The Network View on Input-Output Analysis for Australia*

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This version is preliminary.

Abstract

In this paper we investigate the role played by the Australian input-output structure in the relationship between sectoral shocks and aggregate volatility. Assuming Cobb-Douglas technology and preferences we derive the relations between the intersectoral shocks, outputs in different sectors and aggregate gdp. We show that supply side productivity shocks propagate downstream from supplier to user industries. Next, we investigate the effect of the input-output structure on the aggregate fluctuations. Acemoglu et al. (2012) showed that if out-degree distribution exhibits power law, inter-sectoral sectoral shocks may cause significant volatility. We find that the Australian economy exhibits qualitative features similar to the US data. The outdegree distribution exhibits heavy tails which may lead to significant effects of the sectoral shocks to the aggregate fluctuations.

Keywords: Input-Output Analysis; Intersectoral Linkages.

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1 Introduction

A standard way of explaining business fluctuations in the modern macroeconomy is through the aggregate shocks, i.e., shocks that affect in a similar way many economic sectors. Microeconomic sectoral specific shocks would not be able to generate empirically plausible aggregate fluctuations. Such shocks, the argument goes, being independent of each other would produce fluctuations whose standard deviation scales with factor of $1/\sqrt{n}$, when n denotes the number of sectors. Then when economy is considered at more and more disaggregated level, i.e., when n gets larger, the fluctuations would disappear quickly. This argument, spelled out explicitly in Lucas (1977), may also explain why the focus of most of the theoretical literature is on the model with a representative producer. These models exclude explicit interactions between different sectors or firms.

In reality the sectors are interconnected in different ways and these interconnections may imply that even independent shocks may cause large fluctuations. Importantly, the structure of connections between sectors matter for the consequences of shocks propagation. One of the most natural sort of connections in the production economy is due to input-output linkages. Drawing sectors as nodes and significant flows of intermediate goods from one sector to another as links we can obtain the input-output network as in Fig. 2. In this paper we will analyze the structure of the input-output network for Australia and will show the consequences of the network structure for the aggregate fluctuations.

Our work is inspired by and based upon the recent paper of Acemoglu et al. (2012). There it is shown that the law of large numbers underlying Lucas' argument may not necessarily hold in the large economy. The key idea is that if in the input-output network some sectors occupy the disproportionally large share of inter-sectoral dependencies, sectoral shocks may not disappear even at higher level of disaggregation. In particular, Acemoglu et al. (2012) show that if an out-degree distribution of the input-output network exhibits power law, then the rate of decline of volatility is substantially lower than $\sqrt{n}$. The empirical analysis suggests that the out-degree distribution of the input-output network for the US data does exhibit the power law.

The aim of this paper is to analyze the input-output network of Australian economy, i.e., consider this economy with a specific focus on the detailed structural interactions between different sectors. We are interested in the following set of questions. Is the structure of Australian economy stable over time? Does the input-output network of Australia remind those networks where the idiosyncratic shocks disappear quickly, so that the network approach cannot explain much of the aggregate fluctuations? Or, instead, the structure is such that the shocks to different sectors, even if independent, can generate sizable volatility at the macro level? In this case, what are the specific sectors of Australian economy that affect others disproportionally and so can be dubbed as 'systemically important'? Is the effect of direct input-output links of these sectors capture most of their possible systemic influence, or instead further sectors in the network contribute to the importance of these sectors?

All these questions are important in order to evaluate the prospects of the Australian economic system, whereas the network approach is well suited to address these questions. The logic behind using network approach is clear and intuitive. An economy can be viewed as a network, where possible nodes are firms, banks or industries that are linked with each other. These links are different kinds of connections such as trade flows, loans, etc. The structure of any particular network provides us with crucial information about the system. There was a surge of interest in using network approach to study the issues of systemic risk and shock propagation in economics and finance, see, e.g., Elliott et al. (2014), Acemoglu et al. (2015b), and Glasserman and Young (2015) and the textbook treatments in Vega-Redondo (2007), Jackson (2008) and Goyal (2012). The attention of regulators has also been turned to the importance of networks.[1] A pioneering work of Acemoglu et al. (2012) brought an attention of researchers to the networks with input-output linkages between firms or sectors, see Carvalho (2014) for a review. Acemoglu et al. (2015a) empirically test the implication of the network model discussed in this paper. They confirmed that the demand-side shocks propagated mostly to upstream industries, but not to downstream industries, whereas the technological shocks gives exactly an opposite pattern.

As we explain in this paper, the network approach is closely related and can add to the Input-Output analysis which still remains to be a popular methodology in applied macro-economic analysis. Both approaches use the input-output tables to characterize the technological links between different industries. For example, Rayner and Bishop (2013) used the Input-Output analysis to estimate the mining boom effect on the Australian economy. For this purpose they aggregated data from input-output tables to get three divisions: resource extraction, resource-related activity, non-resource activity. They estimated the growth in gross value added (GVA) in these divisions during the mining boom and showed how GVA of resource extraction is distributed among related industries. The results provided a decomposition of the effects on an employment from the resource extraction and the resource-related activity.

Computable General Equilibrium (Dixon and Parmenter, 1996) is another area of macro which makes extensive use of the Input-Output tables. This literature

[1]See, e.g., the speech of the Executive Director of the Financial Stability department of the Bank of England, Haldane (2009), or more recently the speech of the head of financial stability department of the RBA, Ellis (2015).

specifies an economy at a higher level of disaggregation than a typical macroeconomic model, but at the cost of losing analytical tractability. Also, it does not use the insights from network theory to analyse the structure and systemically important entities.

In this paper we use the most recent input-output tables for years 2012–2013 provided the Australian Bureau of Statistics (ABS). We also looks back in history and compare the most recent data with 7 other input-output tables up to 2001-2002. Similar to the US data, we find that the distribution of the out-degree for the Australian data exhibits heavy tails consistent with power low. We find that the most influential first order suppliers are professional services, wholesale trade, finance, construction services and road transport. However, the maximum weighted out-degree is much smaller in Australia comparing to the US data. This implies that shocks to the Australian "hub" industries will have a smaller aggregate effect.[2]

The remainder of the paper is organized as follows. Section 2 describes the theoretical framework and revokes several useful results from the literature. Section 3 illustrates the network effects using simple examples. The data we use for our empirical analysis are described in Section 4. The empirical results are presented in Section 5. Section 6 concludes. Appendices contain the additional details.

2 Theoretical Framework

As a baseline theoretical model in our analysis we will use the IO (input/output) model similar to those analyzed in Acemoglu et al. (2012) and Acemoglu et al. (2015a). This is, in fact, the single period version of the model presented in Long and Plosser (1983).

