Journal of Empirical Finance 17 (2010) 23–38

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Is there a *symmetric* nonlinear causal relationship between large and small firms?

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A R T I C L E I N F O

Article history:
Received 10 April 2006
Received in revised form 4 August 2009
Accepted 20 August 2009
Available online 28 August 2009

JEL Classification:
C14
G14

Keywords:
Linear causality
Nonlinear causality
BEKK asymmetric GARCH
Small firms
Large firms

A B S T R A C T

This paper uses both linear and nonlinear causality tests to reexamine the causal relationship between the returns on large and small firms. Consistent with previous results, we find that large firms linearly lead small firms. We also find a significant linear causality in the direction from small firms to large firms, particularly in the more recent time period where the impact from small firms to large firms is greater than from large to small. More important, in contrast to the received literature, we find significant *nonlinear* causality that is *bi-directional* and of *the same duration in either direction*. Using the BEKK asymmetric GARCH model we are able to capture most of the detected nonlinear relationship. This indicates that volatility spillovers are largely responsible for the observed nonlinear Granger causality.

1. Introduction

There is now considerable evidence that current returns on small firms are cross-autocorrelated with lagged returns on large firms, but not vice versa.[2] The importance and significance of this evidence is underscored by Lo and MacKinlay's (1990) statement that "…further investigation(s) of (the) mechanisms…" that lead to this result are needed, especially in view of the fact that "…we are still far from having a complete understanding of their nature and sources". This statement, although made several years ago, is still valid. In this paper we conjecture that there is a symmetric nonlinear causal relation between returns on small and large firms and use the modified version of the Baek and Brock (1992) nonlinear Granger causality test to examine the extent to which our conjecture is supported by the data.

We add to the body of research examining causal relationships between large and small capitalization stocks by providing evidence that the causal relationship is *symmetric* and, importantly, both linear and nonlinear. Additionally, we provide evidence that indicates that the nonlinear relationship is due primarily to information flow. This indicates that the previously documented relationship (see, for example, Lo and MacKinlay, 1990; McQueen et al., 1996; and Chordia and Swaminathan, 2000) is more pervasive and complex than previously thought. Our results are relevant to the issue of market efficiency given that Lo and MacKinlay (1990) find that the cross-

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[2] Lo and MacKinlay (1990), Conrad, Gultekin, and Kaul (1991), Brennan, Jegadeesh, and Swaminathan (1993), Mech (1993), Badrinath, Kale, and Noe (1995), McQueen, Pinegar, and Thorley (1996), Chordia and Swaminathan (2000)). Conrad, Gultekin, and Kaul (1991) report similar evidence for volatility.

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doi:10.1016/j.jempfin.2009.08.003

autocorrelation among size-based portfolios may account for at least half of the documented profits associated with a contrarian trading strategy. Thus, the understanding of the dynamic interrelationships between the returns of sized-based portfolios is not only important for theoretical and empirical academic work, but also to investors, regulators, speculators, and hedgers.

Several explanations have been offered for an asymmetric lead–lag relationship between large and small capitalization stocks. For example, Brennan et al. (1993) attribute the lead–lag relationship to the fact that, on average, a significantly greater number of analysts follow large-capitalization stocks than follow small capitalization stocks. Consequently, large firms adjust much faster to information than small firms. In a similar vein, Badrinath et al. (1995), attribute the relationship to trading between informed and uninformed investors, with large-capitalization stocks having more informed investors, proxied by institutional investors, compared to small-capitalization stocks. An underlying assumption of these papers is that some of the information in small firms' stock prices is not relevant for large firms. However, small firms represent a significant share of economic activity and are widely regarded to be important contributors to innovation and economic growth. According to statistics published by the U.S. Small Business Administration, small firms generate over half of non-farm private sector GDP, and over 90% of all employer-firms in the U.S. are small firms. The implication of this is that information contained in small firms should be important for the U.S. economy in general and, therefore, for large firms. It is therefore possible that lagged small firm returns can Granger cause large firm returns.

The existing research on the nature and sources of the causal relationship between firms of different market capitalization has, to date, focused exclusively on a *linear* causal relationship and has ignored the possibility of a nonlinear causal relationship. This is surprising and indicates an important gap in this line of research given the large body of research indicating the importance of nonlinearities in asset pricing.

The issue of nonlinearities in asset pricing has a long history. Rubinstein (1973) points out that if returns are non-normal and investors have non-quadratic utility then investors will generally care about all moments of returns. Kraus and Litzenberger (1976), in their seminal work, developed a three-moment CAPM in which investors show a preference for skewness. Harvey and Siddique (2000) develop a conditional version of the Kraus and Litzenberger model and show that skewness is a priced factor across various sorted portfolios including sized-based portfolios. Others such as Bansal and Viswanathan (1993), Bansal et al. (1993), and Dittmar (2002) also develop asset pricing models in which the pricing kernel is nonlinear. Bansal and Viswanathan (1993) and others find that nonlinear models are better at explaining small firms' returns and cross-sectional variation in expected returns than linear models.

Hiemstra and Jones (1994), and Fujihara and Mougoué (1997), among others, provide evidence of nonlinear causal relationships between the returns on various assets. Hiemstra and Jones, Fujihara and Mougoué, and others, also report the presence of nonlinear causal relations between volume and returns. Gallant et al. (1993) argue that theoretically there is no reason to believe that the relationship between asset prices is only linear, or that the absence or presence of a linear relationship precludes the existence of a nonlinear one. And as they point out, more can be learned about asset pricing by focusing on both linear and nonlinear relationships.

Using data that cover the July 1, 1963 to May 31, 2006 time period, we find, consistent with the existing literature, that lagged returns on large firms statistically significantly *linearly* Granger cause the returns on small firms. However, in contrast to previous studies, we also find that small firms' returns statistically significantly *linearly* Granger cause the returns of large firms. These results hold after controlling for small and large firm returns autocorrelation, possible structural breaks in the data, information flow as proxied by multivariate GARCH models, and nonsynchronous trading. This finding of a symmetric causal relationship between the returns of large and small firms which is different than that found in earlier studies, is more than likely due to (i) the increased role of small firms in the U.S. economy (Audretsch, 2002; and Baumol, 2008); (ii) their increased importance in the U.S. capital markets (Fama and French, 2001) and (iii) the increased level of analyst coverage (Chordia et al., 2006).

The nonlinear results obtained using the modified Baek and Brock (1992) methodology provide evidence of a statistically significant *nonlinear bi-directional* Granger causality between the returns on large and small firms and that these results are not specific to a particular time period as they continue to hold when we partition the sample into several time periods. Further, we find that the predictive power of one portfolio for the other, as measured by the number of lagged returns that are able to forecast current returns is the same irrespective of market capitalization. Thus, our results indicate that the causal relationship between large and small firms is more complex and pervasive than has been previously documented.

We filter the data to examine possible sources of the nonlinearities. Possible sources include differential reaction to information flow (Ross, 1989; and Andersen, 1996) as proxied by GARCH effects, and structural breaks (Hiemstra and Jones, 1994). One potential concern is that the nonlinear causality inferred from the Baek and Brock (1992) test may be affected by the presence or absence of linear relations in the data. Nonlinearities could also potentially affect the inferences about linear causality.

Using a multivariate asymmetric GARCH model (the BEKK model with a leverage term), we obtain essentially the same results for *linear* causality as in the VAR model. This suggests that the evidence of linear causality persists when we control for the GARCH nonlinearity. Diagnostic tests on the residuals of the BEKK model indicate that the nonlinear causal relationships that previously characterized the residuals of the VAR model are substantially reduced.

We find, consistent with Conrad et al. (1991), that volatility shocks to large firms impact the conditional variance of small firms. However, in contrast to their findings we find that volatility shocks to small firms have a statistically significant and economically meaningful impact on the volatility of large firms. We also find that negative shocks to large firms have a larger impact than positive shocks on the conditional variance of small firms. The effect of small firms on large firms becomes stronger over time suggesting that small firms are growing in importance in the U.S. capital markets and therefore, the overall economy. This is consistent with evidence provided by Audretsch (2002) and more recently by Baumol (2008) that shows that small firms have become important players in the U.S. economy as reflected by their contribution to employment and GDP. This increase in importance of small firms in the economy is also reflected in their stock market value where the number of firms in the smallest quintiles that are formed using firms from both the New York Stock Exchange and NASDAQ, has increased from 212 in 1962 to over 3700 by 1990.[3]

The remainder of the paper is organized as follows. Section 2 outlines the methodology, with the data being described in Section 3. Section 4 presents results of both linear and nonlinear Granger causality tests. In Section 5 we carry out several robustness checks. Section 6 summarizes and concludes.

2. Empirical methods

This section presents the methodologies used in the investigation of whether or not large- and small-firm returns are linearly and nonlinearly causally related. The standard linear causality tests proposed by Granger (1969) are outlined in Sub-section 2.1. Sub-section 2.2 contains the modified Baek and Brock (1992) method we used to test for nonlinear Granger causality.

2.1. A linear Granger causality

Consider two variables changing over time, X_t and Y_t. Linear Granger causality investigates whether past values of X_t have significant linear predictive power for current values of Y_t given past values of Y_t. If so, X_t is said to linearly Granger cause Y_t. Bi-directional causality exists if Granger causality runs in both directions.

