Promotions, Multiskilling and Incentives for Skills*

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Abstract

We study incentives for skill acquisition when skills cannot be contracted on. Our framework has two features. First, workers can acquire both firm specific and general skills at a cost, and second, job assignments signal skill acquisition to a less informed labor market. Our key insight is that each skill performs a different function. The firm specific skill helps in revealing information to a labor market whereas the general skill enforces wage promises from employers. As a consequence, both skills are acquired together in equilibrium, sometimes inefficiently. This endogenous complementarity between skills also yields implications for promotions and job design.

1 Introduction

In many jobs, workers acquire skills, firm specific or general, that are costly to themselves but that benefit their employers. Because of this externality, workers have to be given incentives to acquire skills. But a problem arises here. Skills, such as building relationships with clients and acquiring technical knowledge on the job, are often hard to verify by a third party and are thus difficult to contract on. In particular, an employer who promises to pay the worker for acquiring skills, can renege on his promise. The worker anticipates this and does not acquire skills leading to an inefficiency.

How can incentives for skill acquisition be provided when skills cannot be contracted on? The key is to get employers to commit to pay workers when they acquire skills and the literature suggests a few ways to do this. The first approach relies on repeated interaction where workers can threaten to terminate a relationship when employers renege on their promises (Baker, Gibbons, and Murphy (1994), Levin (2003)). The second approach (Prendergast (1993)) allows for contracts to be written on jobs (as opposed to skills). When higher level jobs are more sensitive to skills, employers have an incentive to promote workers with skills to increase output, and

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are thus forced by the contract to reward the worker. Yet another way for firms to commit to repay workers is the use of up or out contracts (Kahn and Huberman (1988)). When the contract specifies that low skilled workers should be fired, firms have less of an incentive to renege on wage promises. Finally, Scoones and Bernhardt (1998) draw on the existence of an outside labor market and heterogeneity (in managerial ability and match quality) of workers. Worker heterogeneity bids wages up, to reflect, at least partially, the worker’s investment in skills.

We suggest an alternative way in which employers can commit to repay workers and our framework has two main features. First, workers can invest in both firm specific and general skills at a cost. Second, current employers have better information about a worker’s skills which they can signal through job assignments to an outside labor market (Waldman (1984)). The key insight from this framework is that each type of skill performs a different function. The firm specific skill gives a share of the surplus to current employers and thus induces them to reveal their private information about a worker’s skills to the labor market. The general skill, on the other hand, forces the current employer to honor wage promises because of competition in the labor market. Using this insight, we show how skills can be acquired by workers even without legally enforceable contracts, and we derive implications for skill acquisition, promotion patterns and job design.

Our model is based on the principal-agent framework of Prendergast (1993) with a worker, a firm (where the worker is currently hired) and an outside labor market, all of whom are risk neutral. Like Prendergast (1993), we assume that skills cannot be contracted on and that higher level jobs are more sensitive to skills. But we add two new features. First, we assume two types of skills, a firm specific and a general skill. Each type of skill has two outcomes, success or failure, and the worker can influence the probability of success at a cost. Second, we assume that the firm observes skill acquisition whereas the outside labor market only observes job assignments (Waldman (1984)).

Our main result is that firm specific and general skills are complements from an incentive viewpoint: when one type of skill is acquired in equilibrium then so is the other type. Interestingly, this result holds even if the firm specific and general skills are not technological complements. Underlying our main result is the insight that each skill performs a distinct function. The firm specific skill helps information to be revealed to a labor market whereas the general skill enables the market to enforce wage promises by an employer. Because both skills are always acquired together there could be situations in which skills are over accumulated relative to their efficient levels.

\[1\] Tournaments with fixed prizes as in Carmichael (1983) and Malcomson (1984) also act as a commitment device.

\[2\] Unlike Prendergast (1993), we assume that contracts written on jobs cannot be enforced by courts of law. Thus in our framework, the labor market is the only enforcement mechanism. This assumption applies well to settings where enforcement of contracts by a court of law is costly (especially to the worker) or involves significant delay.
We then draw on this endogenous complementarity between skills to suggest implications for promotions and job design. Consider promotions first. While both types of skills have to be acquired together in equilibrium, a worker who views these skills as natural substitutes (through a linear cost function) may have an incentive to acquire just one of them. Thus the role of promotions in our framework is to induce the worker to view the skills as complements. One way to do this is to promote the worker if and only if he acquires both skills. In fact, when the worker is relatively certain of acquiring the skill with effort (he can choose a probability of success on a skill close enough to one), this promotion scheme turns out to be the only way to provide incentives. This result suggests that a generalist in an organization, who acquires both types of skills, should be more likely to be promoted than a specialist who acquires only one type of skill.

Next, consider job design where firms bundle skills into jobs. Starting with Adam Smith there is a large literature which emphasizes the benefits of specialization. We also know that the extent of the market (Smith (1937)), coordination costs (Becker and Murphy (1992)) and skill complementarities (Lindbeck and Snower (2000)) are factors that limit specialization. Our main result on endogenous skill complementarity suggests a different constraint on specialization. When skills differ in their functions (information revelation versus enforcement of wage promises) they may have to be combined together into a job, even though these skills do not interact with one another in the production function.

While technological complementarities are not necessary for both types of skills to be always acquired in equilibrium, they do reinforce a worker’s incentive to acquire skills. In particular, because workers are rewarded if and only if they successfully acquire both skills, technological complementarities increase this contingent reward, giving workers less of an incentive to shirk. This allows firms to design jobs with relatively more value on the firm specific skill (and thereby capture a larger fraction of the surplus) without having to worry about a worker shirking. Thus our framework suggests an indirect incentive role for technological complementarities which is distinct from the existing work on job design (see Gibbs and Levenson (2002) for a review of the job design literature).

A number of papers look at the interaction between skill acquisition and asymmetric information in labor markets. The paper that is closest to ours in this area is Scoones and Bernhardt (1998). They consider a framework with costless skills where heterogenous workers (in managerial ability and match quality) have to choose between a firm specific skill and a general skill (not both) and where current employers have better information about managerial ability. They find that workers can choose the firm specific skill even though this does not directly increase wages. There are two reasons for their result. First, heterogeneity in the labor market bids up wages to reflect at least part of the firm specific skill and second, firms are less likely to

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3See Smith (1937). Also see Roy (1950) and Rosen (1978) for models of job assignments with specialization.
under-promote workers for strategic reasons when the firm specific skill is acquired. Their paper is similar to ours in that less informed external labor markets enforce wage promises. The key difference, however, is the source of commitment; in their paper it is worker heterogeneity whereas in ours it is the presence of a general skill. This different source of commitment leads to a different testable implication: the relationship between the firm specific skill and promotion depends on the level of the general skill in our model whereas in Scoones and Bernhardt (1998) the relationship is independent of the general skill. Also, their framework with worker heterogeneity is better suited to studying issues such as turnover whereas our framework with costly multiple skills is better suited to studying incentives and job design within an organization.

