Web Appendix A- Proofs

Proof of Lemma 1: Suppose the optimal contract has $\Delta f_g - \Delta p_g < 0$ with $t_g^* \in \{0, 1\}$. Then consider an alternative contract (f_g', p_g', n_g') with $f_g' = n_g' = p_g' = p_g$. It follows that $\Delta f_g' - \Delta p_g' = 0$. From (MH_g) we know that this contract can implement $t_g = \frac{1}{2}$ which strictly increases the principals revenue without changing any of the costs. Also the incentive compatibility conditions that prevent either specialist from taking a generalist's contract and (IR_g) are not affected. But this contradicts the optimality of (f_g, p_g, n_g). A similar argument can rule out implementing any time other than $t_g = \frac{1}{2}$.

Now consider the case of a specialist. The first thing to notice for both specialist is that $V_s(f_{s1}, p_{s1}, n_{s1}; t_{s1}^*) = V_s(f_{s2}, p_{s2}, n_{s2}; t_{s2}^*)$. Otherwise the incentive compatibility conditions that prevent one specialist from taking the other specialist's contract never hold (because specialists are symmetric).

Now suppose the optimal contract for either specialist sets $\Delta f_s - \Delta p_s > \beta - \alpha$ with $t_s^* < 1$. Define $u_1 = V_s(f_s, p_s, n_s; t_s^*)$. It follows from (IR_s) that $u_1 \geq u_0$. Then consider a new contract (f_s', p_s', n_s') with $\Delta f_s' - \Delta p_s' = 0$ and $V_s(f_s', p_s', n_s'; 1) = V_s(f_s, p_s, n_s; t_s^*) = u_1$. Let $t_s^* = 1 - \epsilon$ where $\epsilon > 0$. Then as u is strictly concave we have

$$u(\epsilon(1-\epsilon)f_s + \epsilon(1-\epsilon)n_s + ((1-\epsilon)^2 + \epsilon^2)p_s) > \epsilon(1-\epsilon)u(f_s) + \epsilon(1-\epsilon)u(n_s) + ((1-\epsilon)^2 + \epsilon^2)u(p_s)$$

$$= u_1 + \alpha(1-\epsilon) + \beta\epsilon$$

Because u is strictly increasing we have

$$\epsilon(1-\epsilon)f_s + \epsilon(1-\epsilon)n_s + ((1-\epsilon)^2 + \epsilon^2)p_s > u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon)$$

It follows that

$$\pi(f_s, p_s, n_s; t_s^*) = \epsilon(1-\epsilon)(Q_h - 2Q_l) + Q_l - C(f_s, p_s, n_s; t_s^*)$$

$$< \epsilon(1-\epsilon)(Q_h - 2Q_l) + Q_l - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon) \qquad (A3)$$

Using Assumption 2 the right hand side of $(A3)$ satisfies

$$\epsilon(1-\epsilon)(Q_h - 2Q_l) + Q_l - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon)$$

$$\leq Q_l + \epsilon(1-\epsilon)\frac{\beta - \alpha}{u'(u^{-1}(u_1 + \alpha))} - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon) \qquad (A4)$$

Define $u^{-1} = h$ and let $g(\epsilon) = u_1 + \alpha(1-\epsilon) + \beta\epsilon$. Using the chain rule we have $h \circ g'(0) = \frac{\beta - \alpha}{u'(u^{-1}(u_1 + \alpha))}$. We can therefore write the right hand side of $(A4)$ as

$$Q_l + \epsilon(1-\epsilon)\frac{\beta - \alpha}{u'(u^{-1}(u_1 + \alpha))} - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon)$$

$$= Q_l + \epsilon(1-\epsilon)h \circ g'(0) - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon) \qquad (A5)$$

Because $h \circ g$ is convex, combining the inequalities $(A3)$, $(A4)$, and $(A5)$ we have

$$\pi(f_s, p_s, n_s; t_s^*) < Q_l + \epsilon(1-\epsilon)h \circ g'(0) - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon)$$

$$\leq Q_l + \frac{\epsilon(1-\epsilon)(h \circ g(\epsilon) - h \circ g(0))}{\epsilon} - u^{-1}(u_1 + \alpha(1-\epsilon) + \beta\epsilon)$$

As $\epsilon < 1$, it follows that

$$\pi(f_s, p_s, n_s; t_s^*) < Q_l - u^{-1}(u_1 + \alpha) = \pi(f_s', p_s', n_s'; 1)$$