The test for linear Granger causality between large and small firms involves the estimation of the following equations in a vector autoregression (VAR) framework:

$$R_{1,t} = \sum_{i=1}^{\theta_1} \alpha_i R_{1,t-i} + \sum_{j=1}^{\theta_2} \beta_j R_{2,t-j} + \varepsilon_{1,t} \tag{1}$$

$$R_{2,t} = \sum_{i=1}^{\theta_3} \delta_i R_{2,t-i} + \sum_{j=1}^{\theta_4} \phi_j R_{1,t-j} + \varepsilon_{2,t} \tag{2}$$

$R_{1,t}$ and $R_{2,t}$ are, respectively, the daily returns on the small and large-firm portfolios on day t; α, β, δ, and ϕ are the parameters to be estimated; $(\varepsilon_1, \varepsilon_2)$ are zero-mean error terms with a constant variance–covariance matrix; the optimal lag lengths are determined using the Bayesian information criterion (BIC).

Linear causal relationships are inferred from Eqs. (1) and (2). To test for linear Granger non-causality at specific lags we examine the statistical significance of the individual β and ϕ coefficient estimates. Furthermore, we test for cumulative linear Granger non-causality by testing the null hypothesis that $\Sigma\beta_j = 0$ in Eq. (1) or $\Sigma\phi_j = 0$ in Eq. (2) using a T-statistic.

2.2. Nonlinear Granger causality[4]

Baek and Brock (1992) developed a nonparametric statistical technique for detecting nonlinear causal relationships from the residuals of linear Granger causality models. Consider the following two series: returns on the small-firm portfolio, $R_{1,t}$, and returns on the large-firm portfolio, $R_{2,t}$. Let the $\boldsymbol{m}$-length lead vector (of future returns) of $R_{1,t}$ be denoted by $R^{\boldsymbol{m}}_{1,t}$, and let $\boldsymbol{Lr1}$ and $\boldsymbol{Lr2}$ be the lengths of the lag vectors $\boldsymbol{R}^{\boldsymbol{Lr1}}_{1,t-\boldsymbol{Lr1}}$ and $\boldsymbol{R}^{\boldsymbol{Lr2}}_{2,t-\boldsymbol{Lr2}}$ of $R_{1,t}$ and $R_{2,t}$, respectively. For given values of $\boldsymbol{m}$, $\boldsymbol{Lr1}$, and $\boldsymbol{Lr2} \geq 1$ and an arbitrarily small constant $d > 0$, $R_{2,t}$ does not strictly nonlinearly Granger cause $R_{1,t}$ if:

$$\begin{aligned} &\Pr(\|R^{\boldsymbol{m}}_{1,t} - R^{\boldsymbol{m}}_{1,s}\| < d \,|\, \|\boldsymbol{R}^{\boldsymbol{Lr1}}_{1,t-\boldsymbol{Lr1}} - \boldsymbol{R}^{\boldsymbol{Lr1}}_{1,s-\boldsymbol{Lr1}}\| < d, \|\boldsymbol{R}^{\boldsymbol{Lr2}}_{2,t-\boldsymbol{Lr2}} - \boldsymbol{R}^{\boldsymbol{Lr2}}_{2,s-\boldsymbol{Lr2}}\| < d) \\ &\quad = \Pr(\|R^{\boldsymbol{m}}_{1,t} - R^{\boldsymbol{m}}_{1,s}\| < d \,|\, \|\boldsymbol{R}^{\boldsymbol{Lr1}}_{1,t-\boldsymbol{Lr1}} - \boldsymbol{R}^{\boldsymbol{Lr1}}_{1,s-\boldsymbol{Lr1}}\| < d), \end{aligned} \tag{3}$$

where $\Pr(\cdot)$ is probability, $\|\cdot\|$ is the maximum norm,[5] and $s, t = \max(\boldsymbol{Lr1}, \boldsymbol{Lr2}) + 1, \ldots, T - \boldsymbol{m} + 1$.

The left hand side of Eq. (3) is the conditional probability that two arbitrary $\boldsymbol{m}$-length lead vectors of $R_{1,t}$ are within a distance d of each other, given that two corresponding $\boldsymbol{Lr1}$-length lag vectors of $R_{1,t}$ and two $\boldsymbol{Lr2}$-length lag vectors of $R_{2,t}$, respectively, are within a distance d of each other. The right hand side of Eq. (3) is the probability that the two $\boldsymbol{m}$-length lead vectors of $R_{1,t}$ are within a distance d of each other, conditional only on their corresponding $\boldsymbol{Lr1}$-length lag vectors being within distance d of each other. Intuitively, Eq. (3) states that if the large-firm returns do not nonlinearly Granger cause the small-firm returns, then the probability that the distance between two conformable vectors of future returns of the small firms being less than d will be the same whether that probability is conditioned on the past returns of both the large and small firms or only on the own past returns of the small firms.

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[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[4] This section is largely based on the work of Hiemstra and Jones (1994).

[5] The maximum norm for vector $\boldsymbol{X} \equiv (X_1, X_2, \ldots, X_K) \in \Re^K$ is defined as $\max(X_i)$, $i = 1,2, \ldots, K$.

The test in Eq. (3) can be restated by expressing the conditional probability in terms of the ratios of joint and conditioning probabilities associated with each part of the test as follows:

$$\frac{CI(\boldsymbol{m}+\boldsymbol{Lr1},\boldsymbol{Lr2},d)}{CI(\boldsymbol{Lr1},\boldsymbol{Lr2},d)}=\frac{CI(\boldsymbol{m}+\boldsymbol{Lr1},d)}{CI(\boldsymbol{Lr1},d)}. \tag{4}$$

The joint probabilities are defined as,

$$CI\,(\boldsymbol{m}+\boldsymbol{Lr1},\boldsymbol{Lr2},d)\equiv\Pr(\|\boldsymbol{R}^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}}-\boldsymbol{R}^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}}\|<d,\|\boldsymbol{R}^{\boldsymbol{Lr2}}_{2t-\boldsymbol{Lr2}}-\boldsymbol{R}^{\boldsymbol{Lr2}}_{2s-\boldsymbol{Lr2}}\|<d), \tag{5}$$

$$CI(\boldsymbol{Lr1},\boldsymbol{Lr2},d)\equiv\Pr(\|\boldsymbol{R}^{\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}}-\boldsymbol{R}^{\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}}\|<d,\|\boldsymbol{R}^{\boldsymbol{Lr2}}_{2t-\boldsymbol{Lr2}}-\boldsymbol{R}^{\boldsymbol{Lr2}}_{2s-\boldsymbol{Lr2}}\|<d),$$

$$CI(\boldsymbol{m}+\boldsymbol{Lr1},d)\equiv\Pr(\|\boldsymbol{R}^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}}-\boldsymbol{R}^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}}\|<d),$$

$$CI(\boldsymbol{Lr1},d)\equiv\Pr(\|\boldsymbol{R}^{\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}}-\boldsymbol{R}^{\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}}\|<d).$$

To test the condition in Eq. (4), we use the correlation-integral estimators of the joint probabilities in Eq. (5).[6] The correlation-integral is a measure of "closeness" of realizations of a possibly multivariate random variable in two different times, and is estimated as a proportion of the number of observations that are within the distance d of each other to the total number of observations.

Assuming that $R_{1,t}$ and $R_{2,t}$ are strictly stationary and meet the required mixing conditions as specified in Denker and Keller (1983), under the null hypothesis that $R_{2,t}$ does not strictly Granger cause $R_{1,t}$, the test statistic T is asymptotically normally distributed. That is,

$$T=\left[\frac{CI(\boldsymbol{m}+\boldsymbol{Lr1},\boldsymbol{Lr2},d,n)}{CI(\boldsymbol{Lr1},\boldsymbol{Lr2},d,n)}-\frac{CI(\boldsymbol{m}+\boldsymbol{Lr1},d,n)}{CI(\boldsymbol{Lr1},d,n)}\right]\sim N\left(0,\frac{1}{\sqrt{n}}\sigma^2(\boldsymbol{m},\boldsymbol{Lr1},\boldsymbol{Lr2},d)\right), \tag{6}$$

where, $n=T+1-\boldsymbol{m}-\max(\boldsymbol{Lr1},\boldsymbol{Lr2})$ and $\sigma^2(\cdot)$ is the asymptotic variance of the modified Baek and Brock test statistic.[7] The test statistic in Eq. (6) is applied to the two estimated residual series from the VAR model in Eqs. (1) and (2), $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, respectively. Hence, if the null hypothesis of Granger non-causality is rejected, the detected causal relationship between the two portfolios must necessarily be nonlinear.

A possible problem arising from the application of the modified Baek and Brock test to estimated residuals is an unaccounted estimation uncertainty that may lead to erroneous inferences.[8] To verify if our nonlinear findings are robust to this problem, in Sub-section 5.2 we follow the re-sampling procedure of Diks and DeGoede (2001) to obtain empirical p-values.

The Baek and Brock (1992) procedure represents an important development in detecting nonlinear Granger causality. Nevertheless, because of the nonparametric nature of the test, it does not provide any information about the functional form or the sign of the detected causal relationship. Therefore, it is important to realize that the Baek and Brock (1992) test should be used as a diagnostic tool that encompasses a modeling process, rather than a major modeling device.