Two other related papers on skill acquisition and asymmetric information in labor markets are Acemoglu and Pischke (1999) and Zabojnik and Bernhardt (2001). Acemoglu and Pischke (1999) show how search and information frictions in the labor market compress wages, which in turn gives the current firm incentives to train workers in general skills. They also show that when firm specific and general skills are technological complements, the existence of a firm specific skill provides incentives for a firm to invest in general training. The key difference in our paper is that the complementarity between firm specific and general skills is endogenous and does not depend on the skills being technological complements. Furthermore, they focus only on the acquisition of general skills within a training context rather than on both types of skills. In Zabojnik and Bernhardt (2001), tournaments convey information about general skill acquisition to an outside labor market and thus provide incentives to workers. As in Acemoglu and Pischke (1999), they only look at the acquisition of general skills and not firm specific skills.

Starting with Becker (1962) and Becker (1964), a large literature studies incentives for investing in specific and general skills, especially within the context of who pays for training. A number of papers also assume asymmetric information about a worker’s productivity. 

Our paper is also related to work on multitasking and multiskilling (Holmstrom and Milgrom (1991), Baker (1992)). This literature suggests that the provision of incentives in multitask settings is difficult because incentives have to be balanced across tasks which differ in the preciseness of their performance measures. Our model, in contrast, suggests that jobs with different types of skills help in the provision of incentives. In this sense, our paper is related to Carmichael and MacLeod (1993) where multiskilling allows firms to commit to retain workers after workers suggest a labor saving innovation. Finally, our result where workers are rewarded if and only if they are successful at both skills has a similar flavor to results in Mac-

Donald and Marx (2001). However, the complementarity of skills in our framework is endogenous.

2 Model

We build on a model by Prendergast (1993) with a risk neutral firm, a risk neutral worker and a risk neutral labor market. There are two skills that the worker can acquire, a firm specific skill and a general skill, with each skill having two outcomes. The outcome for the firm specific skill is denoted by $i$ with $i \in \{0, f\}$ where 0 denotes failure and $f$ denotes success at the firm specific skill. Similarly, the outcome for the general skill is denoted by $j$ with $j \in \{0, g\}$ where 0 denotes failure and $g$ denotes success for the general skill. Thus there are four possible outcomes, failure at both skills $(0, 0)$, success only at the firm specific skill $(f, 0)$, success only at the general skill $(0, g)$ and success at both skills $(f, g)$. The worker can exert effort that is privately observable to influence the probability of success on each skill. To simplify notation, we assume that the worker chooses these probabilities directly. The probability of acquiring the firm specific skill is $p_f \in [p, \bar{p}]$ and the probability of acquiring the general skill is $p_g \in [p, \bar{p}]$ and these probabilities are independent of one another. To ensure that beliefs of the labor market can be determined by Baye’s rule, we assume that $p > 0$ and $\bar{p} < 1$. In the analysis that follows we refer to $p_f$ as the level of the firm specific skill and $p_g$ as the level of the general skill. We assume that the worker has a cost function given by $C(p_f, p_g)$ defined over the domain $[0, 1] \times [0, 1]$ that is strictly positive, strictly increasing, convex and differentiable in the interior of the domain. We also assume that there exists a lower bound $b > 0$ such that for all $p_f$ and $p_g$ in the interior of the domain, 

$$\min\{\frac{\partial C(p_f, p_g)}{\partial p_f}, \frac{\partial C(p_f, p_g)}{\partial p_g}\} \geq b.$$ 

The following definition is also useful in stating the main results of the paper.

**Definition 1.** A worker specializes in a skill if $\max\{p_f, p_g\} > p$ and $\min\{p_f, p_g\} = p$.

The relationship between skills and output depends on the job assignment. There are two jobs, a lower level job, $L$, and a higher level job, $H$, and these jobs vary in their sensitivity to skills. Output for the four possible outcomes is as follows. When the outcome is $(0, 0)$, output is 0. When the worker is only successful at the firm specific skill, output is $\lambda y_L$ for the low job and $\lambda y_H$ for the high job with $0 < y_L < y_H$ and where $\lambda \in [0, 1]$ measures the relative value of firm specific skills to general skills. When the worker is only successful at the general skill, output is $(1-\lambda)y_L$ for the low job and $(1-\lambda)y_H$ for the high job. Finally for the outcome $(f, g)$,
output is given by $y_L + \Delta$ for the low job and $y_H + \Delta$ for the high job with $\Delta > 0$. The notation $\Delta$ measures the extent to which both the skills are complementary. In other words, when we fix a job level, $\Delta$ is the cross partial of expected output with respect to $p_f$ and $p_g$. The table below summarizes the relationship between outcomes and output.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>(0,0)</th>
<th>(f,0)</th>
<th>(0,g)</th>
<th>(f,g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output from Job L</td>
<td>0</td>
<td>$\lambda y_L$</td>
<td>$(1-\lambda)y_L$</td>
<td>$y_L + \Delta$</td>
</tr>
<tr>
<td>Output from Job H</td>
<td>0</td>
<td>$\lambda y_H$</td>
<td>$(1-\lambda)y_H$</td>
<td>$y_H + \Delta$</td>
</tr>
</tbody>
</table>

The timing in the model is as follows. The worker chooses the probabilities of success for both skills which induces a probability distribution over outcomes. The outcomes for both of these skills are then realized. The firm observes these outcomes and decides on the job assignment of the worker. Finally, the labor market observes the job assignment, updates beliefs and offers a wage that is contingent on the assignment. In this setting a strategy for a worker is given by the vector $(p_f, p_g)$. The firm’s strategy is denoted by $(x_0, x_f, x_0, x_f, x_0, x_f)$. The labor market observes job assignments and for each job assigns beliefs to each of the four outcomes. The beliefs associated with each of the outcomes for job $L$ are given by the vector of probabilities $\mu_L = (\mu_{0,0}^L, \mu_{f,0}^L, \mu_{0,g}^L, \mu_{f,g}^L)$ and the beliefs associated with each outcome for job $H$ are given by the vector of probabilities $\mu_H = (\mu_{0,0}^H, \mu_{f,0}^H, \mu_{0,g}^H, \mu_{f,g}^H)$. Wages offered by the labor market are denoted by $w_L$ when job $L$ is observed and $w_H$ when job $H$ is observed.

3 Efficiency

We start our analysis by characterizing efficient levels of skill acquisition. This characterization serves as a benchmark. Let $(p^{eff}_f, p^{eff}_g)$ denote the efficient level of skill acquisition. Then $(p^{eff}_f, p^{eff}_g)$ is the optimal solution to the following problem.

$$\max_{p_f \in [\bar{p}, \bar{p}], p_g \in [\bar{p}, \bar{p}]} p_f (1 - p_g) \lambda y_H + (1 - p_f) p_g (1 - \lambda) y_H + p_f p_g (y_H + \Delta) - C(p_f, p_g)$$

To make the comparison easier with the section on equilibrium skill acquisition, we restrict our attention in this section to a linear cost function, $C(p_f, p_g) = c_f p_f + c_g p_g$ where $c_f$ and $c_g$ are strictly positive constants. We also make the following assumption to ensure that the efficient solution always sets at least one of the skill levels above the minimum level.

Assumption 1. $y_H > c_f + c_g$
The following proposition characterizes the efficient skill acquisition levels. The proofs of all of the propositions and lemmas are in the Appendix.