Assume that the economy consists of n sectors linked via the network of intermediate inputs, with sectors production described by the Cobb-Douglas production function with constant returns to scale. The total production of industry j (with $j = 1, \dots, n$) is given by

$$y_j = e^{z_j} \ell_j^{\alpha_j} \prod_{i=1}^{n} x_{ij}^{a_{ij}} \,, \tag{1}$$

where the factors of production used in industry j are labor, ℓ_j, and the intermediate input from industry i, x_{ij}. Technological coefficients $\alpha_j > 0$ and $a_{ij} \geq 0$

[2]One needs to be careful with comparison as different classifications are used in the US and the Australian input-output tables. The most recent release in Australia includes 114 industries, while in the US there are 417 industries at four-digit SIC (as of 2002).

describe the elasticity of labor and of intermediate inputs, respectively. The production side shock for industry j is denoted by e^z_j. The vector of the log of the shocks is denoted as $\mathbf{z} = (z_1 \; z_2 \; \ldots \; z_n)^T$, and we will assume that its mean is the vector of zeros, $\mathbf{0}$. We also introduce $n \times n$ matrix $\mathbf{A}$ with elements a_{ij}.

We assume the constant returns to scale in the technology function of each sector j, that is, we assume that for every j

$$\alpha_j + \sum_{i=1}^{n} a_{ij} = 1 \,. \tag{2}$$

With this assumption, every squared matrix $\mathbf{A}$ with non-negative elements and where all column-sums are less than 1 fully describes the technological process. For this reason, we call this matrix *production matrix.* Section 3 provides examples of the production matrices. Note that for the sake of simplicity this model assumes that the industries do not use capital as a separate input; rather they use the output of other industries.

The output of industry j, y_j, can be used either in the final consumption, or as an intermediate product for other industry, or being purchased by the government. Let c_j denote the final consumption of the goods produced by industry j, and g_j denote the government purchases from output y_j. Then the market clearing condition for the output of industry j becomes

$$y_j = c_j + \sum_{i=1}^{n} x_{ji} + g_j \,.$$

We populate this economy by a representative agent who gets utility from final consumption of goods and has a disutility of total labor ℓ. The total labor is divided between industries and the labor market clears when $\ell = \sum_{i=1}^{n} \ell_i$. Government spendings do not affect the consumer. Government runs a balanced budget by imposing taxes on the representative consumer equal to the total amount of spending, which can be evaluated at prices p_i in industry i as $T = \sum_{i=1}^{n} p_i g_i$.

As in Acemoglu et al. (2012) we focus on the competitive equilibrium of this economy. We characterize the competitive equilibrium in Appendix A. In particular we show there that the entry a_{ij} of the (i, j) cell of technological matrix $\mathbf{A}$ is equal, in the competitive equilibrium, to the relative cost of input from industry i in the total cost incurred by industry j. This has two important implications. First, this fact implies that the entries of matrix $\mathbf{A}$ are exactly the same as entries of the Direct Requirement table that is computed by the ABS and released as one of the Input-Output tables. Second, it relates the equilibrium analysis in the multi-sectoral macro-model to the network approach. Indeed, any production matrix $\mathbf{A}$ induces the directed, weighted network, where nodes represent various industries and edges represent the flows of intermediate inputs. The weight on the edge from industry i to industry j then corresponds to amount of cents that industry j pays to industry i to produce \$1 of the output. This input-output network for the Australian Economy at quite high level of aggregation is illustrated in Fig. 2. See also Section 3 for the simple examples and Section 5 for the discussion of the Australian input-output network.

The network representing actual economy will be quite complicated to analyze (though all information is captured by $\mathbf{A}$). From the network perspective, however, we may expect that what matters for the shock effect for a given node j are its in- and out-degrees, which are the sums of all the weights on the in-going and out-going edges, respectively. In our case, the in-degree of each node can be determined from (2), and is given by $d_j^{in} = 1 - \alpha_j$, whereas the *out-degree* (or simply *degree*) of the node is

$$d_j = d_j^{out} = \sum_{i=1}^{n} a_{ji}\,. \tag{3}$$

This number gives the relative weight that the other industries play as the customers for the given industry.

For production functions (1) and with standard Cobb-Douglas utility function, there is a unique competitive equilibrium in the model. In this equilibrium the endogenous variables, i.e., prices p_j, wages w_j, inputs ℓ_j, x_{ij} and outputs y_j are random variables whose realizations depend on the realization of the vector of technological shocks, $\mathbf{z} = (z_1\ z_2\ \ldots\ z_n)^T$ and of the vector of government spendings $\mathbf{g} = (g_1\ g_2\ \ldots\ g_n)^T$. It turns out that the equilibrium wages are the same and we denote it as w.

2.1 Empirical Implications

There are two questions which we empirically analyze in this paper. The first question concerns the effect of a given shock to the equilibrium values of various endogenous variables of the model, including GDP. The second question concerns the effect of the distribution of the random variable $\mathbf{z}$ on the volatility of GDP.

Let us start by focusing on the gross domestic product or the gross value added. Since the only primary factor used in the production is labor available in amount ℓ, and since the equilibrium wage is homogeneous across the industries, w, the GDP is equal to $w\ell$. Alternatively it can be computed as the sum of value-added to the primary factor across all industries. Therefore,

$$\text{GDP} = w\ell = \sum_{i=1} \alpha_i p_i y_i \,.$$

In Appendix B we generalize the result of Acemoglu et al. (2012) and show that under a proper price normalization, it is

$$\log \text{GDP} = \mathbf{v}^T \mathbf{z}\,, \tag{4}$$

where vector $\mathbf{z} = (z_1\ z_2\ \ldots\ z_n)^T$ is the vector of technological shocks and $\mathbf{v}^T$ is the transposed on an *influence vector*. The influence vector is defined as

$$\mathbf{v} = \frac{1}{n}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{1}, \tag{5}$$

Thus the log of GDP is a linear combination of the technological shocks hitting different sectors. Matrix $(\mathbf{I} - \mathbf{A})^{-1}$ that appears in (5) is the *Leontief inverse* matrix and it permits the following representation (see Appendix A)

$$\left(\mathbf{I} - \mathbf{A}\right)^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots\,, \tag{6}$$

which allows splitting the effect of shocks on those that are direct and those that are transmitted through the network.

To illustrate, assume that the shock $\mathbf{z}$ has only one non-zero component, the first one, $z_1 > 0$. The effect of this positive technological shock for the first industry to the GDP is captured by the first component of the influence vector, v_1. This can be written using (6) as follows

$$v_1 = \frac{z_1}{n}\left(1 + \sum_{i=1}^{n} a_{1i} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{1i}a_{ij} + \ldots\right)\,.$$

In this representation, the first term, z_1/n, is the direct impact of the shock to the GDP. The second term is, obviously, proportional to d_1, the out-degree of the first industry in the network. It captures the first round of the shock propagation through the input-output network. The third term is proportional to q_1, a so-called second-order degree of sector 1, which we define below in Eq. (8). It captures the second round of the shock propagation. The remaining infinite number of terms describe the shock propagations for the next rounds. We observe that the technological shock to an industry i propagates *downstream* in the sense that the GDP is affected by an amount in which the customer industries depend on industry 1 (the first round), then by an amount in which the customers of those industries depend on them (the second round), and so on.