3. Data

The data used in this study cover the period from July 1, 1963 to May 31, 2006. They consist of daily returns of all stocks covered by the Center for Research in Security Prices (CRSP) database. We divide our sample of stocks into five size-based portfolios based on their market value of equity at the end of the prior year. Thus, portfolio 1 (R_1) contains the quintile of smallest stocks whereas, portfolio 5 (R_2) contains the quintile of largest stocks. The stocks within each quintile are then used to form a value-weighted portfolio.

Systematic calendar effects, such as the day-of-the-week and month-of-the-year effects, are removed from the mean and variance of the daily returns using the procedure outlined in Gallant et al. (1992). This procedure involves the following two steps. First, the mean and the variance of the returns are adjusted for both day-of-the-week and month-of-the-year effects using OLS

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[6] The correlation-integral estimators of the joint probabilities in Eq. (5) are written as

$$CI\,(\boldsymbol{m}+\boldsymbol{Lr1},\boldsymbol{Lr2},d,n)\equiv\frac{2}{n(n-1)}\sum_{t<s}\sum I(r^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}},r^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}},d)\times I(r^{\boldsymbol{Lr2}}_{2t-\boldsymbol{Lr2}},r^{\boldsymbol{Lr2}}_{2s-\boldsymbol{Lr2}},d),$$

$$CI(\boldsymbol{Lr1},\boldsymbol{Lr2},d,n)\equiv\frac{2}{n(n-1)}\sum_{t<s}\sum I(r^{\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}},r^{\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}},d)\times I(r^{\boldsymbol{Lr2}}_{2t-\boldsymbol{Lr2}},r^{\boldsymbol{Lr2}}_{2s-\boldsymbol{Lr2}},d),$$

$$CI(\boldsymbol{m}+\boldsymbol{Lr1},d,n)\equiv\frac{2}{n(n-1)}\sum_{t<s}\sum I(r^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}},r^{\boldsymbol{m}+\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}},d),$$

$$CI(\boldsymbol{Lr1},d,n)\equiv\frac{2}{n(n-1)}\sum_{t<s}\sum I(r^{\boldsymbol{Lr1}}_{1t-\boldsymbol{Lr1}},r^{\boldsymbol{Lr1}}_{1s-\boldsymbol{Lr1}},d),$$

where $t,s=\max(\boldsymbol{Lr1},\boldsymbol{Lr2})+1,\ldots,T-\boldsymbol{m}-1$; $n=T+1-\boldsymbol{m}-\max(\boldsymbol{Lr1},\boldsymbol{Lr2})$.

[7] The asymptotic variance is estimated using the theory of U-statistic for weakly dependent processes (Denker and Keller, 1983). For a complete and detailed derivation of the variance see the appendix in Hiemstra and Jones (1994).

[8] We thank the editor Wayne Ferson for indicating this potential problem to us.

Table 1
Test results of linear causality between small- and large-firm portfolios.

Dependent variable	Independent variable		
	Small	Large	F-stat.
Small	$\alpha_1 = 0.1720$	$\beta_1 = 0.1477$	134.6448[a]
	(12.0681)[a]	(12.8951)[a]	
	$\alpha_2 = 0.0634$	$\beta_2 = 0.0404$	
	(4.3927)[a]	(3.5028)[a]	
	$\alpha_3 = 0.0479$	$\beta_3 = 0.0173$	
	(3.3331)[a]	(2.5102)[a]	
	$\alpha_4 = 0.0963$	$\beta_4 = 0.0318$	
	(6.7353)[a]	(2.7945)[a]	
	$\alpha_5 = 0.0169$	$\beta_5 = 0.0080$	
	(1.2664)	(2.7337)[a]	
	$\Sigma\alpha_i = 0.3966$	$\Sigma\beta_j = 0.2452$	
	(14.65)[a]	(5.1389)[a]	
Large	$\phi_1 = 0.0245$	$\delta_1 = 0.0982$	10.24256[a]
	(2.1810)[b]	(6.8803)[a]	
	$\phi_2 = 0.0016$	$\delta_2 = -0.0291$	
	(2.0881)[b]	(−2.0284)[b]	
	$\phi_3 = 0.0053$	$\delta_3 = -0.0185$	
	(3.2934)[a]	(−1.2932)	
	$\phi_4 = 0.0505$	$\delta_4 = -0.0424$	
	(2.8382)[a]	(−2.9848)[a]	
	$\phi_5 = 0.0622$	$\delta_5 = 0.0206$	
	(3.7329)[a]	(1.5086)	
	$\Sigma\phi_j = 0.1441$	$\Sigma\delta_i = 0.0288$	
	(2.7823)[a]	(1.6701)[c]	

This table reports regression estimates for the following model:

$$R_{1,t} = \sum_{i=1}^{\theta_1} \alpha_i R_{1,t-i} + \sum_{j=1}^{\theta_2} \beta_j R_{2,t-j} + \varepsilon_{1,t}$$

$$R_{2,t} = \sum_{i=1}^{\theta_3} \delta_i R_{2,t-i} + \sum_{j=1}^{\theta_4} \phi_j R_{1,t-j} + \varepsilon_{2,t}$$

where R_1 and R_2 are small and large firm daily returns, respectively. The sample period is from July 1, 1963 to May 31, 2006.

1. T-statistics are reported in parentheses below the sum of the estimated coefficients. The sum represents the cumulative effect.
2. For a given sum of coefficients s, the T-statistic is calculated as $T = s/\sigma_s$, where $s = \Sigma^n a_i$ and n = number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$ then, $s = \Sigma^n a_i = a_1 + a_2 + a_3$ and

$$\sigma_s = \sigma_{(a_1 + a_2 + a_3)} = \sqrt{\sigma_{a_1}^2 + \sigma_{a_2}^2 + \sigma_{a_3}^2 + 2\sigma_{a_1a_2} + 2\sigma_{a_2a_3} + 2\sigma_{a_1a_3}}$$

3. F-stat is the F-statistic testing for the joint significance of the lag coefficients on large (small) firms in the small (large) firms' equation.

[a], [b], and [c] indicate significance at the 1%, 5%, and 10% respectively.

regression equations. For example, using returns for the small-firm portfolio $R_{1,t}$, we estimate the following mean regression $R_{1,t} = \boldsymbol{D}_t\boldsymbol{\beta}_R + \eta_t$ and variance regression, $\ln(\eta_t^2) = \boldsymbol{D}_t\boldsymbol{\varphi}_R + \varepsilon_t$, where $\boldsymbol{D}_t$ is a vector of dummy variables containing the systematic calendar effects,[9] $\boldsymbol{\beta}_R$ and $\boldsymbol{\varphi}_R$ denote parameter vectors, and η_t, ε_t are white noise error terms. Similar equations are estimated for the returns on the large-firm portfolio. Next, using the results from the variance equation the residuals from the mean equation are standardized in the following manner:

$$R_{1,t}^* = \eta_t / exp(\boldsymbol{D}_t\varphi_R / 2), \text{ where } (\boldsymbol{D}_t\varphi_R) \text{ are the fitted values of the variance.}$$

4. Empirical results

4.1. Linear Granger causality

Results of linear Granger causality tests are reported in Table 1. For the small-firm portfolio returns R_{1t}, the null hypothesis of linear Granger non-causality from the large-firm portfolio returns R_{2t} is strongly rejected. This is evidenced by the highly significant summed coefficients of the large-firm returns in the small-firm returns equations ($\Sigma\beta_j = 0.2452$, T-stat. = 5.1389) and by the highly significant F-statistic (F-stat. = 134.6448, p-value < 0.01) that tests the exclusion of the large-firm returns from the

[9] We use dummy variables for the month of January and for every business day of the week.

small-firm returns equation.[10] This finding is consistent with the existing literature (see, e.g., Lo and MacKinlay, 1990; Mech, 1993).

Turning to the results for the portfolio of large firms, we find that lagged returns on small firms are predictors of the returns on large firms ($\Sigma\phi_j = 0.1441$, $t = 2.7823$). This result is also supported by the significant F-statistic (F-stat. $= 10.2456$, p-value < 0.01) that tests the exclusion of the small-firm returns from the large-firm returns equation. This finding is different from the results reported by Lo and MacKinlay (1990), Mech (1993), Chordia and Swaminathan (2000), and Hou (2007). To the extent that stocks of large and small firms are held by different types of investors (see, e.g., Brennan et al., 1993; and Badrinath et al., 1995) these results are consistent with the notion that these different investor groups react heterogeneously to the release of information. Note that the usual asymmetry in the magnitude of the coefficients is still present in that large firms have economically much larger predictive power for small firms than small firms have for large firms.

4.2. Nonlinear Granger causality

Before testing for nonlinear Granger causality, it is important to first determine if the data are characterized by nonlinearities. We perform a formal nonlinear dependence test known as the Brock, Dechert, and Scheinkman (BDS) test. The BDS approach essentially tests for deviations from identically and independently distributed (i.i.d.) behavior in time series.[11] Results of the BDS test, not tabulated here but available upon request, reveal that the vast majority of the estimates of the BDS statistics are statistically significant, indicating significant nonlinearities in the univariate return series.