**Proposition 1.** Let $C(p_f, p_g) = c_f p_f + c_g p_g$ where $c_f$ and $c_g$ are strictly positive constants. Then the efficient levels of skill acquisition are given by

$$p_{eff}^f = \begin{cases} 
  p & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda \in \left[0, \frac{c_f - \bar{p}\Delta}{yH}\right) \\
  [p, \bar{p}] & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda = \frac{c_f - \bar{p}\Delta}{yH} \\
  \bar{p} & \text{if } \Delta \leq \frac{c_f}{p} \text{ and } \lambda \in \left(\frac{c_f - \bar{p}\Delta}{yH}, 1\right] \\
  \bar{p} & \text{if } \Delta > \frac{c_f}{p}
\end{cases}$$

and

$$p_{eff}^g = \begin{cases} 
  \bar{p} & \text{if } \Delta > \frac{c_g}{p} \\
  \bar{p} & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda \in \left[0, 1 - \frac{c_g - \bar{p}\Delta}{yH}\right) \\
  [p, \bar{p}] & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda = 1 - \frac{c_g - \bar{p}\Delta}{yH} \\
  p & \text{if } \Delta \leq \frac{c_g}{p} \text{ and } \lambda \in \left(1 - \frac{c_g - \bar{p}\Delta}{yH}, 1\right]
\end{cases}$$

Proposition 1 highlights two factors that determine efficient skill levels: technological complementarities and the relative importance of both skills. When technological complementarities are high, both skills are acquired at their maximum level. When technological complementarities are low, the relative value of each skill plays an important role. If both skills are important (for intermediate values of $\lambda$), efficiency requires a worker to choose the highest skill level for both skills. If one skill is more important relative to another (around close to 0 or 1), efficiency requires an extreme form of specialization, the highest skill level for the more valuable skill and the lowest skill level for the less valuable skill. Notice that these corner solutions arise because of the linear specification for the cost function. Figure 1 depicts the efficient level of skill acquisition with $\lambda$ on the horizontal axis and $p_{eff}^f$ and $p_{eff}^g$ on the vertical axis when technological complementarities are not too high.
4 Equilibrium

The objective in this section is to compare equilibrium skill levels with the efficient skill levels above. The equilibrium concept we use is Weak Perfect Bayesian Equilibrium and we restrict our attention to pure strategies.

Definition 2. A Weak Perfect Bayesian Equilibrium with skill acquisition level $\delta$ consists of a strategy profile $((p_f, p_g), (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))$, a belief system for the labor market $(\mu^L, \mu^H)$, a pair of wages, one for each job, $(w_L, w_H)$, and a constant $\delta$ with $p - \delta \leq p$, such that the following conditions hold:\footnote{This definition of equilibrium assumes that employers in the labor market have access to the same technology as the current employer and that Bertrand competition results in wages equalling expected output.}

1. The strategy profile $((p_f, p_g), (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))$ is sequentially rational given the wages offered by the market, $(w_L, w_H)$.

2. The belief system $(\mu^L, \mu^H)$ is derived, using Baye’s rule wherever possible, from the strategy profile $((p_f, p_g), (x_{0,0}, x_{f,0}, x_{0,g}, x_{f,g}))$.

3. Competition between the firm and the labor market ensures that the worker is paid his expected output in the labor market. That is $w_L = (1 - \lambda)(\mu^L_{0,g} y_H + \mu^L_{f,g} (y_H + \Delta))$ and $w_H = (1 - \lambda)(\mu^H_{0,g} y_H + \mu^H_{f,g} (y_H + \Delta))$.

4. $\max\{p_f, p_g\} \geq \delta$. 

Figure 1: Efficient Levels of Skill Acquisition.
Henceforth, we refer to a Weak Perfect Bayesian Equilibrium as an equilibrium. Also since $\delta$ is held fixed in the analysis, we refer to a skill acquisition equilibrium with skill level $\delta$ as a skill acquisition equilibrium. The following lemma places restrictions on wages in equilibrium.

**Lemma 1.** Suppose the worker is assigned to job $L$ for any of the outcomes $(f,0)$, $(0,g)$ or $(f,g)$ in equilibrium. Then $w_H \geq w_L$. The inequality is strict if $\lambda \in (0,1)$.

This lemma says that if a worker is assigned to a low job in equilibrium when he is successful at acquiring any skill, it must be the case that wages for the higher level job are greater than wages for the lower level job. The reason for this is that higher level jobs are more sensitive to skills. If $w_L$ was strictly greater than $w_H$, then the firm could always promote the worker, get at least as much output, pay a lower wage and do strictly better.

The next proposition states the main result of the paper, that firm specific and general skills are complements from an incentive viewpoint.

**Proposition 2.** Fix $\delta$ and let $p$ be sufficiently small. Then in a skill acquisition equilibrium, the worker never specializes in a skill.

Proposition 2 says that firm specific skills and general skills are always acquired together in a skill acquisition equilibrium. The interesting feature of this result is that it holds even if the skills are not technological complements ($\Delta = 0$). The logic underlying this result is that both skills perform different functions for the firm. Suppose the firm promoted the worker for the outcome $(0,g)$ and the worker only invested in general skills. Then because $p$ is sufficiently small, the market places significant weight on the outcome $(0,g)$ when it observes the assignment $H$ leading to high wages relative to output for this outcome. The firm can then do better by not promoting the worker for the outcome $(0,g)$. On the other hand, suppose the worker only invested in the firm specific skill. Because $p$ is sufficiently small, the market places very low weight on the general skill being acquired, driving the worker’s wage to 0, and thus taking away his incentive to invest in costly skills.

Next, we study incentives for acquiring skills together and focus on the interesting case where workers view skills as natural substitutes through a linear cost function given by $C(p_f,p_g) = c_f p_f + c_g p_g$ where $c_f$ and $c_g$ are strictly positive constants. To keep things clean, we also normalize the lower and upper limits of the skill levels so that they add up to 1. Thus let $p = \frac{1}{2} - \epsilon$ and $\bar{p} = \frac{1}{2} + \epsilon$ where $0 < \epsilon < \frac{1}{2}$. Also define $\bar{c} = \max\{c_f + c_g, \frac{c_f}{p}, \frac{c_g}{\bar{p}}\}$.

**Proposition 3.** Let $C(p_f,p_g) = c_f p_f + c_g p_g$ where $c_f$ and $c_g$ are strictly positive constants. Also fix $\delta$. Then there exists a skill acquisition equilibrium with skill levels
$(\bar{p}, \bar{p})$ and a promotion scheme $(L, L, L, H)$ if and only if the following condition holds

$$\bar{c} \leq y_H - y_L \leq \frac{y_H + (1 + \bar{p})\Delta}{(1 + \bar{p})}$$

(1)

and $\lambda$ lies in the range specified below.

$$\lambda \in \left[1 - \frac{(1 + \bar{p})(y_H - y_L)}{y_H + (1 + \bar{p})\Delta}, \min\{1 - \frac{(1 + \bar{p})\bar{c}}{y_H + (1 + \bar{p})\Delta}, \frac{y_H + (1 + \bar{p})\Delta}{y_H + (1 + \bar{p})\Delta + (1 + \bar{p})(y_H - y_L)}\} \right].$$

For $\epsilon$ sufficiently close $\frac{1}{2}$, this skill acquisition equilibrium is unique.