In Appendix B we also investigate the effect of the shocks on various endogenous variables. We show, in particular, that in the absence of government spendings, the reaction of the production vector $\mathbf{y} = (y_1\ y_2\ \ldots\ y_n)^T$ to the technological shocks can be described by Eq. (23) as

$$\mathbf{d}\ln\mathbf{y} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1}\mathbf{dz} = \left(\mathbf{I} + \mathbf{A}^T + (\mathbf{A}^T)^2 + \ldots\right)\mathbf{dz}.$$

By examining the first order network effect, we can see that the log-production of an industry j depends on the shocks of only those industries i for which $a_{ij} > 0$. This confirms the statement that the productivity shocks propagate downstream, i.e., from industry i to industry j. We also show that the effect shocks on log of consumption coincides with their effect on the log of production, but that the effect on log prices is exactly an opposite, i.e.,

$$\mathbf{d}\ln\mathbf{p} = -\mathbf{d}\ln\mathbf{c} = -\mathbf{d}\ln\mathbf{y} = -\left(\mathbf{I} - \mathbf{A}^T\right)^{-1}\mathbf{dz}.$$

We now move to the second question raised above and discuss an effect of the shock distribution on the aggregate volatility. By the aggregate volatility we will understand the standard deviation of the log GDP. To focus on the effect of effect of network on volatility, we will assume that the shocks to different sectors are independent. Then from (4) we obtain

$$\sigma_{\mathrm{GDP}} := \sqrt{\mathrm{Var}\,\mathrm{GDP}} = \sqrt{\sum_{i=1}^{n} \sigma_i^2 v_i^2}\,, \tag{7}$$

where σ_i is the standard deviation of shock z_i hitting industry i. The structure of the network thus affects the aggregate volatility through the influence vector $\mathbf{v}$.

Consider now the sequence of economies corresponding to different levels of disaggregation. For a given level of disaggregation the number of sectors is n, and structure of economy is as described above with the production matrix $\mathbf{A}_n$ inducing the input-output network. How will volatility scale when n increases? One can easily show that if all the sectors have the same influences, the aggregate volatility would converge to zero with $n \to \infty$ at the rate $\sqrt{n}$. This is the standard argument to justify that the sectoral-specific or firm-specific shocks are not sufficient to generate sizeable volatility. However, Acemoglu et al. (2012) showed that the rate of convergence may be significantly lower if the network is asymmetric in terms of influences. We describe now their results that motivated our analysis of Australian input-output network.

Definition (5) together with representation (6) imply that the first-order effects of the asymmetry in influences can be captured by asymmetries in the degrees. Consider, therefore, the *distribution* of the out-degrees (3) of the nodes in the network, $\{d_1, d_2, \ldots, d_n\}$ and its counter-cumulative function $P(k) := \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{d_i > k\}}$. Note that when n changes, the counter-cumulative function changes as well. We use notation $P_n(\cdot)$ to stress this dependence. The following result is Corollary 1 in Acemoglu et al. (2012).

Proposition 1. *Assume that for the sequence of economies corresponding to the different levels of aggregation, the counter-cumulative function satisfies the* power law *property with* tail parameter $\beta \in (1, 2)$. *This means that there exist a sequence of numbers* c_n *with a property* $0 < \lim_{n\to\infty} \inf c_n \leq \lim_{n\to\infty} \sup c_n < 1$ *and function* $L(\cdot)$ *with a property that* $\lim_{t\to\infty} L(t)t^{\delta} = \infty$ *and* $\lim_{t\to\infty} L(t)t^{-\delta} = 0$ *for all* $\delta > 0$, *such that, for all* $n \in \mathbb{N}$ *and all* $k < \max_i\{d_i\}$, *it is*

$$P_n(k) = ck^{-\beta}L(k)\,.$$

Then the aggregate volatility vanishes at rate not faster than $n^{-(\beta-1)/\beta}$. *More precisely, for any* $\delta > 0$ *it is*

$$\lim_{n\to\infty} \inf \frac{\sqrt{\mathrm{Var\,GDP}}}{n^{-(\beta-1)/\beta-\delta}} > 0\,.$$

The tail parameter β thus represents a lower bound for the rate at which the aggregate volatility disappears. The smaller this parameter is, i.e., the fatter the tails of the degree distribution, the slower the convergence should be.

Returning again to Eqs. (5) and (6), we observe that the second-order term, related to matrix $\mathbf{A}^2$, is also important for the shock distributions and hence volatility. Therefore, we define the *second-order degree* of sector j as follows

$$q_j = \sum_{i=1}^{n} a_{ji}d_i = \sum_{i=1}^{n}\sum_{k=1}^{n} a_{ji}a_{ik}\,. \tag{8}$$

where the second equality is due to (3). The next result is Corollary 2 in Acemoglu et al. (2012).

Proposition 2. *Assume that for the sequence of economies corresponding to the different levels of aggregation, the counter-cumulative function of the second-degree distribution satisfies the* power law *property with* tail parameter $\zeta \in (1, 2)$. *Then the aggregate volatility vanishes not faster than at rate* $n^{-(\zeta-1)/\zeta}$.

If the second-order degree distribution has power law in the tail, then, similarly to the tail parameter β of the (first-order) degree distribution, the tail parameter ζ of the second-order degree distribution will provide a lower bound for the rate at which the aggregate volatility disappears. It is generally not possible to say which of these two degree distributions have a fatter tail (note that the out-degrees might well be larger than 1). Therefore, $\min\{\beta, \zeta\}$ is the tighter bound. One can continue in a similar way with analysis of higher order degrees of the sectors.

Finally note that the results regarding volatility and shock propagation presented before hold independently of any specific structure of the economy. When an equilibrium configuration is described by a reduced form system such as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{c}} + \tilde{\mathbf{A}}\tilde{\mathbf{y}} + \tilde{\mathbf{z}}\,,$$

where $\tilde{\mathbf{y}}$ is the vector of endogenous variables, vector $\tilde{\mathbf{c}}$ and matrix $\tilde{\mathbf{A}}$ have parameters and exogenous variables and $\tilde{\mathbf{z}}$ is the vector of shocks, then the results about shock propagation and volatility of some aggregate quantity as $\tilde{\mathbf{y}} \cdot \mathbf{1}$ in equilibrium hold. Moreover, one can even obtain this reduced form by reconstructing the matrix of interactions $\tilde{\mathbf{A}}$ from the observed changes in $\tilde{\mathbf{y}}$, see Anufriev and Panchenko (2015). The key advantage of working with the input-output data is that the direction of links can be clearly identified. Without this information only dependence between the different sectors can be deduced.

3 Examples of Input-Output Networks

For better understanding of the shock propagation within our framework let us consider several examples of simple economies consisting of only three industries. As discussed above, we can represent an economy as a network of input-output flows, or, equivalently, as the production matrix. Despite the simplest case with $n = 3$, we can have many different cases of network, see Fig. 1.