Thus (f_s', p_s', n_s') yields higher profits than (f_s, p_s, n_s) and all the four incentive compatibility conditions are satisfied because $V_s(f_s', p_s', n_s'; 1) = u_1$. This contradicts the optimality of (f_s, p_s, n_s).$\blacksquare$

Proof of Lemma 2: Suppose the optimal contract sets $f_g > n_g$. Consider a contract (f_g', p_g', n_g') with $p_g' = p_g$ and $\Delta f_g' - \Delta p_g' = \Delta f_g - \Delta p_g$ and $f_g' = n_g'$. Then it follows that $f_g' = n_g' = u^{-1}(\frac{u(f_g) + u(n_g)}{2})$. This new contract does not affect any of the constraints of the problem. As u is strictly concave we have $u(\frac{f_g + n_g}{2}) > \frac{1}{2}u(f_g) + \frac{1}{2}u(n_g)$ which implies $f_g + n_g > 2u^{-1}(\frac{u(f_g) + u(n_g)}{2}) = f_g' + n_g'$. From Lemma 1 we know that the optimal time allocation for a generalist is $\frac{1}{2}$. Thus $C(f_g, p_g, n_g, \frac{1}{2}) = \frac{1}{2}(f_g + n_g) + \frac{1}{2}p_g > \frac{1}{2}(f_g' + n_g') + \frac{1}{2}p_g = C(f_g', p_g', n_g', \frac{1}{2})$ and hence the principal can earn higher profits which is a contradiction.$\blacksquare$

Proof of Lemma 3: From Lemma 1 we know that the optimal contract sets $t_s = 1$. Thus the incentive compatibility conditions that deal with both types of specialists are given by

$$u(p_{s1}) - \alpha \geq u(p_{s2}) - \alpha$$

and

$$u(p_{s2}) - \alpha \geq u(p_{s1}) - \alpha$$

So $p_{s1} = p_{s2}$.$\blacksquare$

Proof of Lemma 4: Suppose to the contrary that (f_g, p_g, n_g) is an optimal contract with (IR_g) slack. Then the principal can design a new contract (f_g', p_g', n_g') with , $u(f_g') = u(f_g) - \epsilon$, $u(p_g') = u(p_g) - \epsilon$ and $u(n_g') = u(n_g) - \epsilon$. This implies $\Delta f_g' - \Delta p_g' = \Delta f_g - \Delta p_g$ and thus (IC_{sg}) holds. Also for ϵ small enough (IR_g) also holds. Thus the principal can reduce costs without changing the allocation of time and thus earn's higher profits. But this contradicts the optimality of (f_g, p_g, n_g). It follows that (IR_g) must bind for an optimal contract.■

Proof of Proposition 6: Suppose the optimal contract $(f_{s1}, p_{1s1}, p_{2s1}, n_{s1})$ has $t_{s1} < 1$ for specialist 1. Let the expected utility associated with this contract for a specialists at task 1 be given by u_1. Using exactly the same reasoning as in Lemma 1 we can show that an alternative contract given by $(f'_{s1}, p'_{1s1}, p'_{2s1}, n'_{s1})$ with $f'_{s1} = p'_{2s1} = n'_{s1} = 0$ and

$$V_{s1}(f'_{s1}, p'_{1s1}, p'_{2s1}, n'_{s1}; 1) = u_1 \tag{A6}$$

gives the principal higher profits. It has to be checked that a specialist at task 2 does not find this contract more attractive. Notice that from (A6) it follows that $u(p'_{1s1}) = u_1 + \alpha$. So if a specialist at task 2 takes this contract then he gets an expected utility of $u_1 + (\alpha - \beta)$. Let u'_1 be the utility that a specialist at task 2 gets from choosing the contract $(f_{s1}, p_{1s1}, p_{2s1}, n_{s1})$. Then a sufficient condition for $(IC_{s2,s1})$ to hold is $u_1 - u'_1 \leq \beta - \alpha$. Suppose the specialist at task 2 chooses exactly the same time allocation as a specialist for task 1 then

$$u_1 - u'_1(t_{s1}) = (2t_{s1} - 1)(\beta - \alpha) \leq \beta - \alpha$$

Because a specialist at task 2 can choose any other time allocation it follows that $u_1 - u'_1 \leq \beta - \alpha$. Thus $(IC_{s2,s1})$ holds for the new contract. The new contract is more profitable and satisfies all incentive constraints leading to a contradiction. Thus $t_{s1} = 1$. A similar proof applies for a specialist at task 2.