We know from Proposition 2 that for $p$ sufficiently small, skills must be acquired together in equilibrium. The linear cost function in Proposition 3 clearly highlights the incentive problem associated with the joint acquisition of skills. Because costs are linear, corner solutions imply that when skills are acquired together, they are acquired at their maximal levels. However, linear costs also lead to the agent viewing the skills as natural substitutes. Promotion schemes must thus induce the agent to view the skills as complements. This is precisely what the promotion scheme $(L, L, L, H)$ does. In fact when agents are relatively certain about acquiring a skill with their effort ($\bar{p}$ is sufficiently close to 1) this is the only way to provide incentives for acquiring both skills. When skill acquisition is less certain there could be another way to induce both skills via the scheme $(L, H, H, H)$. The reason for this is that a lower $\bar{p}$ limits the weight that an agent can place on the outcomes $(f, 0)$ and $(0, g)$, thereby restricting specialization.

It is also useful to carefully examine the role of condition (1) and the restriction on $\lambda$ in guaranteeing existence of the skill acquisition equilibrium described in Proposition 3. Condition (1) places lower and upper bounds on the extent to which job $H$ is more sensitive to skills relative to job $L$. The lower bound ensures that there are a range of $\lambda$'s for which firms have an incentive to promote workers when the outcome $(f, g)$ is realized and for which workers have an incentive to acquire the maximum level of skills. On the other hand, $y_H - y_L$ should be low enough to deter firms from promoting workers when the outcome $(0, g)$ is realized. From (1), there always exist a range of $\lambda$'s for which the skill levels $(\bar{p}, \bar{p})$ and the scheme $(L, L, L, H)$ is part of a skill acquisition equilibrium.

Finally, we can use Proposition 3 to relate equilibrium skill levels with their efficient counterparts for any given $\lambda$. Because the worker bears all the costs of skill acquisition and the firm gets all of the benefits we would typically expect an under accumulation of skills relative to the efficient level in equilibrium. As Proposition
Figure 2: Equilibrium Levels of Skill Acquisition: \( y_H - y_L \) low.

3 indicates, this is true when the relative value of firm specific skills is high (\( \lambda \) is sufficiently close to 1). The reason for this under accumulation is that wages are not bid up enough by the market to offset costs of the worker. There is also an under accumulation of skills when the relative value of the general skill is high (1 - \( \lambda \) is high) and jobs do not vary much in their sensitivity to skills (when \( y_H - y_L \) is low). The reason here is that firms have little incentive to promote workers because the share of the surplus that the firm can extract is small relative to the output gain. What is unusual about Proposition 3 is that there can be an over accumulation of skills. When jobs vary substantially in their sensitivity to skills (when \( y_H - y_L \) is high) it is possible to have both skills being acquired in equilibrium even though efficiency requires only the general skill. The reason for this over accumulation is that both skills have to be acquired together in equilibrium. Figure’s 2 and 3 depict equilibria where \( y_H - y_L \) is low and high respectively. In the following section we build on Proposition 3 to see how firms can design jobs using technological complementarities.

\[ \frac{(1 + \hat{p})(y_H - y_L)}{y_H + (1 + \hat{p})\Delta} \leq \min\{1 - \frac{(1 + \hat{p}_{c})}{y_H + (1 + \hat{p})\Delta}, \frac{y_H + (1 + \hat{p})\Delta}{y_H + (1 + \hat{p})\Delta + (1 + \hat{p})(y_H - y_L)}\} \]

\[ \frac{(1 + \tilde{p})(y_H - y_L)}{y_H + (1 + \tilde{p})\Delta} \leq \min\{1 - \frac{(1 + \tilde{p}_{c})}{y_H + (1 + \tilde{p})\Delta}, \frac{y_H + (1 + \tilde{p})\Delta}{y_H + (1 + \tilde{p})\Delta + (1 + \tilde{p})(y_H - y_L)}\} \]

\[ \frac{(1 + \tilde{p})(y_H - y_L)}{y_H + (1 + \tilde{p})\Delta} \leq \min\{1 - \frac{(1 + \tilde{p}_{c})}{y_H + (1 + \tilde{p})\Delta}, \frac{y_H + (1 + \tilde{p})\Delta}{y_H + (1 + \tilde{p})\Delta + (1 + \tilde{p})(y_H - y_L)}\} \]

\[ \frac{(1 + \hat{p})(y_H - y_L)}{y_H + (1 + \hat{p})\Delta} \leq \min\{1 - \frac{(1 + \hat{p}_{c})}{y_H + (1 + \hat{p})\Delta}, \frac{y_H + (1 + \hat{p})\Delta}{y_H + (1 + \hat{p})\Delta + (1 + \hat{p})(y_H - y_L)}\} \]

Note that for any \( \lambda \in [0, 1] \) there exist beliefs, such that the pooling strategy where firms promote the worker for all of the outcomes is an equilibrium. Thus when \( \lambda \in \frac{(1 + \hat{p})(y_H - y_L)}{y_H + (1 + \hat{p})\Delta}, \frac{y_H + (1 + \hat{p})\Delta}{y_H + (1 + \hat{p})\Delta + (1 + \hat{p})(y_H - y_L)} \) there are multiple equilibria. Figure 2 only focuses on the skill acquisition equilibria within this interval.
5 Complementarities and Job Design

Two parameters that are of interest to the firm in our model are $\lambda$, which denotes the relative importance of the firm specific to the general skill, and $\Delta$, which specifies the extent to which the skills are technological complements. So far, we have assumed that $\lambda$ and $\Delta$ are exogenous. However, we can think of a setting where firms can design jobs by choosing skills with different $\lambda$'s and $\Delta$'s prior to the worker choosing skill levels.

We know that if the assumptions in Proposition 3 hold then there is a unique equilibrium with skill acquisition where the worker chooses the maximum level of both skills. The expected payoff to the firm from the equilibrium is given by

$$\bar{p}(1 - \bar{p})(y_L - (1 - \lambda)y_H) + \bar{p}^2 \lambda(y_H + \Delta)$$

Notice that this expression is strictly increasing in $\lambda$. Thus the firm would like to set $\lambda$ as large as possible to extract as much surplus as it can. Let $\lambda^*(\Delta)$ be a function that specifies the largest possible $\lambda$ that is consistent with a skill acquisition equilibrium. From Proposition 3, we know that $\lambda^*(\Delta) = \min\{1 - \frac{(1 + \bar{p})c}{y_H + (1 + \bar{p})\Delta}, \frac{y_H + (1 + \bar{p})\Delta}{y_H + (1 + \bar{p})\Delta + (1 + \bar{p})(y_H - y_L)}\}$. The following proposition states that technological complementarities allow a firm to extract a larger share of the surplus.

**Proposition 4.** $\lambda^*$ is strictly increasing in $\Delta$.

There are two channels through which technological complementarities between skills increase profits for a firm. The first is the direct effect which increases output.
by $\Delta$ when both skills are acquired together. Much of the literature on job design focusses on this channel (see Gibbs and Levenson (2002) for a review of the job design literature). A second channel, which Proposition 4 emphasizes, is through the fraction of surplus a firm can extract. To see how this channel works, notice that there are two constraints on the value of the firm specific skill relative to the general skill. First, the more valuable the firm specific skill is relative to the general skill, the less wages get bid up. Thus a worker has greater incentive to deviate and acquire no skills. Second, the more valuable firm specific skills are relative to general skills, firms are more likely to deviate and promote workers when the outcome $(f,0)$ is realized. Increasing the degree to which both skills are technological complements helps to relax both of these constraints. For the first constraint, notice that because workers are rewarded if and only if they acquire both skills, technological complementarities give very strong incentives for a worker to collect both skills. Firms then can use these strong incentives to extract a larger share of the surplus without worrying about workers shirking. For the second constraint, technological complementarities increase the wage for job $H$ and once again act as a deterrent to promote workers when $(f,0)$ is realized.