In the first row we show two opposite examples from the point of view of connectivity of the network. The left panel has a network where all three sectors are isolated. Note that this corresponds to the diagonal production matrix. The right panel shows the example, where all possible connections are present and the production matrix is given by

$$\mathbf{A}_3^{\text{full}} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

In this example we have $a_{ij} > 0$, and also (to satisfy the constant return to scale) $\alpha_j = 1 - a_{1j} - a_{2j} - a_{3j} > 0$ for all $j = 1, 2, 3$.

The second row contains two possible examples when every industry is a supplier or buyer from at most one other industry. The left panel shows network with

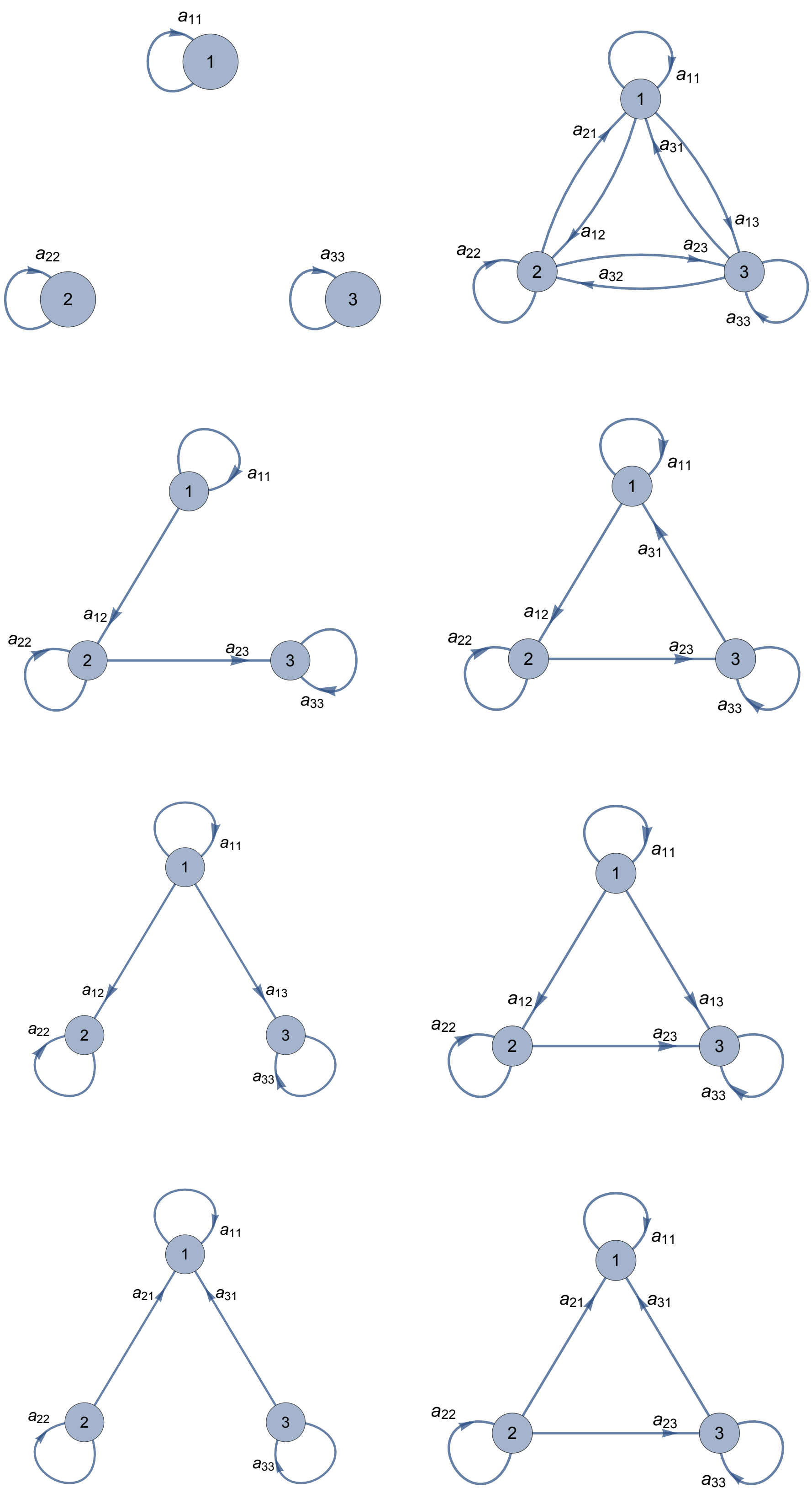

Figure 1: Various Examples of Networks with Three Industries.

'vertical' structure, where industry 1 is a supplier for industry 2, and industry 2 is a supplier for industry 3. The right panel shows the 'ring' structure, where, in addition, industry 3 supplies to industry 1. Note that these simple examples can be easily generalized for the case with n industries. For the case with n industries we can define the *vertical structure* of production, if the production matrix is given by

$$\mathbf{A}_n^{\text{vertical}} = \begin{pmatrix} a_{11} & a_{12} & 0 & \dots & 0 & 0 \\ 0 & a_{22} & a_{23} & \dots & 0 & 0 \\ 0 & 0 & a_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{n-1,n-1} & a_{n-1,n} \\ 0 & 0 & 0 & \dots & 0 & a_{n,n} \end{pmatrix}.$$

This matrix has two nonzero diagonals, the main and the one above the main. The *ring structure* of production is defined with a similar production matrix, but where in addition the element a_{n1} is non-zero. Note that both special networks may have zeros as some or all diagonal elements.

The two examples in the third row have one industry (industry 1) which is a common supplier to the two other industries. The left network corresponds to the matrix

$$\mathbf{A}_3^{\nabla} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix},$$

whereas for the right network, the production matrix has an additional nonzero element a_{23}. Some diagonal elements may be zeros, but generally speaking the upper-diagonal matrix represent the case where the flow generally moves down from some sectors that are pure suppliers to all other industries.

Finally, the examples presented in the fourth row have one industry (industry 1) which is a common buyer from the remaining industries. The left network corresponds to the matrix

$$\mathbf{A}_3^{\Delta} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix}.$$

For the right network, this production matrix has an additional nonzero element a_{32}. In any case, we are dealing with the lower-diagonal matrix in these cases.

4 Data

We first describe the empirical counter-part of the theoretical analysis in Section 2. Consider an economy with n industries, indexed as $i = 1, 2, \dots, n$. Let u_{ij} denote the dollar amount of industry i's products used in the production activity of industry j. The Input-Output Table consists of a square matrix $\mathbf{U} = \{u_{ij}\}_{i,j=1}^{n}$ of these intermediate use supplemented by two vectors. On the right from $\mathbf{U}$ there is a column vector $\boldsymbol{f}$ whose entries f_i correspond to the final demand for products produced by industry i, i.e., final consumption by private sector and government, capital formation and export. Below $\mathbf{U}$ there is a row vector $\boldsymbol{w}$ whose entries w_j describe the monetary equivalent of all other input used in production activity by industry j. It includes the wages, interest on capital, adjustment for taxes and subsidies and import. By construction for every industry i we have that

$$\sum_{j=1}^{n} u_{ij} + f_i = q_i \, ,$$

where q_i is the total output of industry i. Introducing a column vector $\boldsymbol{q}$ with components q_i, we rewrite the last equality as $\mathbf{U} \cdot \mathbf{1} + \boldsymbol{f} = \boldsymbol{q}$, where $\mathbf{1}$ as before denotes the vector of ones. We can now define the direct requirement matrix $\mathbf{A}$ with elements $a_{ij} = u_{ij}/q_j$ and since $\mathbf{U} \cdot \mathbf{1} = \mathbf{A} \cdot \boldsymbol{q}$, the last equality becomes

$$\mathbf{A} \cdot \boldsymbol{q} + \boldsymbol{f} = \boldsymbol{q}.$$

Then $(\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse.[3]

The input-output Tables were obtained from the Australian Bureau of Statistics (ABS). From every release since $2001 - 2002$ we took the "Use Table - Input by Industry and Final Use Category and Supply by Product Group". This table shows intermediate use by using industries (IOIG) and final use by final use categories of products (IOPG) at basic prices with indirect allocation of imports.