To conduct tests for nonlinear causality we use the residuals from the linear VAR model, from which any linear predictive relationship has already been removed. Values for the lead length $\boldsymbol{m}$, the lag lengths $\boldsymbol{Lr1}$ and $\boldsymbol{Lr2}$, and the distance measure d must be selected in order to implement the Baek and Brock test. In contrast to linear causality testing, no methods have been developed for choosing optimal values for lag lengths and distance measure.[12] Consequently, we rely on the results in Hiemstra and Jones (1994) and set the lead length at $\boldsymbol{m} = 1$ and set $\boldsymbol{Lr1} = \boldsymbol{Lr2}$ for all cases. In this study we use common lag lengths of one to five lags and a common distance measure of $d = 1.5\sigma$, where σ denotes the standard deviation of the time series.[13]

In our discussion of the results we focus on p-values for the modified Baek and Brock test as this enables us to compare them with the empirical p-values (reported below) obtained using the re-sampling procedure. The empirical p-values account for estimation uncertainty in the residuals of the VAR model used in the modified Baek and Brock test, thereby, making these results more reliable (see Sub-section 5.2 for more details).

For the full sample, the null hypothesis of no nonlinear Granger causality from small-firm portfolio returns to large-firm portfolio returns is strongly rejected (all the p-values are less than 0.01). The null hypothesis of no nonlinear Granger causality from large-firm portfolio returns to small-firm portfolio returns is also rejected (p-values are less than 0.02).[14] Taken together, these results are strong indicators of *bi-directional nonlinear* Granger causality between the returns on small and large firms. Importantly, the duration of small firms' predictive power for the returns on large firms is almost the same as the predictive power of large firms for small firms. More specifically, there is persistence in the predictive power of large firms for small firms and of small firms for large firms. These results are in sharp contrast to those reported in previous studies that show *unidirectional linear* predictability from large to small-firm returns.

Thus far, our results provide new evidence on the causal relationship between the returns on large and small firms on two accounts: (i) it is also nonlinear, and (ii) it is symmetric both in terms of statistical significance and the persistence of the effect as measured by the number of significant lags. Therefore, our results indicate that the lead–lag effect between returns on size-based portfolios is stronger and more complex than previously thought. These findings underscore the importance of testing for general nonlinear relationships in investigations of the dynamic relationship between the returns on different market capitalization stocks.

The finding that small firms' returns Granger causes large firms' returns deserves further articulation. These findings are consistent with the notion that small firms have grown in importance in the U.S. economy. This is borne out by various statistics. For instance, as of 2007, small firms provide about 55% of jobs in the U.S. economy and since the latter part of the 1970s, small businesses have provided two out of every three net new jobs in the U.S. economy. In addition, statistics provided by the U.S. Small Business Association, show that small businesses account for more than 50% of GDP with this amount increasing over time.[15] Audretsch (2002) also provides supporting evidence. He finds that while large firms often produce a large number of patents per

[10] The F statistic is calculated as

$$F_{(d_R - d_{UR},\, d_R)} \approx \left(\frac{SSE_R - SSE_{UR}}{SSE_{UR}}\right) \div \left(\frac{d_R - d_{UR}}{d_{UR}}\right),$$

where, SS_R and SSE_{UR} are, respectively, sum of squared errors for the restricted and unrestricted versions of Eqs. (1) and (2) and d is the degree of freedom.

[11] See Fujihara and Mougoué (1997) for a formal description of the test.

[12] Recent work by Diks and Panchenko (2006) sheds some light on the optimal asymptotic rate of adjustment of the distance measure d with the sample size. However, their study does not fully solve the problem of optimal choice of d in finite samples.

[13] In the estimation we also considered $d = 0.5\sigma$ and 1.0σ. There were no qualitative differences in our results.

[14] Recent finding by Diks and Panchenko (2005, 2006) suggests that the rejection of the null in this case may also indicate the presence of conditional heteroscedasticity in the data. We address this issue in the next section.

[15] These statistics were obtained from the website http://411sbfacts.com/speeches.html, which compiled information from official sources such as, the U.S. Bureau of the Census, the U.S. Bureau of Labor Statistics, and the U.S. Small Business Association.

firm, the patenting rate for small firms has been typically higher than that for large firms when measured on a per-employee basis. Finally, this importance is also reflected in small firms' growing share of stock market value. As pointed out above the number of firms belonging to the smallest quintile has increased from 212 firms in 1962 to over 3700 by 1990. While large firms have stayed relatively constant, increasing from 217 to 265. An implication of this, and supported by Chordia et al. (2006) is that analysts' coverage and institutional ownership have also increased over time, suggesting increasing information flow from these firms to large firms.

5. Robustness of empirical findings and the nature of the nonlinear Granger causality

The nonlinear Granger causality test we used in the previous analysis does not identify the underlying source of causality. In this section we verify the robustness of our earlier findings and try to shed some light on the nature of the detected nonlinear bi-directional causal relationship. Specifically, we examine whether the nonlinear relationship is due to structural breaks in the data (Baek and Brock, 1992; Andersen, 1996) or to a differential reaction to information flow as proxied by volatility (Ross, 1989) or some combination of the two. We also apply a re-sampling scheme to verify whether the results of the modified Baek and Brock (1992) test hold when applied to residuals.

5.1. Time-variation in the Granger causality

Baek and Brock (1992) suggest that a weakness of their test is that it could spuriously reject the null hypothesis of Granger non-causality due to the presence of non-stationarity induced by structural breaks in the data and heteroscedasticity. To investigate whether our results are sample-period sensitive, we partition our sample into three sub-periods, namely July 1, 1963 to December 31, 1979; January 2, 1980 to December 30, 1994; and January 3, 1995 to May 31, 2006. This partition is based on an examination of the S&P 500 index that appears to reveal two breaks roughly corresponding with the dates used to split our sample. The first sub-sample represents the stable "good old times" marked by the prevalence of "traditional companies". The second sub-period, which includes the 1987 stock market crash, is characterized by the biotechnology boom and is generally dominated by riskier companies, and the third period coincides with the internet stock bubble and its aftermath.

Another rationale for splitting our sample around 1980 is provided by Fama and French (2001), who show that a structural break occurred in the stock market around the end of 1979 with a decline in the quality of firms going public and a decrease in the propensity of firms to pay dividends. A greater percentage of riskier and smaller firms going public suggests that these types of firms were also growing in importance in the overall economy and consequently were being followed by more analysts and held by (more) institutional investors (Chordia et al., 2006). The importance of this is that, to the extent that the lead–lag relationship is significantly impacted by analysts following and institutional investors (Brennan et al., 1993; Badrinath et al., 1995), we would expect that returns in small firms could also Granger cause returns of large firms during this time period.

To conserve space we tabulate the sub-period *p*-values only for the nonlinear causality tests based on re-sampling procedure in Sub-section 5.2. The untabulated results are qualitatively similar to the full-sample results discussed in Sub-section 4.2. More specifically, the *p*-values that test the hypothesis of no nonlinear causality from small firms to large firms are uniformly greater than the corresponding *p*-values testing the reverse hypothesis. This finding is interesting as it suggests that, contrary to the existing literature, the impact of the predictive power of small firms on large firms is strong and persistent.

In sum, our sub-sample results imply that time-variation in the causal relationship does not explain the documented nonlinear bi-directional causal relation between small-firm returns and large-firm returns. For comparison, we also provide the results of the linear test for the three sub-samples (Table A1 of the Appendix). They reveal a unidirectional causality from large firms to small firms in the first sub-period and significant bi-directional causality in the second and third sub-periods. However, it is worth noting that while the size of the summed impact of small-firm returns on large-firm returns decreases from 0.341 to 0.25198 from the second sub-period to the third sub-period, the impact of the summed impact of large-firm returns on small-firm returns diminishes more dramatically, falling from 0.5240 to 0.0684. Additionally, the statistical significance of the summed impact of small-firm returns on large-firm returns also increases from the second sub-period to the third sub-period.

Next, we conduct additional sub-sample analysis to relate our research better to earlier work. More specifically, we rerun all of our empirical tests based on the sub-samples of Lo and MacKinlay (1990), Mech (1993), and Hou (2007).[16] Overall, using different sub-samples do not change our earlier findings. That is, causality generally runs from large to small firms during the first sub-samples but then becomes bi-directional during the second sub-samples.

Taken together, these results are consistent with our argument outlined above that with the growing importance of small firms in the U.S. capital markets and with them being increasingly held by institutional investors and followed by more analysts, small-firms returns would become more important in the predictability of large-firm returns.

5.2. Re-sampling procedure for nonlinear Granger causality testing

In this sub-section we further ascertain the robustness of our empirical findings. We mentioned before that the modified Baek and Brock test for Granger non-causality is applied to the residuals of the VAR model, rather than to original untreated

[16] These sub-samples are July 6, 1962 to April 3, 1975 and April 4, 1975 to December 31, 1987 (Lo and MacKinlay), January 5, 1973 to July 3, 1980 and July 11, 1980 to December 26, 1986 (Mech), and July 1963 to June 1982 and July 1982 to December 2001 (Hou).

observations. This may lead to erroneous inferences because of an unaccounted estimation uncertainty. The reason for this is the potential difference of the null distribution when the test is applied to residuals rather than to original observations (Randles, 1984). This difference arises because the parameter estimation uncertainty is not reflected in the test statistics. To eliminate any erroneous inference we use a re-sampling scheme that incorporates parameter estimation uncertainty. We continue to use the test statistics of the modified Baek and Brock test and modify the re-sampling procedure of Diks and DeGoede (2001) to determine empirical p-values of the nonlinear Granger causality tests. The test statistics T is given in Eq. (6).