6 Conclusion

We study incentives for skill acquisition when skills cannot be contracted on. The problem here is that firms find it difficult to commit to repay workers when they acquire costly skills. Our framework has two features which allows firms to commit. First, there are two types of skills, a firm specific skill and a general skill and workers can acquire both at a cost. Second, job assignments signal skill acquisition to a less informed labor market. In this framework each type of skill plays a distinct role; firm specific skills give a larger share of the surplus to current employers and thus reveal information to a labor market whereas general skills, via competition in the labor market, bid wages up, forcing current employers to honor their wage promises.

We find three main results. First, both types of skills are complements from an incentive viewpoint; if one skill is acquired in equilibrium then so is the other type. This leads to a potential over accumulation of skills. Second, when workers view skills as natural substitutes (they have linear cost functions) they are promoted if and only if they successfully acquire both skills. Finally, we study job design and show how technological complementarities allow firms to capture a larger share of the surplus.

To be sure, there are other ways to provide incentives for skills that cannot be contracted on: legally enforceable contracts can be written on jobs (Prendergast (1993)), firms can strategically under promote workers based on their natural abilities (Scoones and Bernhardt (1998)) and firms can resort to up or out contracts (Kahn and Huberman (1988)). Our challenge lies in identifying settings where each of these commitment mechanisms are used. One way to think about this question is
to introduce a benefit from specialization into the model. Using multiple skills as a commitment device is then costly and firms may have to resort to other mechanisms such as up or out contracts. We leave this as an area for future research.
Appendix

Proof of Proposition 1: The first order necessary conditions for the optimal solution are

\[
\left( \frac{\lambda y_H + p_g \Delta - c_f}{(1 - \lambda) y_H + p_f \Delta - c_g} \right) = \mu_f \left( \begin{array}{c} 1 \\ 0 \end{array} \right) + \mu_g \left( \begin{array}{c} -1 \\ 0 \end{array} \right) + \bar{\mu}_f \left( \begin{array}{c} 0 \\ 1 \end{array} \right) + \bar{\mu}_g \left( \begin{array}{c} 0 \\ -1 \end{array} \right)
\]

where \( \bar{\mu}_f \) is the non negative multiplier associated with the constraint \( p_f \leq \bar{p} \) and \( \mu_f \) is the non negative multiplier associated with the constraint \( p_f \geq \bar{p} \). Likewise, \( \bar{\mu}_g \) and \( \mu_g \) are non negative multipliers associated with the inequality constraints for \( p_g \).

In the following claim we establish that the efficient skill level has at least one of the skills at its maximum level.

Claim The efficient level of skills has either \( p_f = \bar{p} \) or \( p_g = \bar{p} \) or both.

Proof Suppose the optimal solution had \( p_f < \bar{p} \) and \( p_g < \bar{p} \). Then the first order conditions imply \( \lambda y_H + p_g \Delta - c_f \leq 0 \) and \( (1 - \lambda) y_H + p_f \Delta - c_g \leq 0 \). Adding both inequalities and rearranging we get \( y_H + (p_g + p_f) \Delta \leq c_f + c_g \). But this contradicts Assumption 1.  

Now suppose \( \Delta > \frac{c_f}{\bar{p}} \) and suppose to the contrary that \( p_f < \bar{p} \) at the optimum. From the claim above it follows that \( p_g = \bar{p} \) at the optimum. Since \( \Delta > \frac{c_f}{\bar{p}} \) the first order condition with respect to \( p_f \) can be written as \( \lambda y_H + \bar{p} \Delta - c_f = \bar{\mu}_f - \mu_f > 0 \).

From the complementary slackness conditions we have \( p_f = \bar{p} \) at the optimum which is a contradiction. Similarly when \( \Delta > \frac{c_g}{\bar{p}} \) we must have \( p_g = \bar{p} \) at the optimum.

Next suppose \( \Delta \leq \frac{c_f}{\bar{p}} \) and \( \Delta \leq \frac{c_g}{\bar{p}} \) and consider the following partition of the interval \([0, 1]\) into five cases.

First, let \( 0 \leq \lambda < \frac{c_f - \bar{p} \Delta}{y_H} \). Rearranging the second inequality we get \( \lambda y_H + \bar{p} \Delta - c_f < 0 \). Combining this inequality with the constraint \( p_g \leq \bar{p} \), we can write the first order condition with respect to \( p_f \) as \( \lambda y_H + p_g \Delta - c_f = \bar{\mu}_f - \mu_f < 0 \). Since the multipliers are non-negative, it follows that \( \mu_f > 0 \). The complementary slackness conditions yield

\[
\left( \frac{\lambda y_H + p_g \Delta - c_f}{(1 - \lambda) y_H + p_f \Delta - c_g} \right) = \mu_f \left( \begin{array}{c} 1 \\ 0 \end{array} \right) + \mu_g \left( \begin{array}{c} -1 \\ 0 \end{array} \right) + \bar{\mu}_f \left( \begin{array}{c} 0 \\ 1 \end{array} \right) + \bar{\mu}_g \left( \begin{array}{c} 0 \\ -1 \end{array} \right)
\]
conditions imply that \( p_f = p \) at the optimum. It follows from the claim above that \( p_g = \bar{p} \) at the optimum.

Second, let \( \lambda = \frac{c_f - \bar{p}\Delta}{y_H} \). Rearranging, we get \(-\lambda y_H = \bar{p}\Delta - c_f\). Adding \( y_H \) to both sides of the equality we get

\[
(1 - \lambda)y_H = y_H + \bar{p}\Delta - c_f
\]  \hspace{1cm} (2)

Using equation (2) and Assumption 1, the first order condition with respect to \( p_g \) is

\[
\lambda y_H + p_g\Delta - c_f = \bar{p}\Delta - c_f + \mu_f - \mu_g = 0
\]  \hspace{1cm} (3)

Substituting \( \lambda y_H = c_f - \bar{p}\Delta \) and \( p_g = \bar{p} \) into (3) we get

\[
\lambda y_H + p_g\Delta - c_f = c_f - \bar{p}\Delta + \bar{p}\Delta - c_f = 0
\]  \hspace{1cm} (4)

Since the objective function is linear with respect to \( p_f \) when \( p_g \) is held fixed it follows from (4) that any \( p_f \) in the interval \([p, \bar{p}]\) is an optimal solution.

Third, let \( \frac{c_f - \bar{p}\Delta}{y_H} < \lambda < 1 - \frac{c_g - \bar{p}\Delta}{y_H} \). Rearranging the first inequality we get \( \lambda y_H > c_f - \bar{p}\Delta \). Substituting this inequality into the first order condition with respect to \( p_f \) we get \( \lambda y_H + p_g\Delta - c_f = \mu_f - \mu_g > 0 \). Since the multipliers are non-negative, it follows that \( \mu_f > 0 \). The complementary slackness conditions imply that \( p_f = \bar{p} \) at the optimum. Similar reasoning can be used to show that \( p_g = \bar{p} \) at the optimum.

Fourth, let \( \lambda = 1 - \frac{c_g - \bar{p}\Delta}{y_H} \). Using reasoning similar to the second case we can show that \( p_f = \bar{p} \) at the optimum along with any \( p_g \) from the interval \([p, \bar{p}]\).