Now consider the case of a generalist. His expected utility function can be written down as

$$V_g(f_g, p_{1g}, p_{2g}, n_g; t_g) = t_g(1 - t_g)(u(f_g) + u(n_g)) + t_g^2 u(p_{1g}) + (1 - t_g)^2 u(p_{2g}) - 1$$

Using similar reasoning to that of Lemma 1 it follows that $f_g = n_g$. Now suppose the optimal contract $(f_g, p_{1g}, p_{2g}, n_g)$ sets $f_g \leq max\{p_{1g}, p_{2g}\}$ and suppose without any loss of generality that $p_{1g} > p_{2g}$. Then it follows from the moral hazard constraint for a generalist that $t_g = 1$. The principal's profit in this case is

$$\frac{1}{2}(Q_l - u^{-1}(u_0 + 1)) + \frac{1}{4}\pi_{s1} + \frac{1}{4}\pi_{s2} \tag{A7}$$

where $Q_l - u^{-1}(u_0 + \alpha) \geq \pi_{s2}$ and $Q_l - u^{-1}(u_0 + \alpha) \geq \pi_{s1}$. Consider an alternative contract $(f'_g, p'_{1g}, p'_{2g}, n'_g)$ with $f'_g = p'_{1g} = p'_{2g} = n'_g = u^{-1}(u_0 + 1)$. This contract yields a total profit to the principal of

$$\frac{1}{2}(\frac{1}{4}Q_h + \frac{1}{2}Q_l - u^{-1}(u_0 + 1)) + \frac{1}{2}(Q_l - u^{-1}(u_0 + 1)) \tag{A8}$$

Notice that as Q_h gets sufficiently large the expression in $(A8)$ exceeds the expression in $(A7)$ which results in a contradiction. This leaves us with two possible cases. In the first case $f_g > max\{p_{1g}, p_{2g}\}$ and $p_{1g} = p_{2g}$ in which case we are done. In the second case $f_g = p_{1g} = p_{2g} = n_g$. But we know from Proposition 5 that this contract is not optimal. Thus it follows that the optimal contract sets $f_g > max\{p_{1g}, p_{2g}\}$.■

Proof of Lemma 5: The proof is divided into a series of claims.

Claim 1 If $t'_s > 0$ and the specialist's moral hazard constraint holds then $t_s + t'_s = 1$.

Proof Let $t'_s > 0$ and suppose a specialist's moral hazard constraint holds. Suppose to the contrary that $t_s + t'_s < 1$. Then from (3) it must be the case that

$$\Delta f_s - \Delta p_s > 0 \tag{A9}$$

If not then by setting $t'_s = 0$ a specialist can strictly increase his utility. Likewise from (3) we must have

$$t_s(\Delta p_s - \alpha) + t'_s(\Delta p_s - \beta) + t_s t'_s(\Delta f_s - \Delta p_s) \geq 0 \tag{A10}$$

Otherwise a specialist could put in no time in either task and strictly increase his utility. Now consider a new time allocation that scales each time allocation by a factor $\epsilon > 1$ and such that $\epsilon(t_s + t'_s) < 1$. From (3), ($A9$) and ($A10$) we see that this new time allocation strictly increases a specialist's utility leading to a contradiction. ■

Claim 1 is useful because it tells us that a specialist chooses one of three options. The first is to spend all his time on his good task. The second is to spend a unit of time, with positive amounts on both tasks and the third is to not work at all.

Using Claim 1 and using (2) and (3), we can show that the moral hazard constraints for a generalist and specialist can be written as follows.