Diks and DeGoede (2001) conduct several experiments to determine the best randomization procedure for obtaining empirical p-values. They find that the best finite sample properties of the tests are obtained when only the causing series were bootstrapped. Hence, we adopt this methodology. The Stationary bootstrap of Politis and Romano (1994) is used to preserve potential serial dependence in the causing series. The re-sampling scheme which is robust with respect to parameter estimation uncertainty is implemented as follows.

1. Estimate a parametric model and obtain the fitted values of the conditional mean and the estimated residuals.[17]
2. Resample the residuals in such a way that satisfies the null hypothesis.[18] Let N denote the length of the series and P_S is the stationary bootstrap switching probability. We start a new bootstrapped sequence from a random position in the initial series selected from the uniform distribution between 1 and N. With probability $1-P_S$ the next element in the bootstrapped sequence corresponds to the next element in the initial series. With probability P_S we randomly select an element from the initial sequence and put it as the next element in the bootstrapped sequence. The procedure continues until we obtain a bootstrapped sequence of length N. To ensure stationarity of the bootstrapped sequence, we connect the beginning and the end of the initial sequence.
3. Create artificial data series using the fitted values and the re-sampled residuals.
4. Re-estimate the model using the artificial data and obtain new series of the residuals.
5. Compute test statistics T_i for the artificial residuals.

By repeating the bootstrap B-times and calculating test statistic T_i for each bootstrap $i=1 \ldots B$, we are able to obtain empirical distribution of the test statistics under the null. To obtain p-values of the test we compare the test statistics computed from the initial data T_0 with the test statistics under the null T_i:

$$p = \frac{\sum_{i=0}^{B} \#(T_0 \leq T_i)}{B+1},$$

where, $\#(\cdot)$ denotes the number of events in the brackets. The test rejects the null hypothesis in the direction of nonlinear Granger causality whenever T_0 is large. We set the number of bootstraps $B=99$.[19] The bootstrap switching probability P_S is set to 0.05.

The results based on the bootstrapped empirical p-values are reported in Table 2 for the full sample (Panel A) as well as the three sub-periods (Panels B, C, and D). Panel A shows that the p-values testing the hypothesis of no nonlinear causality from small-firm returns to large-firm returns and from large-firm returns to small-firm returns, are at most equal to 0.10, implying bi-directional nonlinear causality between the returns of small firms and large firms over the entire sample period. These results are, therefore, qualitatively similar to the ones discussed in Sub-section 4.2 for the full-sample period.

The results reported in Panels B, C and D show significant variation across the various sub-periods. Panel B, which contains the results for the 1963 to 1979 sub-period, shows no evidence of nonlinear causality between small-firm returns and large-firm returns. However, the reported results for Panel C (1980 to 1994) and Panel D (1995 to May 1996) reveal a significant bi-directional nonlinear causality between the returns of small and large firms, indicating that the ability of past returns of small firms to predict the returns of large firms has ameliorated over time.[20] These results are consistent with those reported for the linear causality tests where during the later sub-periods small-firms returns Granger-caused large-firm returns. They also provide support for our argument that with increasingly younger and smaller firms going public, coupled with investors' tremendous demand for dot.com companies of the 1990s, smaller firms took on a larger, more important, role in the U.S. stock market. As pointed out earlier, evidence provided by Chordia et al. (2006) indicate that since the start of the 1990s smaller firms are increasingly being held by institutional investors and followed by significantly more analysts. This finding provides support for the argument that smaller firms are now characterized by more information for the price formation of large firms, thereby enabling their returns to have predictive power for large firms' returns.

[17] We account for estimation uncertainty of the calendar effects by starting with the unadjusted returns and explicitly including the calendar dummies in the conditional mean equation.

[18] The re-sampling procedure imposes a more restrictive null hypothesis of conditional independence. However, the test detects the deviations from the null in the direction of interest, that is, Granger causality.

[19] $B=99$ is the smallest commonly suggested number of bootstrap replications (see Davidson and MacKinnon, 2000). Because of computational limitations we were unable to increase B, which may possibly result in some loss of power for our tests.

[20] Although not reported, the significance and duration of the predictability of both large and small firms is robust to partitioning the data into several other sub-periods.

Table 2
Test results for nonlinear Granger causality between small- and large-firm portfolios.

	H_0: small does not cause large	H_0: large does not cause small
Panel A: July 1, 1963–May 31, 2006		
$L_{\text{Large}} = L_{\text{Small}}$	*p*-value	*p*-value
1	0.01	0.01
2	0.10	0.01
3	0.07	0.01
4	0.10	0.01
5	0.06	0.03
Panel B: July 1, 1963–December 31, 1979		
$L_{\text{Large}} = L_{\text{Small}}$	*p*-value	*p*-value
1	0.15	0.12
2	0.30	0.08
3	0.46	0.11
4	0.30	0.12
5	0.15	0.12
Panel C: January 2, 1980–December 30, 1994		
$L_{\text{Large}} = L_{\text{Small}}$	*p*-value	*p*-value
1	0.01	0.01
2	0.01	0.01
3	0.01	0.01
4	0.03	0.02
5	0.05	0.07
Panel D: January 3, 1995–May 31, 2006		
$L_{\text{Large}} = L_{\text{Small}}$	*p*-value	*p*-value
1	0.04	0.01
2	0.08	0.01
3	0.01	0.06
4	0.03	0.07
5	0.03	0.05

This table reports *parametric bootstrap p-values* for the standard Baek and Brock nonlinear Granger causality test given in Eq. (6). The test is applied to the estimated VAR residuals from Eqs. (1) and (2). $L_{\text{Large}} = L_{\text{Small}}$ denotes the number of lags on the residuals series used in the test. In all cases, the tests are applied to the unconditional standardized residuals. The lead length, $\boldsymbol{m}$, is set to unity, and the distance measure, d, is set to 1.5.

5.3. Granger causality in volatility: bi-variate-GARCH model

Hiemstra and Jones (1994), among others, provide evidence that the nonlinear structure characterizing univariate daily stock return series is related to ARCH errors. Moreover, Diks and Panchenko (2005, 2006) show that the modified Baek and Brock (1992) test described in Sub-section 2.2 could produce spurious results in the presence of conditional heteroscedastic structures in the data. This suggests that we need to account for conditional heteroscedasticity before applying the test for nonlinear Granger causality. Further, Ross (1989), Andersen (1996), and others suggest that the volatility of a time series can measure the rate of information flow. Thus, we control for the rate of information flow and then assess whether the nonlinear bi-directional predictability between small-firm returns and large-firm returns changes.

To model nonlinear Granger causality in volatility we use the BEKK model of Engle and Kroner (1995).[21] The bi-variate mean and variance equations are estimated *simultaneously* to enhance the efficiency of the parameter estimates. The flexibility of the BEKK approach allows us to introduce a leverage term in the conditional variance equation as in Kroner and Ng (1998) thus, enabling us to capture the effect of negative past returns on the strength of the causal relationship.

In implementing the BEKK model, we make the assumption that the joint distribution of current returns conditional on past returns is multivariate normal with conditional mean μ_t, and conditional variance H_t, i.e., $R_t|I_{t-1} \sim N(\mu_t, H_t)$. The mean equation is specified by the VAR model, that is, Eqs. (1) and (2). The variance equation, H_t, in matrix form is given as follows:

$$H_t = \mathbf{C}_0\mathbf{C}_0' + \mathbf{A}\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A}' + \mathbf{B}\zeta_{t-t}\zeta_{t-1}'\mathbf{B}' + \mathbf{G}H_{t-1}\mathbf{G}', \tag{7}$$

where $\mathbf{C}_0$ is a lower triangular matrix; $\mathbf{A}$ is a (2×2) matrix, the diagonal elements of which capture the impact of unexpected shocks to large- and small-firm past returns on the current conditional volatility of large and small firm, and the off-diagonal elements measure the corresponding cross effects. Both the diagonal and off-diagonal elements determine the conditional

[21] The acronym BEKK stands for Baba, Engle, Kraft, and Kroner, the original authors who pioneered the work that was later refined and published by Engle and Kroner (1995).

Table 3
Test results of linear causality between small- and large- firm portfolios for the asymmetric BEKK model.

Dependent variable	Independent variable	
	Small	Large
Panel A: July 1, 1963–May 31, 2006		
Small	$\sum_{1}^{5}\alpha_i = \underset{(352.221)^a}{0.4917}$	$\sum_{1}^{5}\beta_j = \underset{(119.798)^a}{0.1307}$
Large	$\sum_{1}^{5}\phi_j = \underset{(131.163)^a}{0.1793}$	$\sum_{1}^{5}\delta_i = \underset{(1.592)}{0.0078}$
Panel B: July 1, 1963–December 31, 1979		
Small	$\sum_{1}^{5}\alpha_i = \underset{(32.780)^a}{0.5118}$	$\sum_{1}^{5}\beta_j = \underset{(18.590)^a}{0.1424}$
Large	$\sum_{1}^{5}\phi_j = \underset{(17.601)^a}{0.2354}$	$\sum_{1}^{5}\delta_i = \underset{(5.686)^a}{0.0727}$
Panel C: January 2, 1980–December 30, 1994		
Small	$\sum_{1}^{5}\alpha_i = \underset{(82.421)^a}{0.3779}$	$\sum_{1}^{5}\beta_j = \underset{(49.958)^a}{0.1792}$
Large	$\sum_{1}^{5}\phi_j = \underset{(1.064)}{0.0051}$	$\sum_{1}^{5}\delta_i = \underset{(-2.236)^b}{-0.0127}$
Panel D: January 3, 1995–May 31, 2006		
Small	$\sum_{1}^{5}\alpha_i = \underset{(249.441)^a}{0.4687}$	$\sum_{1}^{5}\beta_j = \underset{(64.085)^a}{0.0753}$
Large	$\sum_{1}^{5}\phi_j = \underset{(63.784)^a}{0.1453}$	$\sum_{1}^{5}\delta_i = \underset{(-75.346)^a}{-0.1036}$

This table reports regression estimates for the following model:

$$R_{1,t} = \sum_{i=1}^{\theta_1} \alpha_i R_{1,t-i} + \sum_{j=1}^{\theta_2} \beta_j R_{2,t-j} + \varepsilon_{1,t}$$

$$R_{2,t} = \sum_{i=1}^{\theta_3} \delta_i R_{2,t-i} + \sum_{j=1}^{\theta_4} \phi_j R_{1,t-j} + \varepsilon_{2,t}$$

where R_1 and R_2 are small and large firm daily returns, respectively. The sample period is from July 1, 1963 to May 31, 2006.

