - c_f p - c_g p \]

Now suppose, \(p_f = \bar{p}\) and \(p_g \neq \bar{p}\) in equilibrium. Then there are two possibilities. First suppose \(p_g\) is interior. Then from the linearity with respect to \(p_g\) it follows that \((\bar{p}, p)\) yields exactly the same expected payoff for the worker and this is given by
\[ \bar{p} p (w_H - w_L) - c_f \bar{p} - c_g p \]

By deviating to \((p, p)\) the worker’s expected payoff is
\[ p p (w_H - w_L) - c_f p - c_g p \]

Taking limits as \(\epsilon\) tends to \(1/2\) we see that the worker does strictly better from the deviation leading to a contradiction. If \(p_g = \bar{p}\) a similar reasoning holds.
Using the same reasoning above we can rule out \( p_g = \bar{p} \) and \( p_f \neq \bar{p} \) as an optimal choice for the worker.

Next suppose the equilibrium strategy \((p_f, p_g)\) was interior. Since the expected payoff of the worker is linear in \( p_f \) holding \( p_g \) fixed it follows that by choosing \( p_f = p \) and \( p_g \) the worker gets exactly the same payoff. But by the reasoning above the worker can do strictly better by deviating and choosing \((\bar{p}, \bar{p})\) leading to a contradiction.

Finally consider the cases \( p_f = p \) with \( p_g \neq p \) or \( p_g = p \) with some \( p_f \neq p \) as equilibrium strategies. Once again, using the reasoning above it can be shown that by deviating to the strategy \((\bar{p}, \bar{p})\) the worker gets a strictly higher payoff which is a contradiction.

Thus the only possibility left to consider for a skill acquisition equilibrium is \((\bar{p}, \bar{p})\). Notice that a necessary and sufficient condition for optimality of \((\bar{p}, \bar{p})\) is

\[
\bar{p}^2 (w_H - w_L) - (c_f + c_g)\bar{p} \geq \bar{p}^2(w_H - w_L) - (c_f + c_g)p
\]

which can be written as

\[
w_H - w_L \geq c_f + c_g \tag{5}
\]

The necessity is straightforward. To see why the condition in (5) is sufficient for optimality consider any \((p_f, p_g)\) besides \((\bar{p}, \bar{p})\) and \((\bar{p}, \bar{p})\). Notice from (5) that we have

\[
(w_H - w_L)(\bar{p}^2 - p_fp_g) \geq (c_f + c_g)(\bar{p}^2 - p_fp_g) \tag{6}
\]

Also for \( \epsilon \) sufficiently close to \( \frac{1}{2} \) we have

\[
(c_f + c_g)(\bar{p}^2 - p_fp_g) \geq c_f(\bar{p} - p_f) + c_g(\bar{p} - p_g) \tag{7}
\]

Combining inequalities (6) and (7) and rearranging we get

\[
\bar{p}^2 (w_H - w_L) - (c_f + c_g)\bar{p} \geq p_fp_g(w_H - w_L) - c_fp_f - c_gp_g
\]

Thus the condition in (5) is sufficient for the optimality of \((\bar{p}, \bar{p})\).

Next, consider the beliefs of the market. Since beliefs are consistent, the following must hold

\[
w_H = (1 - \lambda)(y_H + \Delta)
\]
\[ w_L = \frac{p(1-p)(1-\lambda)y_H}{1-p^2} \]

Thus \( w_H - w_L = \frac{(1-\lambda)(y_H + \Delta(1+p))}{(1+p)} \). Also for \((L,L,L,H)\) to be an equilibrium strategy it must be the case that the firm does not have any incentive to deviate at each of the four outcomes \((0,0), (f,0), (0,g)\) and \((f,g)\). Notice that the firm never deviates from \(L\) to \(H\) when \((0,0)\) is realized because it gets the same output but must pay a higher wage. The conditions that prevent deviations for the outcomes \((f,0), (0,g)\) and \((f,g)\) respectively are

\[ \lambda(y_H - y_L) \leq w_H - w_L \quad (8) \]

and

\[ (1-\lambda)(y_H - y_L) \leq w_H - w_L \quad (9) \]

and

\[ (y_H + \Delta) - (y_L + \Delta) \geq w_H - w_L \quad (10) \]

Substituting \( w_H - w_L \) from above, condition (9) can be written as \( y_H - y_L \leq \frac{y_H + (1+p)\Delta}{(1+p)} \). Also, combining (5), (8) and (10) and rearranging we get

\[ 1 - \frac{(1+p)(y_H - y_L)}{y_H + (1+p)\Delta} \leq \lambda \leq \min\left\{1 - \frac{(1+p)(c_f + c_g)}{y_H + (1+p)\Delta}, \frac{y_H + (1+p)\Delta}{y_H + (1+p)\Delta + (1+p)(y_H - y_L)} \right\} \]

To show that the set of \( \lambda \)'s satisfying the inequality is non empty notice that \( c_f + c_g \leq y_H - y_L \) implies \( 1 - \frac{(1+p)(y_H - y_L)}{y_H + (1+p)\Delta} \leq 1 - \frac{(1+p)(c_f + c_g)}{y_H + (1+p)\Delta} \). Also notice that \( 1 - \frac{(1+p)(y_H - y_L)}{y_H + (1+p)\Delta} \leq \frac{y_H + (1+p)\Delta}{y_H + (1+p)\Delta + (1+p)(y_H - y_L)} \) if and only if \((1+p)^2(y_H - y_L)^2 \geq 0\) which always holds.

**Proof of Proposition 3:** Notice that the derivatives of \( 1 - \frac{(1+p)(c_f + c_g)}{y_H + (1+p)\Delta} \) and \( \frac{y_H + (1+p)\Delta}{y_H + (1+p)\Delta + (1+p)(y_H - y_L)} \) with respect to \( \Delta \) are strictly positive. Thus for any \( \Delta' > \Delta \) it must be the case that

\[ 23 \]
\[
\lambda^*(\Delta) \leq 1 - \frac{(1 + \bar{p})(c_f + c_g)}{y_H + (1 + \bar{p})\Delta} < 1 - \frac{(1 + \bar{p})(c_f + c_g)}{y_H + (1 + \bar{p})\Delta'}
\]

and

\[
\lambda^*(\Delta) \leq \frac{y_H + (1 + \bar{p})\Delta}{y_H + (1 + \bar{p})\Delta + (1 + \bar{p})(y_H - y_L)} < \frac{y_H + (1 + \bar{p})\Delta'}{y_H + (1 + \bar{p})\Delta' + (1 + \bar{p})(y_H - y_L)}
\]

Combining the inequalities above it follows that

\[
\lambda^*(\Delta) < \min\{1 - \frac{(1 + \bar{p})(c_f + c_g)}{y_H + (1 + \bar{p})\Delta'}, \frac{y_H + (1 + \bar{p})\Delta'}{y_H + (1 + \bar{p})\Delta' + (1 + \bar{p})(y_H - y_L)}\} = \lambda^*(\Delta')
\]

Thus \(\lambda^*\) is strictly increasing in \(\Delta\). \(\blacksquare\)

References