First in the case of a generalist we have

$$t_g = \frac{1}{2}, t'_g = \frac{1}{2} \text{ if } \Delta f_g - \Delta p_g > 0 \text{ and } \Delta p_g - 1 \geq 0$$

$$t_g = [0,1], t'_g = 1 - t_g \text{ if } \Delta f_g - \Delta p_g = 0 \text{ and } \Delta p_g - 1 \geq 0$$

$$t_g \in \{0,1\}, t'_g = 1 - t_g \text{ if } \Delta f_g - \Delta p_g < 0 \text{ and } \Delta p_g - 1 \geq 0$$

$$t_g = \frac{1}{2}, t'_g = \frac{1}{2} \text{ if } \Delta f_g - \Delta p_g > 0, \ \Delta p_g - 1 < 0 \text{ and } \Delta f_g - \Delta p_g \geq 4(1 - \Delta p_g)$$

$$t_g = 0, t'_g = 0 \text{ for all other cases}$$

Likewise the moral hazard constraint for a specialist can be written as

$$t_s = \frac{1}{2}(1 + \frac{(\beta - \alpha)}{(\Delta f_s - \Delta p_s)}), \ t'_s = 1 - t_s \text{ if } \Delta f_s - \Delta p_s > \beta - \alpha \text{ and } \Delta p_s - \alpha \geq 0$$

$$t_s = 1, t'_s = 0 \text{ if } \Delta f_s - \Delta p_s \leq \beta - \alpha \text{ and } \Delta p_s - \alpha \geq 0$$

$$t_s = \frac{1}{2}(1 + \frac{(\beta - \alpha)}{(\Delta f_s - \Delta p_s)}),\ t'_s = 1 - t_s \text{ if } \Delta f_s - \Delta p_s > \beta - \alpha \text{ and}$$

$$\frac{1}{2}(\alpha + \beta - \frac{(\beta - \alpha)^2}{\Delta f_s - \Delta p_s} - \frac{(\Delta f_s - \Delta p_s)^2}{2} + \frac{(\beta - \alpha)^2}{2}) \leq \Delta p_s < \alpha$$

$$t_s = 0, t'_s = 0 \text{ for all other cases}$$

The next six claims deal with the problem of a generalist and the last claim focusses on the problem of a specialist.

Claim 2: Any contract that implements $t_g = \frac{1}{2}$ and $t_g + t'_g = 1$ sets $f_g > n_g$ and $f_g > p_g$.

Proof There are two possible cases to consider. Consider the first case with $\Delta f_g - \Delta p_g \geq 0$ and $\Delta p_g \geq 1$. It follows that $\Delta f_g \geq \Delta p_g \geq 1$. Using the definition of Δf_g and Δp_g we have $u(f_g) - u(p_g) \geq u(p_g) - u(n_g) \geq 1$. This in turn implies $u(f_g) \geq u(p_g) + 1 \geq u(n_g) + 2$. Thus $f_g > n_g$ and $f_g > p_g$.

Now consider the second case with $\Delta f_g - \Delta p_g > 0$ and $\Delta p_g < 1$ and $\Delta f_g - \Delta p_g \geq 4(1 - \Delta p_g)$. There are two sub cases. First, if $\Delta p_g \geq 0$ then $\Delta f_g > 0$ and the result follows. The second sub case has $\Delta p_g < 0$. Because $\Delta f_g \geq 4 - 3\Delta p_g$ we have $\Delta f_g \geq 4 + 3|\Delta p_g| > |\Delta p_g| > 0$. This implies that $f_g > p_g$ and $f_g > n_g$. ■

Claim 3: The individual rationality constraint for the generalist binds.

Proof Suppose the generalist's individual rationality constraint does not bind. Then the principal can reduce utility equally across all outcomes by a very small amount. This does not change any of the incentives and thus leaves the time allocations unchanged. But the new contract is less costly thereby increasing profits. Thus the individual rationality constraint for a generalist must bind. ■

Claim 4: $u(n_g) \leq u_0$.

Proof Suppose $u(n_g) > u_0$. Then by setting $t_g = t'_g = 0$ a generalist can always guarantee himself a payoff strictly greater than his reservation utility. But this contradicts Claim 3. Thus it must be the case that $u(n_g) \leq u_0$.■

Claim 5: The least cost contract that implements $t_g = \frac{1}{2}$ and $t_g + t'_g = 1$ sets $u(n_g) = u_0$.

Proof Suppose the least cost contract sets $u(n_g) < u_0$. Then there are three possible cases.