1. T-statistics are reported in parentheses below the sum of the estimated coefficients. The sum represents the cumulative effect.
2. For a given sum of coefficients s, the T-statistic is calculated as $T = s/\sigma_s$, where $s = \Sigma^n a_i$ and $n =$ number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$ then, $s = \Sigma^n a_i = a_1 + a_2 + a_3$ and

$$\sigma_s = \sigma_{(a_1 + a_2 + a_3)} = \sqrt{\sigma_{a_1}^2 + \sigma_{a_2}^2 + \sigma_{a_3}^2 + 2\sigma_{a_1 a_2} + 2\sigma_{a_2 a_3} + 2\sigma_{a_1 a_3}}$$

3. F-stat is the F-statistic testing for the joint significance of the lag coefficients on large (small) firms in the small (large) firms' equation.

[a], [b], and [c] indicate significance at the 1%, 5%, and 10% respectively.

variance; $\mathbf{B}$ is a (2×2) matrix that measures the impact of negative past returns on current conditional volatilities which is known as the leverage effect; $\zeta_t = \max(\varepsilon_t, 0)$. Decomposition of the constant matrix $\mathbf{C} = \mathbf{C}_0\mathbf{C}_0'$ ensures its positive definiteness.

The model given by Eq. (7) is identified given the following restrictions:

1. Diagonal elements of $\mathbf{C}_0$ are positive;
2. The elements a_{11}, b_{11}, and g_{11} of the matrixes $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{G}$ respectively are positive.

The estimation is carried out *simultaneously* using a maximum likelihood (ML) procedure.[22] The results of the simultaneous estimation of the VAR-asymmetric BEKK model for the causality in means tests are reported in Table 3. Consistent with our earlier results, we find significant bi-directional linear Granger causality between the portfolios of large and small firms over the entire sample as well as over each of the three sub-samples. The magnitudes of the coefficient estimates are slightly different and the standard errors (not reported) are generally smaller than in the single VAR model, reflecting the improved efficiency of the simultaneous procedure and accounting for heteroscedasticity in variance. In the first sub-period, the causality impact of small firms on large firms ($\Sigma\phi_j = 0.2354$) is greater than that of large firms on small firms ($\Sigma\beta_j = 0.1424$). However, during the second sub-period the causality impact of small firms on large firms while still positive ($\Sigma\phi_j = 0.0051$) is insignificantly different from zero (T-stat. = 1.064), whereas the causality impact of large-firm returns on small-firm returns is positive ($\Sigma\beta_j = 0.1792$) and highly significant (T-stat. = 49.958). Interestingly, the causality impact of small firms on large firms is positive ($\Sigma\phi_j = 0.1453$) and highly significant (T-stat. = 63.784) during the third sub-period while that of large firms on small firms is also positive ($\Sigma\beta_j = 0.0753$) and significant (T-stat. = 64.085). This is supportive of our earlier arguments of the growing importance of younger and smaller firms during this period.

[22] We use the modified MatLab code of Kevin Sheppard to carry out our estimations.

Table 4 reports the estimated coefficients of matrices **A**, **B**, and **G** in Panels A, B, and C, respectively. As noted earlier, off-diagonal elements of matrix **A** capture the impact of shocks to small-firm lag-one returns on the current conditional volatility of large-firm returns and vice versa. Panel A shows that the causality effect of large firms' lag-one squared returns on the current volatility of small firms is statistically significant and economically meaningful (0.15, 0.15, and 0.05 for the 1963–1979, 1980–1994, and 1995–2006 sub-periods, respectively). This finding is consistent with Conrad et al. (1991) who also find that shocks to large-firm returns affect future small-firm return volatility. As for the causality effect of small firms' past returns on the current volatility of large firms, Panel A shows that it is rather substantial for the sub-period from 1963 to1979 (0.15), becomes relatively stronger during the second sub-period (0.18) and increases significantly during the most recent sub-period (0.36). Overall, the results in Panel A show that not only do shocks to large-firm returns' affect the future volatility of small-firm returns but that shocks to small-firms' returns *also* affect the future volatility of large-firm returns. To the best of our knowledge, this bi-directional causality in variance has not been previously documented in the literature.

The finding of bi-directional causality merits further elaboration. Panel B presents the estimates of matrix **B** that capture the additional effects of negative past returns or the leverage effect. The results in Panel B show that the additional effect of negative large-firm past returns on the current volatility of small firms is relatively moderate for the period 1963–1979 and practically goes to zero for the later periods (0.03, 0.00, and 0.01, respectively). In contrast, however, the additional effect of negative small-firm past returns on the current volatility of large firms is highly significant for years 1963–1979 (0.11), remains statistically significant during the second sub-period (0.04) and increases significantly during the last sub-period (0.24). Overall, while the findings in Panel B imply that bad news to small firms affect large-firm volatility and similarly that large firms affect small firms, the impact of bad news evolves with the passage of time. As just noted, the additional impact of negative small-firm past returns on the volatility of large firms increases over time, and the opposite is true regarding the additional impact of negative large-firm past returns on the volatility of small firms.

Finally, Panel C reports the empirical estimates of matrix **G** that examines the impact of past volatilities on current volatilities. The findings in Panel C show that the impact of small-firm past returns' volatility on the current volatility of large-firm (−0.06) and the impact of large-firm past returns' volatility on the current volatility of small-firm volatility (0.04) are practically negligible over the full-sample period. With the exception of the second sub-sample period, we continue to observe this pattern for the remaining sub-sample periods.

5.4. Testing for Granger causality on residuals of the BEKK model

In this section, we applied the nonparametric Granger causality test to residuals of the BEKK model. The test is used as a diagnostic device to establish whether BEKK model is capable of fully capturing the Granger causal relationship. The rejection of the test would mean either that there is some Granger causality left beyond the second moment, or that the BEKK specification does not fully reflect the true relationship. We apply the test based on the re-sampling procedure (Sub-section 5.2) to account for

Table 4
Estimates of the parameters of the asymmetric BEKK model.

		Panel A: matrix **A**		Panel B: matrix **B**		Panel C: matrix **G**	
		Independent variable		Independent variable		Independent variable	
Sample period	Dependent variable	Small	Large	Small	Large	Small	Large
1963–2006	Small	0.35[a] (0.0028)	0.10[a] (0.0010)	0.44[a] (0.0022)	0.01[a] (0.0002)	0.87[a] (0.0004)	0.04[a] (0.0001)
	Large	0.09[a] (0.0019)	0.15[a] (0.0009)	0.15[a] (0.0013)	0.15[a] (0.0027)	−0.06[a] (0.0001)	0.99[a] (0.0000)
1963–1979	Small	0.42[a] (0.0034)	0.15[a] (0.0011)	0.45[a] (0.0108)	0.03[b] (0.0102)	0.85[a] (0.0007)	0.04[a] (0.0002)
	Large	0.15[a] (0.0010)	0.01[a] (0.0004)	0.11[a] (0.0035)	0.27[a] (0.0045)	−0.05[a] (0.0002)	0.99[a] (0.0001)
1980–1994	Small	0.53[a] (0.0265)	0.15[a] (0.0051)	0.47 (0.0296)	0.00 (0.0034)	0.68[a] (0.0122)	0.12[a] (0.0021)
	Large	0.18[a] (0.0112)	0.14[a] (0.0032)	0.04[a] (0.0103)	0.17[a] (0.0169)	−0.13[a] (0.0049)	1.02[a] (0.0009)
1995–2006	Small	0.23[a] (0.0042)	0.05[a] (0.0011)	0.48[a] (0.0013)	0.01[a] (0.0002)	0.89[a] (0.0002)	0.02[a] (0.0000)
	Large	0.36[a] (0.0032)	0.15[a] (0.0020)	0.24[a] (0.0018)	0.13[a] (0.0024)	−0.06[a] (0.0002)	0.99[a] (0.0001)

This table reports the coefficient estimates of matrices **A**, **B**, and **G** for the following BEKK variance equation:

$$\mathbf{H}_t = \mathbf{C}_0\mathbf{C}_0^{'} + \mathbf{A}\varepsilon_{t-1}\varepsilon_{t-1}^{'}\mathbf{A}^{'} + \mathbf{B}\zeta_{t-t}\zeta_{t-1}^{'}\mathbf{B}^{'} + \mathbf{G}\mathbf{H}_{t-1}\mathbf{G}^{'}.$$

Matrix **A** captures the impact of unexpected shocks to small-firm (large-firm) past returns on the current conditional volatility of large-firm (small-firm) returns; matrix **B** measures the impact of negative past returns on current conditional volatilities or the so-called leverage effect. **G** captures the impact of past volatility on current volatility, and $\mathbf{H}_t$ is the conditional variance–covariance matrix. The *p*-values are reported beneath the parameter estimates.
[a], [b], and [c] indicate statistical significance at the 1%, 5%, and 10%, respectively.