Finally for the fifth case suppose \( 1 - \frac{c_g - \bar{p}\Delta}{y_H} < \lambda \leq 1 \). Using reasoning similar to the first case we can show that \( p_f = \bar{p} \) and \( p_g = p \) at the optimum.

\begin{proof}
Suppose to the contrary that the firm assigns the worker to job \( L \) for any of the outcomes \((f,0), (0,g), \) and \((f,g)\) in equilibrium with \( w_H < w_L \).
\end{proof}
Then the firm can change the workers assignment from \( L \) to \( H \) whenever any one of the outcomes \( (f,0), (0,g) \) or \( (f,g) \) is realized. Since \( y_H > y_L \) the firm makes a strictly higher profit leading to a contradiction. Similar reasoning can be used to establish the strict inequality, \( w_H > w_L \), when \( \lambda \in (0,1) \).

**Proof of Proposition 2:** The proof of the proposition is divided into a series of claims. In the first nine claims, we rule out all strategies for a firm besides \((L,H,H,H)\) and \((L,L,L,H)\) as potential skill acquisition equilibrium strategies. The remaining two claims establish that when either \((L,H,H,H)\) or \((L,L,L,H)\) is part of a skill acquisition equilibrium, the worker cannot specialize. Note for this proof that we only consider the case with \( \lambda < 1 \). When \( \lambda = 1 \), the worker’s wage, irrespective of the job assignment, always equals 0 and the worker has no incentive to acquire costly skills.

**Claim 1** The strategies \((L,L,L,L)\) and \((H,H,H,H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** Suppose \((L,L,L,L)\) or \((H,H,H,H)\) was part of a skill acquisition equilibrium. Then, no matter what the beliefs of the market are, the workers could do strictly better by choosing \((p,p)\) and saving on costs which is a contradiction.


**Proof** Suppose \((H,H,L,L)\) or \((H,H,H,L)\) were equilibrium strategies. Since beliefs of the market have to be consistent with the firm’s and worker’s strategies it follows that \( w_L \geq (1 - \lambda)y_H > w_H \). But this contradicts Lemma 1.

Next, suppose \((H,L,L,L)\) or \((L,H,L,L)\) were equilibrium strategies. Since beliefs of the market have to be consistent with the firm’s and worker’s strategies it follows that \( w_L > w_H = 0 \) for each of the strategies above. But this contradicts Lemma 1.

**Claim 3** The strategies \((H,L,H,H)\) and \((H,H,L,H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** Consider the firm’s strategy, \((H,L,H,H)\), first. The worker’s expected payoff given the firm’s strategy \((H,L,H,H)\) is

\[
w_H - p_f(1 - p_g)(w_H - w_L) - C(p_f,p_g)
\]

Suppose \( p_f > p \) in equilibrium. From Lemma 1, we know that \( w_H - w_L \geq 0 \). Since \( C \) is strictly increasing, the worker can do strictly better by choosing \( p_f = p \) leading to a contradiction.
Next, suppose \( p_f = p \) and \( p_g \geq \delta \) in equilibrium. If the worker deviates and chooses \((p, p)\) the difference in the worker’s expected payoff is

\[-(p_g - p)p(w_H - w_L) + C(p, p) - C(p, p)\]

Let \( p \) be sufficiently small so that \( p(y_H + \Delta) < b \). Then

\[ C(p, p) - C(p, p) \geq (p_g - p)b > (p_g - p)p(y_H + \Delta) \geq (p_g - p)p(w_H - w_L) \]

where the first inequality follows from the fact that \( C \) is convex with partial derivatives bounded below by \( b \) and the last inequality follows from the fact that \( w_H - w_L \) is bounded above by \( y_H + \Delta \). Thus the worker does strictly better from the deviation leading to a contradiction.

Similar reasoning can be used to show that \((H, H, L, H)\) cannot be part of a skill acquisition equilibrium.

**Claim 4** The strategy \((H, L, H, L)\) cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy \((H, L, H, L)\) is

\[ w_H - p_f(w_H - w_L) - C(p_f, p_g) \]

Suppose \( p_f \geq \delta \) or \( p_g \geq \delta \) in equilibrium. From Lemma 1, we know \( w_H - w_L \geq 0 \). Since \( C \) is strictly increasing, the worker can do strictly better by choosing \( p_f = p \) and \( p_g = p \) leading to a contradiction.

**Claim 5** The strategy \((H, L, L, H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy \((H, L, L, H)\) is

\[ w_H - (p_f(1 - p_g) + p_g(1 - p_f))(w_H - w_L) - C(p_f, p_g) \]

Suppose we have an equilibrium with skill acquisition with either \( p_f \geq \delta \) or \( p_g \geq \delta \). Without any loss of generality assume \( p_f \geq \delta \). If the worker deviates and chooses \((p, p)\) the difference in the worker’s expected payoff is

\[ (p_f(1 - p_g) + p_g(1 - p_f) - 2p(1 - p))(w_H - w_L) + C(p_f, p_g) - C(p, p) \]

Set \( p \) sufficiently small so that \((\delta - p)b > 2p(1 - p)(y_H + \Delta)\). Then
\[ C(p_f, p_g) - C(p, p) \geq (\delta - p)b > 2p(1 - p)(y_H + \Delta) \geq 2p(1 - p)(w_H - w_L) \]

where the first inequality follows from the fact that \( C \) is convex with partial derivatives bounded below by \( b \) and the last inequality follows from the fact that \( w_H - w_L \) is bounded above by \( y_H + \Delta \). Since \( w_H - w_L \geq 0 \) from Lemma 1 and since \( p_f(1 - p_g) + p_g(1 - p_f) \geq 0 \) it follows that the worker does strictly better from the deviation leading to a contradiction.\[ \square \]

**Claim 6** The strategy \((L, L, H, L)\) cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy \((L, L, H, L)\) is

\[ w_L + (1 - p_f)p_g(w_H - w_L) - C(p_f, p_g) \]

Suppose \( p_f > p \) in equilibrium. From Lemma 1, we know that \( w_H - w_L \geq 0 \). Since \( C \) is strictly increasing, the worker can do strictly better by choosing \( p_f = p \) leading to a contradiction.

Next, suppose \( p_f = p \) and \( p_g \geq \delta \) in equilibrium. Since beliefs are consistent in equilibrium it follows that \( w_H = (1 - \lambda)y_H \) and \( w_L = \frac{pp_g(1 - \lambda)(y_H + \Delta)}{1 + p_g(p - 1)} \). If the firm deviates from \( H \) to \( L \) when the outcome \((0, g)\) is realized the difference in expected payoff is given by

\[ (1 - \lambda)(y_L - \frac{pp_g(y_H + \Delta)}{1 + p_g(p - 1)}) \]

Let \( p \) be sufficiently small so that \( y_L - \frac{p\bar{p}(y_H + \Delta)}{1 + \bar{p}(p - 1)} > 0 \). Then since \( \frac{pp_g(y_H + \Delta)}{1 + p_g(p - 1)} \) is strictly increasing in \( p_g \) it follows that the firm always has a profitable deviation, leading to a contradiction.\[ \square \]

**Claim 7** The strategy \((L, L, H, H)\) cannot be part of an equilibrium with skill acquisition.

**Proof** Suppose \((L, L, H, H)\) was an equilibrium strategy. Since beliefs are consistent in equilibrium it follows that \( w_H \geq (1 - \lambda)y_H \) and \( w_L = 0 \). Now suppose the
outcome is $(0,g)$. The firm’s payoff is non positive. Whereas if the firm deviates it gets a payoff of $(1 - \lambda)y_L > 0$. Thus the firm has a profitable deviation and $(L, L, H, H)$ cannot be an equilibrium strategy.\[\square\]

**Claim 8** The strategy $(L, H, L, H)$ cannot be part of an equilibrium with skill acquisition.

**Proof** The worker’s expected payoff given the firm’s strategy $(L, H, L, H)$ is

$$w_L + p_f(w_H - w_L) - C(p_f, p_g)$$

Suppose $p_g > p$ in equilibrium. Since $C$ is strictly increasing, the worker can do strictly better by choosing $p_g = p$ leading to a contradiction.