First consider the case where $\Delta f_g - \Delta p_g \geq 0$ and $\Delta p_g - 1 > 0$. Consider a new contract (f'_g, p'_g, n'_g) such that $u(f'_g) = u'_{fg} = u(f_g) - \epsilon$, $u(n'_g) = u'_{ng} = u(n_g) + \epsilon$ and $u(p'_g) = u'_{pg} = u(p_g)$. This keeps the generalist at exactly his same level of utility with incentive complementarities remaining unchanged. For ϵ sufficiently small, a generalist's time allocation and the principal's revenue also stay the same. As $f_g > n_g$ and u^{-1} is strictly convex it follows that

$$u^{-1}(u'_{ng}) - u^{-1}(u_{ng}) < u^{-1}(u_{fg}) - u^{-1}(u'_{fg})$$

which implies

$$t_g(1-t_g)u^{-1}(u'_{fg}) + (t_g^2 + (1-t_g)^2)u^{-1}(u'_{pg}) + t_g(1-t_g)u^{-1}(u'_{ng})$$

$$< t_g(1-t_g)u^{-1}(u_{fg}) + (t_g^2 + (1-t_g)^2)u^{-1}(u_{pg}) + t_g(1-t_g)u^{-1}(u_{ng})$$

Thus a principal reduces his cost, leading to a contradiction.

For the second case consider $\Delta f_g - \Delta p_g > 0$ and $\Delta p_g - 1 = 0$. Once again consider a new contract (f'_g, p'_g, n'_g) such that $u(f'_g) = u(f_g) - \epsilon$, $u(p'_g) = u(p_g)$ and $u(n'_g) = u(n_g) + \epsilon$ where $\epsilon > 0$. For ϵ sufficiently small it will be the case that $\Delta f_g - \Delta p_g \geq 4(1 - \Delta p_g)$ and thus a generalists time allocation and utility remains exactly the same. Once again as illustrated above a principal can increase his profits leading to a contradiction.

A similar argument holds for the case where $\Delta f_g - \Delta p_g > 0$, $\Delta p_g - 1 < 0$ and $\Delta f_g - \Delta p_g > 4(1 - \Delta p_g)$.

■

Claim 6: The optimal time allocation for a generalist sets $t_g = t'_g = \frac{1}{2}$.

Proof Suppose $t_g \neq \frac{1}{2}$. Then from the moral hazard constraint it follows that contracts must satisfy one of the following three cases. First, it could be the case that $\Delta f_g - \Delta p_g = 0$ and $\Delta p_g - 1 = 0$ with an interior $t_g \neq \frac{1}{2}$. From Claim 3 we know that $n_g = u^{-1}(u_0)$, $p_g = u^{-1}(u_0 + 1)$, and $f_g = u^{-1}(u_0 + 2)$. In this case the principal's profit can be written as

$$t_g(1-t_g)Q_h+(t_g^2+(1-t_g)^2)Q_l-t_g(1-t_g)u^{-1}(u_0+2)+(t_g^2+(1-t_g)^2)u^{-1}(u_0+1)+t_g(1-t_g)u^{-1}(u_0)$$

Differentiating with respect to t_g we get

$$(1 - 2t_g)(Q_h - 2Q_l - (u^{-1}(u_0 + 2) + u^{-1}(u_0) - 2u^{-1}(u_0 + 1)))$$

The first order conditions imply that the optimal solution sets $t_g = \frac{1}{2}$. From Assumption 3 it follows that the second order conditions hold as well. Thus by leaving the contract as it is and choosing $t_g = \frac{1}{2}$, the principal can increase profits, leading to a contradiction.

Now consider the other two cases where t_g is either 0 or 1. In this case the profit is $Q_l - u^{-1}(u_0 + 1)$. Once again we can see that by setting $u(n_g) = u_0$, $u(p_g) = u_0 + 1$ and $u(f_g) = u_0 + 2$ the principal earns a higher profit from Assumption 3.

■

Claim 7: The optimal contract for a generalist is $\Delta f_g - \Delta p_g = 0$ and $\Delta p_g - 1 = 0$.

Proof Suppose the cost minimizing contract set$\Delta f_g - \Delta p_g > 0$ and $\Delta p_g < 1$. Consider a new contract (f_g', p_g', n_g') with $u(f_g') = u(f_g) - \epsilon = u_{fg}'$, and $u(p_g') = u(p_g) + \epsilon' = u_{pg}'$, and $n_g' = n_g$, such that a generalist is at the same level of utility. It follows that $\epsilon = 2\epsilon'$. Define a new function g which is linear and passes through the coordinates $(u_{fg}, u^{-1}(u_{fg}))$ and $(u_{pg}, u^{-1}(u_{pg}))$. As u^{-1} is strictly convex, it follows that

$$\frac{1}{4}(u^{-1}(u_{fg}) - u^{-1}(u_{fg} - 2\epsilon')) > \frac{1}{4}(g(u_{fg}) - g(u_{fg} - 2\epsilon'))$$

$$= \frac{1}{2}(g(u_{pg} + \epsilon') - g(u_{pg})) > \frac{1}{2}(u^{-1}(u_{pg} + \epsilon') - u^{-1}(u_{pg}))$$