The ABS uses the so-called input-output industry group (IOIG) classification and input-output product classification (IOPC). There are some minor variations in classifications from release to release. For our major source of data, i.e., release of $2012 - 2013$ there are 114 industries in IOIG classification.[4]

[3] Among four main basic tables produced and published by the Australian Bureau of Statistics, the Use table (Table 2) has a similar structure with matrix $\mathbf{U}$ except that it has product groups in the rows. The flow table 5, which is derived from the Use and Supply basic tables, has exactly the same structure as matrix $\mathbf{U}$. Import in this case is allocated directly to the industry that uses it. Tables 6 and 7 are the direct requirement matrix $\mathbf{A}$ and the Leontief inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$.

[4] Note that US data is available at a higher level of disaggregation which may affect our comparison later.

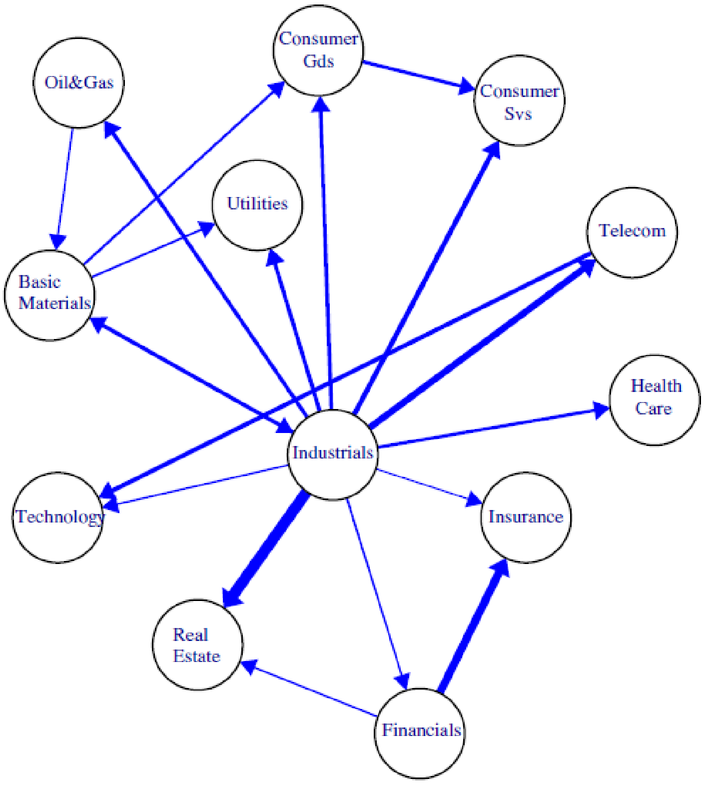

Figure 2: Aggregate level network.

Figure 2 shows an aggregate view of the Australian input-output network. It is apparent from the figure that Industrials take a central position in this network.

5 Network statistics and results

Weighted in-degree shows a share of intermediary inputs from all sectors in the production of a given sector. Using nonparametric estimation we obtained weighted in-degree distributions for all considered periods which is shown in Fig. 3. The average weighted in-degree in 2012/2013 was 0.57. This statistics was stable over other periods and varied between 0.56 to 0.58. Acemoglu et al. (2012) based on the US data finds that the average weighted in-degree across considered years and industries was 0.55.

Weighted in-degree also provides information on labor and capital shares in industry's production. Industries with low weighted in-degree are labor-intensive. The most labor-intensive industries (with weighted in-degree less than 0.3) in 2012/2013 were iron ore mining, knitted product manufacturing and, predictably, various services such as finance, ownership of dwellings, education, health care, residential care, information services and waste collection.

Considering top labor-intensive industries over different years we can notice

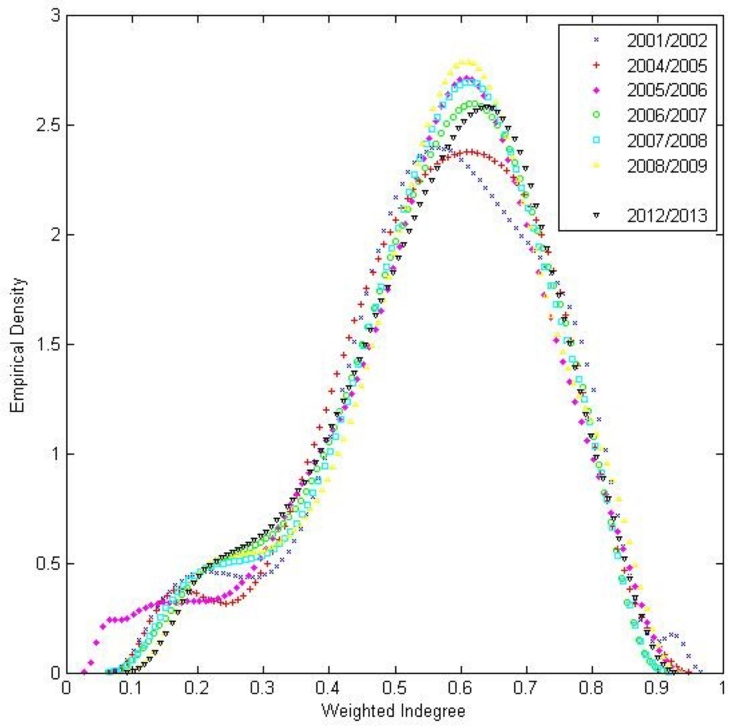

Figure 3: Empirical densities of in-degrees.

that oil and gas extraction sector was also among them in a period $2006-2010$. For the first two periods most labor-intensive services were a bit different, namely legal and business services, radio and television service, sport and gambling. Other periods are pretty similar to the 2012/13 period in terms of top labor-intensive services.

As we can observe in Fig. 3, the weighted in-degree distributions of the latest periods are quite close to each other, whereas distribution of earlier periods are slightly different.