Next, suppose $p_g = p$ and $p_f \geq \delta$ in equilibrium. If the worker deviates and chooses $(p, p)$ the difference in the worker’s expected payoff is

$$-(p_f - p)(w_H - w_L) + C(p_f, p) - C(p, p)$$

Since beliefs are consistent in equilibrium it follows that $w_H = p(1 - \lambda)(y_H + \Delta)$ and $w_L = p(1 - \lambda)y_H$. Thus the difference in expected payoff can be written as

$$-(p_f - p)(1 - \lambda)p\Delta + C(p_f, p) - C(p, p)$$

Let $p$ be sufficiently small so that $(1 - \lambda)p\Delta < b$. Then

$$C(p_f, p) - C(p, p) \geq (p_f - p)b > (p_f - p)(1 - \lambda)p\Delta$$

where the first inequality follows from the fact that $C$ is convex with partial derivatives bounded below by $b$. Thus the worker does strictly better from the deviation leading to a contradiction.\[\square\]

**Claim 9** The strategy $(L, H, H, L)$ cannot be part of an equilibrium with skill acquisition.

**Proof** Suppose to the contrary that $(L, H, H, L)$ was part of an equilibrium with skill acquisition. Then since the firm should be optimizing at the outcomes $(f, 0), (0, g)$, and $(f, g)$, it follows that the following condition must hold

$$y_H - y_L \leq w_H - w_L \leq \min\{\lambda(y_H - y_L), (1 - \lambda)(y_H - y_L)\}$$

Notice that the inequality above cannot hold which is a contradiction.\[\square\]
**Claim 10** If \((L, H, H, H)\) is part of a skill acquisition equilibrium then \(\min\{p_f, p_g\} > \frac{1}{2}\).

**Proof** The worker’s expected payoff given the firm’s strategy \((L, H, H, H)\) is

\[
w_L + (p_f + p_g - p_fp_g)(w_H - w_L) - C(p_f, p_g)
\]

First, suppose \(p_f = \frac{1}{2}\) and \(p_g \geq \delta\) in equilibrium. Then since beliefs are consistent it follows that \(w_L = 0\) and \(w_H = \frac{p_g(1 - \lambda)(p\Delta + y_H)}{p + p_g - pp_g}\). If the firm deviates from \(H\) to \(L\) when the outcome \((0, g)\) is realized the difference in expected payoff is given by

\[
(1 - \lambda)(y_L - (\frac{p_g(p\Delta + y_H)}{p + p_g - pp_g}))
\]

Let \(p\) be sufficiently small so that \(y_H - \frac{p_g(p\Delta + y_H)}{p + \delta - pp_g} < y_L\). Then since \(\frac{p_g(p\Delta + y_H)}{p + p_g - pp_g}\) is strictly increasing in \(p_g\) it follows that the firm always has a profitable deviation, leading to a contradiction.

Next consider the case with \(p_g = \frac{1}{2}\) and \(p_f \geq \delta\) in equilibrium. If the worker deviates and chooses \((p, p)\) the difference in the worker’s expected payoff is

\[
-(1 - p)(p_f - p)(w_H - w_L) + C(p_f, p) - C(p, p)
\]

Since beliefs are consistent it follows that \(w_L = 0\) and \(w_H = \frac{p(1 - \lambda)(p_f\Delta + y_H)}{p_f + p - p_fp_f}\).

Thus the difference in expected payoff can be written as

\[
-(1 - p)(p_f - p)\frac{p(1 - \lambda)(p_f\Delta + y_H)}{p_f + p - p_fp_f} + C(p_f, p) - C(p, p)
\]

Let \(p\) be sufficiently small so that \((1 - p)\frac{p(1 - \lambda)(\delta\Delta + y_H)}{\delta + p - \delta p} < b\) and so that \(p\Delta - y_H(1 - p) < 0\). Then
\[
C(p_f, p) - C(p_f, p) \geq (p_f - p)b > (1 - p)(p_f - p)\frac{p(1 - \lambda)(\delta \Delta + y_H)}{\delta + p - \delta p} \geq (1 - p)(p_f - p)\frac{p(1 - \lambda)(p_f \Delta + y_H)}{p_f + p - p_f p}
\]

where the first inequality follows from the fact that \(C\) is convex with partial derivatives bounded below by \(b\) and the last inequality follows from the fact that \(p(1 - \lambda)(p_f \Delta + y_H)\) is decreasing in \(p_f\) when \(p\Delta - y_H(1 - p) < 0\). Thus the worker does strictly better from the deviation leading to a contradiction.

**Claim 11** If \((L, L, L, H)\) is part of a skill acquisition equilibrium then \(\min\{p_f, p_g\} > p\).

**Proof** The worker’s expected payoff given the firm’s strategy \((L, L, L, H)\) is

\[
p_f p_g (w_H - w_L) - C(p_f, p_g)
\]

First suppose \(p_f = p\) and \(p_g \geq \delta\) in equilibrium. If the worker deviates and chooses \((p, \delta)\) the difference in the worker’s expected payoff is

\[-p(p_g - p)(w_H - w_L) + C(p, p_g) - C(p, p)\]

Let \(p\) be sufficiently small so that \(p(y_H + \Delta) < b\). Then

\[C(p, p_g) - C(p, p) \geq (p_g - p)b > (p_g - p)p(y_H + \Delta) \geq (p_g - p)p(w_H - w_L)\]

where the first inequality follows from the fact that \(C\) is convex with partial derivatives bounded below by \(b\) and the last inequality follows from the fact that \(w_H - w_L\) is bounded above by \(y_H + \Delta\). Thus the worker does strictly better from the deviation leading to a contradiction.

Similar reasoning can be used to rule out \(p_f \geq \delta\) and \(p_g = p\). ■

**Proof of Proposition 3:** From the proof of Proposition 2, we can restrict our attention to the strategies \((L, H, H, H)\) and \((L, L, L, H)\) when looking for skill acquisition equilibria.

Consider the strategy \((L, H, H, H)\) first. The worker’s expected payoff given the firm’s strategy \((L, H, H, H)\) is

\[
w_L + (p_f + p_g - p_f p_g)(w_H - w_L) - c_f p_f - c_g p_g
\]

\[22\]
We know from Proposition 2 that there cannot be skill acquisition equilibria with the minimal skill level acquired for any of the skills. Consider the remaining cases.