Because the time allocations for a generalist remain unchanged for the new contract, it follows that the new contract is less costly without changing the revenue of the principal from a generalist. This is a contradiction. From Claim 3 it must be the case that $n_g = u^{-1}(u_0)$, $p_g = u^{-1}(u_0 + 1)$, and $f_g = u^{-1}(u_0 + 2)$. ■

Claim 8: The optimal time allocation for a specialist is $t_s = 1$.

Proof This is because the full information contract and allocation can be implemented in this case by setting $f_g = n_g = 0$ and $p_g = u^{-1}(u_0 + \alpha)$. ■

Proof of Proposition 7: Once again we split this proposition into a series of claims.

Claim 1: The individual rationality constraint for a generalist always binds.

Proof Suppose not. Then the principal can always reduce utilities equally across outcomes for a generalist by a small amount without violating any of the constraints or changing the incentives. This reduces the expected wages paid to a generalist and the information rents paid to a specialist which increases overall profits. ■

Claim 2: $u(n_g) \leq u_0$.

Proof Suppose not. Then the generalist's IR constraint does not bind. ■

Claim 3: The least cost contract that implements $t_g = \frac{1}{2}$ and $t_g + t'_g = 1$ sets $u(n_g) = u_0$.

Proof First consider the case where $\Delta f_g - \Delta p_g \geq 0$ and $\Delta p_g - 1 > 0$. Suppose $u(n_g) < u_0$. From Claim 5 in Lemma 5 we know that the new contract (f'_g, p'_g, n'_g) such that $u(f'_g) = u(f_g) - \epsilon$, $u(n'_g) = u(n_g) + \epsilon$ and $u(p_g) = u(p'_g)$ leaves a principal's revenue from a generalist and incentive complementarities unchanged and strictly reduces costs associated with a generalist. We need to check that this new contract does not increase a specialists information rents. There are two subcases here. First if $\Delta f_g - \Delta p_g \leq \beta - \alpha$ then with the new contract a specialist continues to specialize and as $p'_g = p_g$ the incentive compatibility constraint is unchanged and thus the information rents paid to a specialist are unchanged. Now suppose $\Delta f_g - \Delta p_g > \beta - \alpha$. For ϵ sufficiently small this does not change a specialist's time allocation and once again the constraint remains the same.

Now consider the second case with $\Delta f_g - \Delta p_g \geq 0$ and $\Delta p_g - 1 = 0$. Once again by decreasing f_g and increasing n_g a principal can reduce the expected wages of a generalist. Once again a specialist's time allocation remains unchanged for this contract.

A similar reasoning hold for the case where $\Delta f_g - \Delta p_g > 0$ and $\Delta p_g - 1 < 0$.

■

Claim 4: The optimal time allocation for a generalist is $t_g = t'_g = \frac{1}{2}$.

Proof There are three possible cases to consider. First suppose $\Delta f_g - \Delta p_g = 0$ and $\Delta p_g - 1 = 0$ with an interior t_g which is not a half. Then from Claim 6 in Lemma 5 we know that the principal can increase his profit without changing the contract. Because the contract remains the same this does not affect the incentive compatibility condition for a specialist.

For the next case t_g can be a corner solution. In this case a specialist specializes completely if he takes the generalist's contract. Consider a new contract with $u(n_g) = u_0$, $u(p_g) = u_0 + 1$ and $u(f_g) = u_0 + 2$. This new contract increases a principal's payoff from a generalist and once again does not affect the incentive compatibility conditions of a specialist.

■

Claim 5: The optimal time allocation for a specialist is $t_s = 1$.

Proof The proof is similar to Lemma 1 and is thus omitted. ■

Claim 6: Suppose $u'(u^{-1}(u_0 + 1)) - u'(u^{-1}(u_0 + 2))$ is sufficiently small. Then the optimal contract for a generalist exhibits incentive complementarities.