Table 5
Test results of nonlinear Granger causality between small- and large-firm portfolios for the asymmetric BEKK model.

	H_0: small does not cause large	H_0: large does not cause small
Panel A: July 1, 1963–May 31, 2006		
$L_{\text{Large}} = L_{\text{Small}}$	p-value	p-value
1	0.02	0.11
2	0.40	0.03
3	0.60	0.01
4	0.16	0.01
5	0.23	0.01
Panel B: July 1, 1963–December 31, 1979		
$L_{\text{Large}} = L_{\text{Small}}$	p-value	p-value
1	0.06	0.01
2	0.63	0.05
3	0.33	0.03
4	0.22	0.01
5	0.13	0.02
Panel C: January 2, 1980–December 30, 1994		
$L_{\text{Large}} = L_{\text{Small}}$	p-value	p-value
1	0.08	0.14
2	0.32	0.12
3	0.45	0.46
4	0.52	0.31
5	0.60	0.51
Panel D: January 3, 1995–May 31, 2006		
$L_{\text{Large}} = L_{\text{Small}}$	p-value	p-value
1	0.78	0.29
2	0.81	0.33
3	0.90	0.47
4	0.55	0.21
5	0.62	0.42

This table reports *parametric bootstrap p-values* for the standard Baek and Brock nonlinear Granger causality test given in Eq. (6) for the full and three sub-samples. The test is applied to the estimated VAR-BEKK residuals using nonsynchronous portfolios. $L_{\text{Large}} = L_{\text{Small}}$ denotes the number of lags on the residuals series used in the test. In all cases, the tests are applied to the unconditional standardized residuals. The lead length, $\boldsymbol{m}$, is set to unity, and the distance measure, d, is set to 1.5.

any biases in the test arising from estimation uncertainty. Step 1 of the re-sampling procedure now includes the conditional variance–covariance equation from the BEKK model.[23]

Table 5 reports the p-values of the results of the nonparametric Granger causality test based on the residuals of the BEKK model. Comparing these results with the results of the test applied to residuals of the VAR model (Table 2), we notice a significant reduction in the number of rejections of the null hypotheses. This suggests that overall the BEKK model captures most of the nonlinear Granger causality detected from the residuals of the VAR model. Panel A reveals that there is still some unidirectional causality from large firms to small firms for the full sample that the BEKK model does not account for. Examination of the sub-periods indicates that the findings for the full-sample period are mostly driven by the second and third sub-period results. Panel B reports the results for the first sub-period (1963–1979) and shows mostly unidirectional causality from large-firm returns to small-firm returns. This implies that the BEKK model is not fully capturing the causal relationship during this period. By contrast, the results for the second sub-period (Panel C) and the third sub-period (Panel D) reveal large p-values, failing to reject the null hypothesis of no Granger causality between large (small) and small (large) firms. Thus, we conclude that in the most recent sub-periods the simultaneous VAR-asymmetric BEKK model is able to adequately describe the Granger causality between the returns of large and small firms.

5.5. Nonsynchronous trading

In this final sub-section we investigate whether or not our results are due to a difference in the non-trading rates between the size-based portfolios. It must, however, be pointed out that since it is well established that smaller firms tend to trade less frequently than larger firms, the impact of differential non-trading rates would bias the results against finding that the returns of small firms reliably predict large firm returns. To control for nonsynchronous trading we examine the individual stocks, isolate

[23] As before, we start with the unadjusted returns and explicitly include the calendar dummies in the conditional mean and conditional variance–covariance equations.

Table 6
Test results of linear causality between small- and large-firm portfolios: synchronous data.

Dependent variable	Independent variable		
	Small	Large	F-Stat.
Small	$\alpha_1=0.2005$	$\beta_1=0.1393$	129.9000[a]
	(13.6141)[a]	(10.4636)[a]	
	$\alpha_2=0.0629$	$\beta_2=0.0573$	
	(4.2008)[a]	(2.2043)[b]	
	$\alpha_3=0.0413$	$\beta_3=0.0181$	
	(2.7631)[a]	(4.2629)[a]	
	$\alpha_4=0.0942$	$\beta_4=0.0389$	
	(6.3500)[a]	(2.9387)[a]	
	$\alpha_5=0.0044$	$\beta_5=0.0148$	
	(0.3193)	(2.1680)[b]	
	$\Sigma\alpha_i=0.4033$	$\Sigma\beta_j=0.2684$	
	(12.5342)[a]	(3.2315)[a]	
Large	$\phi_1=0.0105$	$\delta_1=0.0900$	10.0018[a]
	(1.9812)[b]	(6.10566)[a]	
	$\phi_2=0.0020$	$\delta_2=-0.0315$	
	(2.1184)[b]	(−2.1289)[b]	
	$\phi_3=0.0047$	$\delta_3=-0.0197$	
	(2.2851)[b]	(−1.3419)	
	$\phi_4=0.0440$	$\delta_4=-0.0436$	
	(2.1797)[b]	(−2.9814)[a]	
	$\phi_5=0.0583$	$\delta_5=0.0235$	
	(2.8109)[a]	(1.6758)[c]	
	$\Sigma\phi_j=0.1195$	$\Sigma\delta_i=0.0186$	
	(2.2341)[b]	(0.5614)	

This table reports regression estimates for the following model:

$$R_{1,t} = \sum_{i=1}^{\theta_1} \alpha_i R_{1,t-i} + \sum_{j=1}^{\theta_2} \beta_j R_{2,t-j} + \varepsilon_{1,t}$$

$$R_{2,t} = \sum_{i=1}^{\theta_3} \delta_i R_{2,t-i} + \sum_{j=1}^{\theta_4} \phi_j R_{1,t-j} + \varepsilon_{2,t}$$

where R_1 and R_2 are small and large firm daily returns, respectively. The sample period is from July 1, 1963 to May 31, 2006.

1. *T*-statistics are reported in parentheses below the sum of the estimated coefficients. The sum represents the cumulative effect.
2. For a given sum of coefficients *s*, the *T*-statistic is calculated as $T=s/\sigma_s$, where $s=\Sigma^n a_i$ and $n=$ number of lags on the independent variable whose impact is being investigated. For example, if $n=3$ then, $s=\Sigma^n a_i=a_1+a_2+a_3$ and

$$\sigma_s = \sigma_{(a_1+a_2+a_3)} = \sqrt{\sigma_{a_1}^2+\sigma_{a_2}^2+\sigma_{a_3}^2+2\sigma_{a_1a_2}+2\sigma_{a_2a_3}+2\sigma_{a_1a_3}}$$

3. *F*-stat is the *F*-statistic testing for the joint significance of the lag coefficients on large (small) firms in the small (large) firms' equation.

[a], [b], and [c] indicate significance at the 1%, 5%, and 10% respectively.

those for which the closing quote was not a trade, and remove those from our sample. We then construct value-weighted portfolios using only the sub-sample of stocks that traded.[24]

Table 6 reports the results of the linear Granger causality tests with the nonlinear Granger causality tests given in Table 7. In both tables, we continue to find significant bi-directional causality between the returns on large and small firms. Overall, these findings suggest that, even in the absence of nonsynchronous trading, small-firm returns are as important and reliable nonlinear predictor of the future returns on large firms as are the large-firm returns for small firms.

6. Summary and conclusions

In this paper we reexamine the causal relationship between the returns of large and small firms. Previous studies have shown that there is an asymmetric linear causal relationship between the returns of large and small firms' portfolios, with the causality going from large to small. We argue, in this paper, that this picture is incomplete. This argument is based primarily on two reasons. First, over the past decade there has been a significant increase in the importance of small firms in the economy which could result in small firms having a significant causal effect on large firms. Second, the existing studies have, in general, only tested for a linear relationship. However, Rubinstein (1973), and more recently Bansal and Viswanathan (1993), Bansal

[24] We thank the Editor Wayne Ferson for suggesting this approach to us.

Table 7
Test results for nonlinear Granger causality between small- and large-firm portfolios (sub-samples): synchronous data.