Another network statistics is weighted out-degree, which shows how important the industry is in terms of supply of the intermediate product to other industries. It is obtained as a sum of all inputs shares provided by the industry. Nonparametric estimates of empirical densities of the first and second order weighted out-degree in 2012/13 are presented in Fig. 4. The left and right panels suggest that both the first and second degree distributions are fat-tailed, which implies that a relatively large number of industries are supplying to a lot of other industries. In comparison with US data, maximum of weighted out-degree is much smaller, 7.8 against 30. This indicates that Australian "hubs" industries are not as influential as in the US and in case of shocks to these industries the effect on other industries would be smaller.

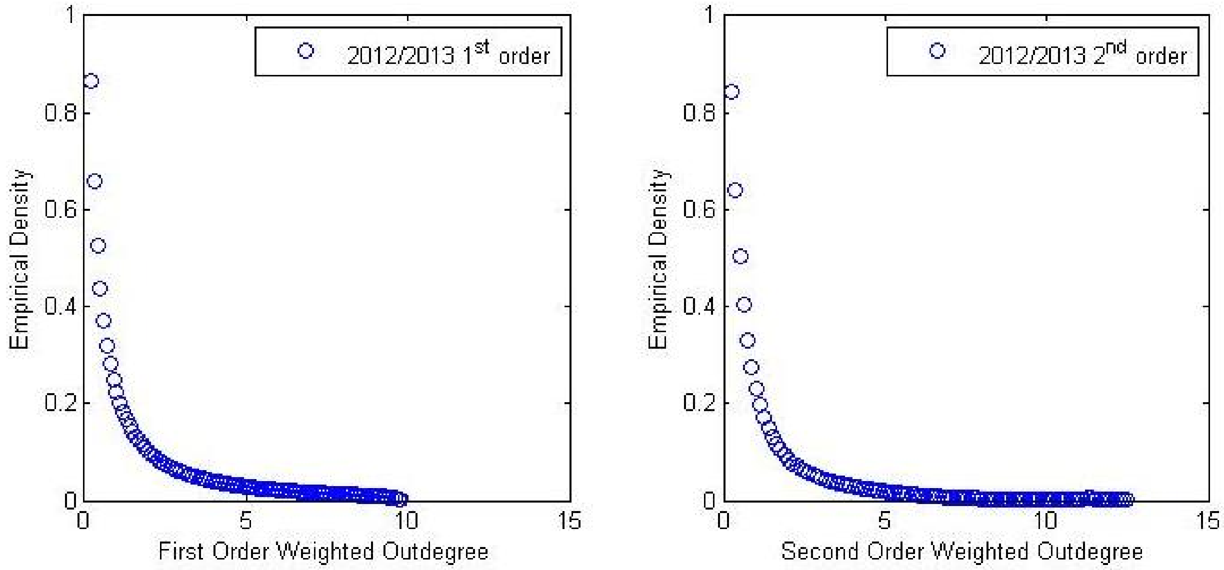

Figure 4: Empirical densities of first- and second-order degrees.

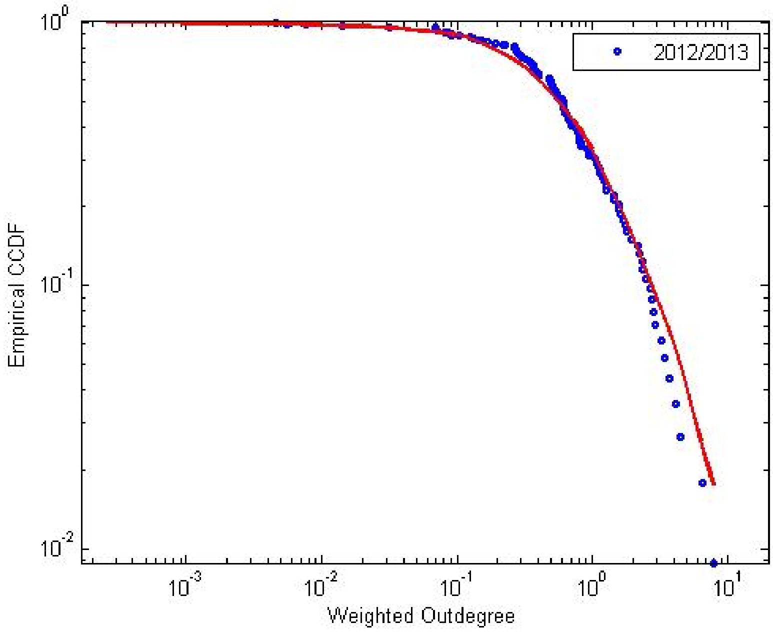

Figure 5: The Weighted Out-degree Distribution in 2012/13.

To demonstrate that the tail of the first order weighted out-degree distribution can be described by the power law, we will draw the empirical counter-cumulative distribution of the first order weighted out-degree on a log-log scale, see Fig. 5. These estimates were obtained using Nadaraya-Watson kernel regression with a bandwidth selected using least squares cross-validation. According to the red line, which is visualization of linear relationship, the tail does have a power-law distribution. The industries that are located in the tail of this distribution are the most influential as the first-order suppliers. Top six among them (with weighted

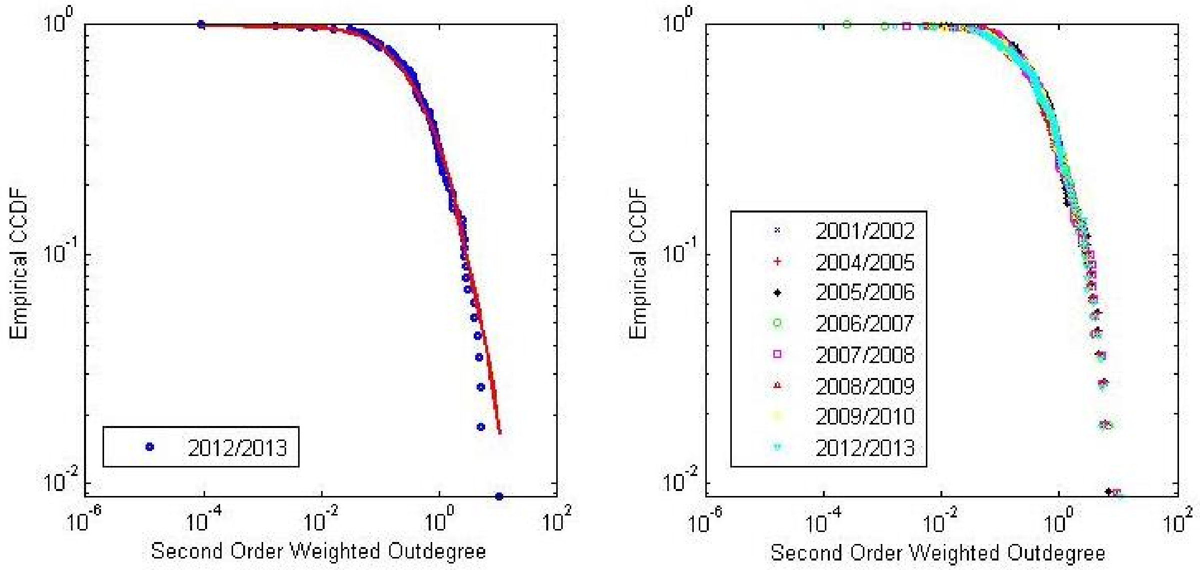

Figure 6: Empirical counter-cumulative distribution function of the second-order out-degrees.

out-degree greater than 3) are the following: professional services, wholesale trade, finance, construction services, road transport and sheep, grains, beef, dairy cattle.[5]

Similarly, Fig. 6 shows the counter-cumulative distribution functions of the second-order weighted out-degree. The second panel of the figure shows the second-order weighted out-degree distribution for the other years, which are close to each other.