Proof Suppose the optimal contract (f_g, p_g, n_g) had no incentive complementarities. Then from Claim's 1 and 3 it must be the case that $u(n_g) = u_0$, $u(p_g) = u_0 + 1$ and $u(f_g) = u_0 + 2$. From Claim 4, the cost associated with a generalist is

$$\frac{1}{4}u^{-1}(u_0) + \frac{1}{2}u^{-1}(u_0 + 1) + \frac{1}{4}u^{-1}(u_0 + 2)$$

Because the incentive compatibility condition for a specialist must bind, and from the specialist's moral hazard constraint it follows that the cost associated with a specialist is

$$u^{-1}(u_0 + 1)$$

Now consider an alternative contract with $u'_{pg} = u_{pg} - \epsilon$ and $u'_{fg} = u_{fg} + 2\epsilon$. This new contract keeps a generalist at the same level of utility. Then the cost associated with a generalist is given by

$$\frac{1}{4}u^{-1}(u_0) + \frac{1}{2}u^{-1}(u_0 + 1 - \epsilon) + \frac{1}{4}u^{-1}(u_0 + 2 + 2\epsilon)$$

and the cost associated with a specialist is

$$u^{-1}(u_0 + 1 - \epsilon)$$

So the cost from introducing incentive complementarities for a generalist is given by

$$\frac{1}{4}(u^{-1}(u_0 + 2 + 2\epsilon) - u^{-1}(u_0 + 2)) - \frac{1}{2}(u^{-1}(u_0 + 1) - u^{-1}(u_0 + 1 - \epsilon))$$

and the benefit from reducing a specialist's information rents is given by

$$u^{-1}(u_0 + 1) - u^{-1}(u_0 + 1 - \epsilon)$$

Because u^{-1} is differentiable and strictly increasing, using the inverse function theorem we can rewrite the cost from incentive complementarities as

$$\frac{1}{4}(\frac{2\epsilon}{u'(u^{-1}(u_0 + 2))} + R(2\epsilon)) - \frac{1}{2}(\frac{\epsilon}{u'(u^{-1}(u_0 + 1 - \epsilon))} + R(\epsilon))$$

and the gain as

$$\frac{\epsilon}{u'(u^{-1}(u_0 + 1 - \epsilon))} + R(\epsilon)$$

Dividing by 2ϵ and taking limits as ϵ tends to 0, we find that as $u'(u^{-1}(u_0+1)) - u'(u^{-1}(u_0+2))$ gets sufficiently small the gains from incentive complementarities dominate the costs.$\blacksquare$

Web Appendix B- Variable List and Data Construction

- PROXIES FOR RESEARCH ABILITY
  - $PATINV_t$ = a dummy variable which takes the value 1 if a person is named as an inventor on a patent application from period $t-5$ to t;
  - $PATGRT_t$ = number of patents that are granted to an individual from period $t-5$ to t ;
  - $PATCOM_t$ = number of patents that are granted to an individual from period $t-5$ to t which result in a commercial product;
  - $PUBLISH_t$ = a dummy variable which takes the value 1 if an individual has articles published in refereed journals from period $t-5$ to t. This variable is only defined for individuals who were granted patents.
- PROXIES FOR SUPERVISORY ABILITY
  - $SUPDIR_t$= the number of people that an individual supervises directly at date t;
- INDUSTRY

  These industry codes associated with each individual are based on the 2002 Census Industry Codes.
  - $SCISERV_t$= a dummy variable which takes the value 1 if an individual belongs to the Professional and Scientific Services category (base group).
  - $MANUFAC_t$= a dummy variable which takes the value 1 if an individual belongs to the Manufacturing category. There are nine sub categories.
    * Pharmaceutical and Medicines
    * Industrial and Miscellaneous Chemicals
    * Computers and Peripheral Equipment
    * Navigational, Measuring, Electromedical, and Control Instruments
    * Communications Equipment
    * Motor Vehicles and Motor Vehicle Equipment