First, suppose \( p_f = \bar{p} \) and \( p_g \neq \bar{p} \) in equilibrium. Consider a deviation where the worker chooses \((\bar{p}, p)\). The difference in expected payoff from choosing \((\bar{p}, p)\) for the worker is

\[-(1 - \bar{p})(p_g - \bar{p})(w_H - w_L) + c_g(p_g - \bar{p})\]

Set \( \bar{p} \) sufficiently close to 1 (or \( \epsilon \) sufficiently close to \( \frac{1}{2} \)) so that \((1 - \bar{p})(y_H + \Delta) < c_f\). Then since \( w_H - w_L \) is bounded above by \( y_H + \Delta \) it follows that the worker can do strictly better by deviating leading to a contradiction.

Similar reasoning can be used to show that \( p_g = \bar{p} \) and \( p_f \neq p \) cannot be an equilibrium strategy.

Finally suppose the equilibrium strategy \((p_f, p_g)\) was interior. Since the expected payoff of the worker is linear in \( p_f \) holding \( p_g \) fixed it follows that \((\bar{p}, p)\) yields exactly the same expected payoff for the worker and this is given by

\[-pp_g(p_g - \bar{p})(w_H - w_L) + c_g(p_g - \bar{p})\]

If the worker deviates and chooses \((\bar{p}, p)\) the difference in the worker’s expected payoff is
Let $p$ be sufficiently small ($\epsilon$ be sufficiently close to $\frac{1}{2}$) so that $p(y_H + \Delta) < c_f$. Then the worker can do strictly better from the deviation leading to a contradiction.

Using the same reasoning above we can rule out $p_g = \bar{p}$ and $p < p_f \neq \bar{p}$ as an optimal choice for the worker.

Next suppose the equilibrium strategy $(p_f, p_g)$ was interior. Since the expected payoff of the worker is linear in $p_f$ holding $p_g$ fixed it follows that by choosing $p_f = \bar{p}$ and $p_g$ the worker gets exactly the same payoff. But by the reasoning above the worker can do strictly better by deviating and choosing $(\bar{p}, \bar{p})$ leading to a contradiction.

Thus the only possibility left to consider for a skill acquisition equilibrium is $(\bar{p}, \bar{p})$. Notice that a worker chooses $(\bar{p}, \bar{p})$ if and only if for all $(p_f, p_g)$ the following condition holds

\[-p(p - \bar{p})(w_H - w_L) + c_f(p - \bar{p})\]

Since the condition above must hold for all $(p_f, p_g)$, it must hold for the following cases, i) $p_f = \bar{p}$, $p_g = \bar{p}$, ii) $p_f < \bar{p}$, $p_g = \bar{p}$, iii) $p_f = \bar{p}$, $p_g < \bar{p}$, iv) $p_f < \bar{p}$, $p_g < \bar{p}$. Substituting the values of skills in the first three cases gives us the following conditions respectively

\[w_H - w_L \geq c_f + c_g\]  \hspace{1cm} (5)

\[w_H - w_L \geq \frac{c_f}{\bar{p}}\]  \hspace{1cm} (6)

\[w_H - w_L \geq \frac{c_g}{\bar{p}}\]  \hspace{1cm} (7)

For case iv) notice that because the agents expected payoff is linear in one skill when the other is fixed it follows that condition (6) is sufficient for $(\bar{p}, \bar{p})$ to be optimal.

Next, consider the beliefs of the market. Since beliefs are consistent, the following must hold

\[w_H = (1 - \lambda)(y_H + \Delta)\]
Thus \( w_{H} - w_{L} = \frac{(1 - \lambda)(y_{H} + \Delta(1 + \bar{p}))}{(1 + \bar{p})} \). Also, for \((L, L, L, H)\) to be an equilibrium strategy it must be the case that the firm does not have any incentive to deviate at each of the four outcomes \((0, 0)\), \((f, 0)\), \((0, g)\) and \((f, g)\). Notice that the firm never deviates from \(L\) to \(H\) when \((0, 0)\) is realized because it gets the same output but must pay a higher wage. The conditions that prevent deviations for the outcomes \((f, 0)\), \((0, g)\) and \((f, g)\) respectively are

\[
\lambda(y_{H} - y_{L}) \leq w_{H} - w_{L} \tag{8}
\]

and

\[
(1 - \lambda)(y_{H} - y_{L}) \leq w_{H} - w_{L} \tag{9}
\]

Substituting \( w_{H} - w_{L} \) from above, condition (9) can be written as

\[
y_{H} + (1 + \bar{p})\Delta \leq y_{H} + (1 + \bar{p})\Delta + (1 + \bar{p})(y_{H} - y_{L}) \tag{10}
\]

To show that the set of \( \lambda \)'s satisfying the inequality is non empty notice that

\[
\bar{c} \leq y_{H} - y_{L} \text{ implies } 1 - \frac{(1 + \bar{p})(y_{H} - y_{L})}{y_{H} + (1 + \bar{p})\Delta} \leq 1 - \frac{(1 + \bar{p})\bar{c}}{y_{H} + (1 + \bar{p})\Delta} \tag{11}
\]

Also notice that

\[
1 - \frac{(1 + \bar{p})(y_{H} - y_{L})}{y_{H} + (1 + \bar{p})\Delta} \leq \frac{y_{H} + (1 + \bar{p})\Delta}{y_{H} + (1 + \bar{p})\Delta + (1 + \bar{p})(y_{H} - y_{L})} \tag{12}
\]

if and only if \((1 + \bar{p})^{2}(y_{H} - y_{L})^{2} \geq 0 \) which always holds. 

**Proof of Proposition 4:** Notice that the derivatives of \( 1 - \frac{(1 + \bar{p})\bar{c}}{y_{H} + (1 + \bar{p})\Delta} \) and \( \frac{y_{H} + (1 + \bar{p})\Delta}{y_{H} + (1 + \bar{p})\Delta + (1 + \bar{p})(y_{H} - y_{L})} \) with respect to \( \Delta \) are strictly positive. Thus for any \( \Delta' > \Delta \) it must be the case that

\[
1 - \frac{(1 + \bar{p})\bar{c}}{y_{H} + (1 + \bar{p})\Delta} \leq \min\{1 - \frac{(1 + \bar{p})\bar{c}}{y_{H} + (1 + \bar{p})\Delta}, \frac{y_{H} + (1 + \bar{p})\Delta}{y_{H} + (1 + \bar{p})\Delta + (1 + \bar{p})(y_{H} - y_{L})}\}.
\]
\[ \lambda^*(\Delta) \leq 1 - \frac{(1 + \bar{p})c}{y_H + (1 + \bar{p})\Delta} < 1 - \frac{(1 + \bar{p})c}{y_H + (1 + \bar{p})\Delta'} \]

and

\[ \lambda^*(\Delta) \leq \frac{y_H + (1 + \bar{p})\Delta}{y_H + (1 + \bar{p})\Delta + (1 + \bar{p})(y_H - y_L)} < \frac{y_H + (1 + \bar{p})\Delta'}{y_H + (1 + \bar{p})\Delta' + (1 + \bar{p})(y_H - y_L)} \]

Combining the inequalities above it follows that

\[ \lambda^*(\Delta) < \min \left\{ 1 - \frac{(1 + \bar{p})c}{y_H + (1 + \bar{p})\Delta'}, \frac{y_H + (1 + \bar{p})\Delta'}{y_H + (1 + \bar{p})\Delta' + (1 + \bar{p})(y_H - y_L)} \right\} = \lambda^*(\Delta') \]

Thus \( \lambda^* \) is strictly increasing in \( \Delta \).■

References