	H_0: small does not cause large	H_0: large does not cause small
Panel A: July 1, 1963–May 31, 2006		
$L_{Large}=L_{Small}$	*p*-value	*p*-value
1	0.01	0.01
2	0.04	0.01
3	0.05	0.01
4	0.07	0.01
5	0.05	0.01
Panel B: July 1, 1963–December 31, 1979		
$L_{Large}=L_{Small}$	*p*-value	*p*-value
1	0.01	0.01
2	0.05	0.05
3	0.05	0.03
4	0.01	0.03
5	0.01	0.03
Panel C: January 2, 1980–December 30, 1994		
$L_{Large}=L_{Small}$	*p*-value	*p*-value
1	0.01	0.01
2	0.01	0.01
3	0.04	0.01
4	0.06	0.01
5	0.11	0.01
Panel D: January 3, 1995–May 31, 2006		
$L_{Large}=L_{Small}$	*p*-value	*p*-value
1	0.07	0.01
2	0.02	0.03
3	0.03	0.08
4	0.06	0.02
5	0.06	0.03

This table reports *parametric bootstrap p-values* for the standard Baek and Brock nonlinear Granger causality test given in Eq. (6) for three sub-samples. The test is applied to the estimated VAR residuals from Eqs. (1) and (2). $L_{Large}=L_{Small}$ denotes the number of lags on the residuals series used in the test. In all cases, the tests are applied to the unconditional standardized residuals. The lead length, $\boldsymbol{m}$, is set to unity, and the distance measure, *d*, is set to 1.5.

et al. (1993), Harvey and Siddique (2000) and Dittmar (2002) develop theoretical models in which asset returns are characterized by nonlinearities. While, Hiemstra and Jones (1994), and Fujihara and Mougoué (1997), among others, provide evidence of a nonlinear causal relationship between the returns on various assets while rejecting the presence of a linear relationship.

Using both linear and nonlinear specifications to examine the relationship between sized-based portfolios, we find that the relationship is much more pervasive and complex than previously shown. Similar to the previous studies, we find that that lagged returns on large firms *linearly* Granger cause the returns on small firms. In contrast to previous studies, however, we find that the returns of small firms *linearly* Granger cause the returns of large firms. This finding is robust to returns autocorrelation, structural breaks in the data, nonsynchronous trading, and information flow as proxied by multivariate GARCH models.

Our nonlinear tests are even more striking. We find evidence of significant *nonlinear bi-directional* Granger causality between the returns on large and small firms' portfolios. Further, we find that unlike the linear causal relationship where the predictive power, as measured by the number of lagged returns that are able to forecast current returns, of large firms is greater than small, the predictive power of one portfolio for the other is the same irrespective of market capitalization. Importantly these findings remain after controlling for structural breaks, autocorrelation in the returns, and unaccounted estimation uncertainty in our models.

Thus, our results indicate that the relationship between large and small firms is much broader, time varying and, present at a deeper level than has been previously shown in the literature. And although, we have made some progress since Lo and McKinlay argued, and further articulated by Campbell, Lo and MacKinlay (1997), that "…further investigation(s) of (the) mechanisms…" that lead to this result are needed, there is still work to be done before we have a complete understanding of the relationship between large and small firms.

Acknowledgements

The authors would like to acknowledge and thank without implicating Delroy Hunter for his significant contribution to this article. The authors also thank Prashob Viruthiamparambath and Brandon Chi-Wei Chen for their able research assistance.

Appendix A

Table A1
Test results of linear causality between small- and large- firm portfolios (sub-samples).

Panel A: July 1, 1963–December 31, 1979				Panel B: January 2, 1980–December 30, 1994				Panel C: January 3, 1995–May 31, 2006			
Dep. variable	Independent variable			Dep. variable	Independent variable			Dep. variable	Independent variable		
	Small	Large	*F*-stat.		Small	Large	*F*-stat.		Small	Large	*F*-stat.
Small	$\alpha_1 = 0.3677$	$\beta_1 = 0.1339$	97.9623[a]	Small	$\alpha_1 = 0.0317$	$\beta_1 = 0.2653$	71.8470 [a]	Small	$\alpha_1 = 0.1570$	$\beta_1 = 0.0342$	14.3791[a]
	(14.6939)[a]	(5.6955)[a]			(1.4008)	(17.2663)[a]			(5.5179)[a]	(1.4824)	
	$\alpha_2 = -0.0565$	$\beta_2 = 0.0676$			$\alpha_2 = 0.1518$	$\beta_2 = 0.0599$			$\alpha_2 = 0.0468$	$\beta_2 = 0.0005$	
	(−2.1319)[b]	(2.8443)[a]			(6.7233)[a]	(3.7286)[a]			(1.6253)	(0.0219)	
	$\alpha_3 = 0.0808$	$\beta_3 = 0.0463$			$\alpha_3 = -0.0027$	$\beta_3 = 0.0244$			$\alpha_3 = 0.1072$	$\beta_3 = 0.0221$	
	(3.0851)[a]	(1.9578)[b]			(−0.1193)	(1.5243)			(3.7376)[a]	(0.9669)	
	$\alpha_4 = 0.0530$	$\beta_4 = 0.0119$			$\alpha_4 = 0.1427$	$\beta_4 = 0.1563$			$\alpha_4 = 0.0479$	$\beta_4 = 0.0007$	
	(2.0365)[b]	(0.5045)			(6.3922)[a]	(3.5589)[a]			(1.6789)[c]	(0.0295)	
	$\alpha_5 = 0.0403$	$\beta_5 = 0.0186$			$\alpha_5 = 0.0317$	$\beta_5 = 0.0181$			$\alpha_5 = -0.0102$	$\beta_5 = 0.0109$	
	(1.7299)[c]	(0.8298)			(1.5544)	(1.2037)			(−0.3692)	(0.4990)	
	$\Sigma\alpha_i = 0.4853$	$\Sigma\beta_j = 0.2783$			$\Sigma\alpha_i = 0.3553$	$\Sigma\beta_j = 0.5240$			$\Sigma\alpha_i = 0.3487$	$\Sigma\beta_j = 0.0684$	
	(8.5357)[a]	(2.5684)[a]			(7.1726)[a]	(5.4687)[a]			(5.4881) [a]	(2.4762)[a]	
Large	$\phi_1 = 0.0071$	$\delta_1 = 0.2558$	23.4589[a]	Large	$\phi_1 = 0.1084$	$\delta_1 = 0.1223$	5.6960 [a]	Large	$\phi_1 = 0.0164$	$\delta_1 = -0.0033$	2.0538 [a]
	(0.2649)	(10.2134)[a]			(3.2325)[a]	(5.3778)[a]			(0.4680)	(−0.1172)	
	$\phi_2 = 0.0328$	$\delta_2 = -0.0452$			$\phi_2 = 0.0273$	$\delta_2 = -0.0344$			$\phi_2 = 0.0258$	$\delta_2 = -0.0199$	
	(1.1623)	(−1.7849)[c]			(2.8156)[a]	(−1.4476)			(0.7295)	(−0.7025)	
	$\phi_3 = 0.0103$	$\delta_3 = 0.0485$			$\phi_3 = 0.0541$	$\delta_3 = -0.0026$			$\phi_3 = 0.0875$	$\delta_3 = -0.0844$	
	(0.3696)	(1.9257)[c]			(1.9881)[b]	(−0.1083)			(2.4808)[b]	(−2.9965)[a]	
	$\phi_4 = 0.0360$	$\delta_4 = -0.0270$			$\phi_4 = 0.1113$	$\delta_4 = -0.0839$			$\phi_4 = 0.0287$	$\delta_4 = 0.0079$	
	(1.2997)	(−1.0751)			(3.3690)[a]	(−3.5797)[a]			(0.8155)	(0.2836)	
	$\phi_5 = 0.0142$	$\delta_5 = -0.0141$			$\phi_5 = 0.0404$	$\delta_5 = 0.0380$			$\phi_5 = 0.0935$	$\delta_5 = 0.0044$	
	(0.5713)	(−0.5916)			(1.3358)	(1.7042)[c]			(2.7524)[a]	(0.1631)	
	$\Sigma\phi_j = 0.1004$	$\Sigma\delta_i = 0.2179$			$\Sigma\phi_j = 0.34152$	$\Sigma\delta_i = 0.0395$			$\Sigma\phi_j = 0.25198$	$\Sigma\delta_i = -0.0953$	
	(0.2367)	(3.7745)[a]			(2.2756)[b]	(0.7621)			(3.9871)[a]	(−1.52)	

This table reports regression estimates for the following model:

$$R_{1,t} = \sum_{i=1}^{\theta_1} \alpha_i R_{1,t-i} + \sum_{j=1}^{\theta_2} \beta_j R_{2,t-j} + \varepsilon_{1,t}$$

$$R_{2,t} = \sum_{i=1}^{\theta_3} \delta_i R_{2,t-i} + \sum_{j=1}^{\theta_4} \phi_j R_{1,t-j} + \varepsilon_{2,t}$$

where R_1 and R_2 are small and large firm daily returns, respectively. The sample period is from July 1, 1963 to May 31, 2006.

1. *T*-statistics are reported in parentheses below the sum of the estimated coefficients. The sum represents the cumulative effect.
2. For a given sum of coefficients s, the *T*-statistic is calculated as $T = s/\sigma_s$, where $s = \Sigma^n a_i$ and $n =$ number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$ then, $s = \Sigma^n a_i = a_1 + a_2 + a_3$ and

$$\sigma_s = \sigma_{(a_1 + a_2 + a_3)} = \sqrt{\sigma_{a_1}^2 + \sigma_{a_2}^2 + \sigma_{a_3}^2 + 2\sigma_{a_1 a_2} + 2\sigma_{a_2 a_3} + 2\sigma_{a_1 a_3}}$$

3. *F*-stat is the *F*-statistic testing for the joint significance of the lag coefficients on large (small) firms in the small (large) firms' equation.

[a], [b], and [c] indicate significance at the 1%, 5%, and 10% respectively.

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