    - Aircraft and Parts
    - Medical Equipment and Supplies
    - Other Manufacturing
  - $PUBLIC\ ADMINISTRATION_t$= a dummy variable which takes the value 1 if an individual belongs to the Public Administration Category. This is a code that the NSF adds to the census industry code.
  - $OTHER\ INDUSTRIES_t$= a dummy variable which takes the value 1 if an individual belongs to none of the industry categories listed above.
- $SALARY_t$= basic annual salary before deductions at date t. Excludes bonuses;
- $SAL10000_t = \frac{SALARY}{10000}$;
- AGE_t= the age of an individual at date t;
- AGE DUMMIES
  - $AGE30_t$= a dummy variable which takes the value 1 if $AGE_t \leq 30$ (base group);
  - $AGE35_t$= a dummy variable which takes the value 1 if $31 \leq AGE_t \leq 35$;
  - $AGE40_t$= a dummy variable which takes the value 1 if $36 \leq AGE_t \leq 40$;
  - $AGE45_t$= a dummy variable which takes the value 1 if $41 \leq AGE_t \leq 45$;
  - $AGE50_t$= a dummy variable which takes the value 1 if $46 \leq AGE_t \leq 50$;
  - $AGE55_t$= a dummy variable which takes the value 1 if $51 \leq AGE_t \leq 55$;
  - $AGE60_t$= a dummy variable which takes the value 1 if $56 \leq AGE_t \leq 60$;
  - $AGE61_t$= a dummy variable which takes the value 1 if $AGE_t \geq 61$;
- JOB ACTIVITIES
  - $SUPIND_t$= the number of people that an individual supervises indirectly at date t;
  - $MGMT_t$= a dummy variable which takes a value of 1 if an individual spends at least 10 per cent of his time on supervision and management at date t;

- $ADMIN_t$= a dummy variable which takes a value of 1 if an individual spends at least 10 per cent of his time on other administrative activities such as accounting, finance, contracts, employee relations, sales, purchasing, marketing and quality management at date t;
- $COMP_t$= a dummy variable which takes a value of 1 if an individual spends at least 10 per cent of his time on computer applications at date t;
- $APPLIED_t$= a dummy variable which takes a value of 1 if an individual's primary and secondary activities at date t are design or development;
- $HOURSWORKED_t$= the number of hours worked per week;
- $STRTYR_t$= the year that an individual started the job that he holds at date t;
- $DIFFEMP_t$= a dummy variable which takes a value of 1 if an individual changed employers from date $t-2$ to date t.

- $DEGREE\ FIELD_t$= dummies based on the major field of degree. The five major fields are
  - Computer and Mathematical Sciences;
  - Biological, Agricultural and Environmental Sciences;
  - Physical and Related Sciences;
  - Engineering (base group);
  - Science and Engineering Related Fields.
- $TASTE_t$= a dummy variable which takes the value 1 if intellectual challenge is very important to an individual when thinking about a job at date t.
- $SATSAL_t$= a dummy variable which takes the value 1 if an individual is very satisfied with job's salary at date t.
- $SATCHAL_t$= a dummy variable which takes the value 1 if an individual is very satisfied with job's intellectual challenge at date t.
- $SATBEN_t$= a dummy variable which takes the value 1 if an individual is very satisfied with job's benefits at date t.
- EMPLOYER SIZE

- $EMSIZE1_t$= a dummy variable which takes a value of 1 if the employer size is less than 24 at date t;
- $EMSIZE2_t$= a dummy variable which takes a value of 1 if the employer size is between 25 and 99 at date t;
- $EMSIZE3_t$= a dummy variable which takes a value of 1 if the employer size is between 100 and 499 at date t;
- $EMSIZE4_t$= a dummy variable which takes a value of 1 if the employer size is between 500 and 999 at date t;
- $EMSIZE5_t$= a dummy variable which takes a value of 1 if the employer size is between 1000 and 4999 at date t;
- $EMSIZE6_t$= a dummy variable which takes a value of 1 if the employer size is above 5000 at date t (base group);

• $NEW\ BUSINESS_t$= a dummy variable which takes the value 1 if employer was a new business;

Data Construction

All the steps listed below are for both years 2001 and 2003. To begin with I drop individuals belonging to the following occupations: economists, psychologists, social scientists and other non science and engineering occupations. I then remove individuals whose main employer was an educational institution or those who were self employed. To remove outliers, I drop the top 1 percent of all the patent and publication variables and the supervisory variables. Finally I drop the bottom 5 percent and top 1 percent of salaries. All of the variables that are significant in Table 4, remain so when we exclude the four highest observations for the patent, publication, supervisory and salary variables and the bottom 5 percent for salaries. Including the bottom 5 percent of observations for salaries, however, leads to insignificant results for some of the variables.