Table 1 presents ML estimates of the tail exponents of the first- and second-order out-degree distribution, β and ζ. Recall from Propositions 1 and 2 that the tail exponents between 1 and 2 would correspond to the power low distribution and would imply the sizable aggregate volatility stemming from the network structure. The lower values of these parameters correspond to the fatter tails and lower speed of convergence. Interestingly, we observe a decline in the tail exponents over time indicating that the Australian economy becomes more susceptible to the shocks in the individual sectors. Note, however, that the estimation results depend on the method that is used for estimation. The estimates based on the OLS (not shown here for brevity) are larger and careful further analysis is required to reconcile the discrepancies. In comparison to the US data, our numbers are somewhat lower.

[5]In addition to the second-order weighted out-degree, we computed the corresponding eigenvector centralities. Eigenvector centrality is a self-referential measure which captures infinite order interactions between industries. The centrality takes into account influence on all industries that depend on this industry through others. Our estimates show that the industries with the highest eigenvector centralities are similar to the industries with highest weighted out-degree.

Table 1: ML Estimates of $\hat{\beta}$ and $\hat{\zeta}$

	01/02	04/05	05/06	06/07	07/08	08/09	09/10	12/13
$\hat{\beta}$	1.880	1.652	1.352	1.206	1.218	1.062	1.136	1.150
	(0.349)	(0.297)	(0.182)	(0.169)	(0.172)	(0.131)	(0.139)	(0.139)
$\hat{\zeta}$	1.333	1.167	1.161	1.190	1.136	1.506	1.122	1.084
	(0.229)	(0.184)	(0.164)	(0.179)	(0.169)	(0.337)	(0.159)	(0.158)

6 Conclusion

In this paper we addressed an important question of the role played by the Australian input-output structure in the relationship between sectoral shocks and aggregate volatility. We find that the Australian economy exhibits qualitative features similar to the US data. The outdegree distribution exhibits heavy tails which may lead to significant effects of the sectoral shocks to the aggregate fluctuations. There are many open questions remaining. In particular, we would like to quantify the amount of variation in the output of different sectors explained by the IO structure relatively to unexplained or "exogenous" variation. It would be also interesting to investigate whether the exogenous shocks are iid or there are any dependence patters between them.

In this paper we focused only on the input-output network, but of course interactions between sectors happen on many different levels as well. Hence, multilayer networks may arise. We leave this for further analysis.

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APPENDIX

A Competitive Equilibrium

Similarly to Acemoglu et al. (2012) and Acemoglu et al. (2015a), we assume that economy consists of a single representative household with Cobb-Douglas preferences over n final products, c_i with $i = 1, \ldots, n$, providing ℓ units of labor. The utility function of the household is

$$U(c_1, \ldots, c_n, \ell) = \gamma(\ell) \prod_{i=1}^{n} c_i^{\beta_i}, \tag{9}$$

where $\gamma(\cdot)$ is a decreasing differentiable function expressing disutulity from work, and $\beta_i > 0$ (with $\sum_{i=1}^{n} \beta_i = 1$) are the weights of good i in the utility of the consumer.

Total labor ℓ is divided between n industries, and ℓ_i denote the labor employed in industry i. Every industry operates according to the Cobb-Douglas production function (1), i.e.,

$$y_j = e^{z_j} \ell_j^{\alpha_j} \prod_{i=1}^{n} x_{ij}^{a_{ij}} ,$$

as described in the main text. Recall that y_j is the output of industry j, z_j denote the log of the industry-specific technological shock, x_{ij} is the amount of good i used in production process in industry j. Technological parameters $\alpha_j > 0$ and $a_{ij} \geq 0$ describe the shares of labor and intermediate goods in production of industry j, respectively. The technology in each industry is assumed to have constant returns to scale, so that relation (2) holds:

$$\alpha_j + \sum_{i=1}^{n} a_{ij} = 1 .$$

Therefore the production matrix $\mathbf{A} = (a_{ij})_{i,j=1}^{n}$ is non-negative and has all column-sums less than 1. It will be convenient to introduce the vector of labor elasticities $\boldsymbol{\alpha} = (\alpha_1\ \alpha_2 \ldots \alpha_n)^T$ and the vector of all ones, $\mathbf{1}$. Note that (2) implies that

$$\boldsymbol{\alpha} = \mathbf{1} - \mathbf{A}^T \mathbf{1} = \left(\mathbf{I} - \mathbf{A}^T\right) \mathbf{1} . \tag{10}$$

The goods produced in the industry j can be used for final consumption, intermediate production or government purchases. In order to finance its spendings, government taxes the consumer by amount T. We impose the condition that the budget is balanced, i.e.,

$$T = \sum_{i=1}^{n} p_i g_i . \tag{11}$$

Competitive Equilibrium: Definition. We consider the competitive equilibrium of this economy. The competitive equilibrium is the set of wages and prices for goods, w_i and p_i, such that

(1) given these prices, the representative consumer chooses c_i and ℓ_i to maximize utility function (9) subject to the budget constraint

$$\sum_{i=1}^{n} p_i c_i = \sum_{i=1}^{n} w_i \ell_i - T\,, \tag{12}$$

(2) given these prices, the representative firm in each industry j chooses ℓ_j and x_{ij} to maximize profit

$$\pi_j = p_j y_j - \sum_{i=1}^{n} p_i x_{ij} - w_j \ell_j\,, \tag{13}$$

(3) markets for goods and labor clear, i.e.,

$$y_j = c_j + \sum_{i=1}^{n} x_{ji} + g_j \quad \text{and} \quad \sum_{i=1}^{n} \ell_i = \ell\,. \tag{14}$$

Competitive Equilibrium: Characterization. To characterize the competitive equilibrium we begin with the household's problem. Lagrange function for this problem is as following:

$$L = \gamma(\ell) \prod_{i=1}^{n} c_i^{\beta_i} - \lambda \left(\sum_{i=1}^{n} p_i c_i - \sum_{i=1}^{n} w_i \ell_i + T \right)\,.$$

Differentiation of this function gives the following set of the first-order conditions:

$$\begin{aligned} \ell_i :& \quad \frac{\gamma'(\ell) U(\mathbf{c}, \ell)}{\gamma(\ell)} + \lambda w_i = 0 \\ c_i :& \quad \frac{\beta_i U(\mathbf{c}, l)}{c_i} - \lambda p_i = 0 \end{aligned}$$

where $\mathbf{c}$ denotes the vector of consumption. From the first equation we conclude that wage is constant, i.e.,

$$w_1 = \cdots = w_n =: w\,.$$

The constant wage is a consequence of the assumption that the marginal disutility of labour is the same across all the industries. Moreover, expressing λ from both conditions and equating them, we obtain

$$-\frac{\gamma'(\ell)}{\gamma(\ell)} \cdot \frac{1}{w} = \frac{\beta_i}{c_i p_i}.$$

This, in particular, implies that $p_i c_i/\beta_i$ is the same for every good i. Substituting this fact to the budget constraint and simplifying we obtain that

$$\frac{p_1 c_1}{\beta_1} = \cdots = \frac{p_n c_n}{\beta_n} = w\ell - T\,. \tag{15}$$

This is just a representation of a known property of Cobb-Douglas preferences that in the optimal point, the total budget is divided proportionally to the elasticities of consumption. Using (15), we can rewrite the previous equation as

$$-\frac{\gamma(\ell)w}{\gamma'(\ell)} = w\ell - T\,. \tag{16}$$

This equality implicitly defines the optimal labor as a function of all prices. When $T = 0$, i.e., government is not affecting the economy, optimal labor does not depend on wage and is completely determined by the disutility of labor.

Next we turn to the industry optimization problem. Maximization of profit in Eq. (13) gives the following first-order conditions, which we rewrite in terms of elasticities of inputs

$$\begin{aligned} x_{ij}: &\quad \frac{a_{ij}}{x_{ij}} p_j y_j - p_i = 0 &\quad\Rightarrow\quad& a_{ij} = \frac{p_i x_{ij}}{p_j y_j} \\ \ell_j: &\quad \frac{\alpha_j}{\ell_j} p_j y_j - w = 0 &\quad\Rightarrow\quad& \alpha_j = \frac{w\ell_j}{p_j y_j} \end{aligned}$$

Since the profit of firms are zero in the competitive equilibrium with constant return to scale, we have that $p_j y_j = \sum_{i=1}^n p_i x_{ij} + w\ell_j$. This allows us to rewrite the last two conditions as follows

$$a_{ij} = \frac{p_i x_{ij}}{\sum_{i=1}^n p_i x_{ij} + w\ell_j} \quad \text{and} \quad \alpha_j = \frac{w\ell_j}{\sum_{i=1}^n p_i x_{ij} + w\ell_j}\,. \tag{17}$$

Thus, these conditions express a well-known property of the Cobb-Douglas function with the constant return to scale. Namely, the technological elasticities of inputs are, in equilibrium, equal to the fractions of the cost of the corresponding input in the total costs (or, equivalently, in the total sales) of a given industry. This property, as we explain in the main text, allows us to connect the exogenous technological coefficients a_{ij} with the entries of the Direct Requirement Table used in the Input-Output analysis.

Connection with Leontief Matrix. The market clearing condition (14) for industry j together with the the first-order condition for industry with respect to x_{ij} imply that in the competitive equilibrium

$$p_j y_j = p_j c_j + \sum_{i=1}^n a_{ji} p_i y_i + p_j g_j\,. \tag{18}$$

This equality can be rewritten in the matrix form as follows

$$\begin{pmatrix} 1-a_{11} & -a_{12} & -a_{13} & \dots & -a_{1n} \\ -a_{21} & 1-a_{22} & -a_{23} & \dots & -a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & -a_{n3} & \dots & 1-a_{nn} \end{pmatrix} \begin{pmatrix} p_1 y_1 \\ p_2 y_2 \\ \vdots \\ p_n y_n \end{pmatrix} = \begin{pmatrix} p_1 c_1 + p_1 g_1 \\ p_2 c_2 + p_2 g_2 \\ \vdots \\ p_n c_n + p_n g_n \end{pmatrix}$$

The matrix in the left-hand side, $\mathbf{I}-\mathbf{A}$, has two important properties. First, it is a column diagonally dominant matrix, because for every column i we have

$$1 - a_{ii} - \sum_{j \neq i} |-a_{ji}| = 1 - a_{ii} - \sum_{j \neq i} a_{ji} = \alpha_i > 0\,,$$

and so $|1-a_{ii}| > \sum_{j\neq i} |-a_{ji}|$. Therefore, this matrix is non-singular and its inverse $(\mathbf{I}-\mathbf{A})^{-1}$ exists. Second, since the maximum of the column sums is less than 1, for the corresponding norm we have that $\|\mathbf{A}\|_1 < 1$, and therefore, all the eigenvalues of $\mathbf{A}$ are inside the unit circle. Therefore, the matrix can be written as the infinite sum as in Eq. 6 of the main text

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \dots\,.$$

Matrix $(\mathbf{I}-\mathbf{A})^{-1}$ is often called the Leontief inverse in the Input-Output analysis.

The previous calculations imply that the vectors of *sales* of industries can be computed as follows

$$\mathbf{s} := \begin{pmatrix} p_1 y_1 \\ p_2 y_2 \\ \vdots \\ p_n y_n \end{pmatrix} = (\mathbf{I}-\mathbf{A})^{-1} \begin{pmatrix} p_1 c_1 + p_1 g_1 \\ p_2 c_2 + p_2 g_2 \\ \vdots \\ p_n c_n + p_n g_n \end{pmatrix}\,. \tag{19}$$

This equation holds in equilibrium in our model. It connects the competitive equilibrium with Cobb-Douglas production function and the Leontief Input-Output analysis. We showed that the sales in industry j are affected by the total equilibrium demand in all the industries via the elements in the jth row of the Leontief inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}$. Representation (6) illustrates the idea of multiplicator, as it shows that not only direct but also all indirect network effects of the change in demand of an industry j should be taken into account.

Summing up equalities (18) for all the industries, we have that

$$\sum_{j=1}^{n} p_j y_j = \sum_{j=1}^{n} (p_j c_j + p_j g_j) + \sum_{j=1}^{n}\sum_{i=1}^{n} a_{ji} p_i y_i = w\ell + \sum_{i=1}^{n} (1-\alpha_i) p_i y_i$$

where we used the budget constraint (12), the balanced budget condition, the labor market clearing, and the constant returns to scale assumption (2). Therefore,

$$w\ell = \sum_{i=1}^{n} \alpha_i p_i y_i = \boldsymbol{\alpha} \cdot \mathbf{s}\,.$$

Note that $w\ell$ is the total value added in the economy or GDP. (Since all the produced goods are either used in other production or consumed, the only factor that adds to the value is the labor. The product $w\ell$ is then the monetary equivalent of the value added. Alternatively, we obtain the same quantity by summing up the labor fractions of the total sales over all the industries, i.e., as a scalar product of vectors $\boldsymbol{\alpha}$ and $\mathbf{s}$.)

Gabaix (2011) define the so-called 'granular residual' by weighting the shocks to the individual firms via their relative sales vector (defined as the vectors of shares of various firms' sales in the GDP). From (19) we can also obtain this vector in our framework as

$$\begin{pmatrix} \frac{p_1 y_1}{w\ell} \\ \frac{p_2 y_2}{w\ell} \\ \vdots \\ \frac{p_n y_n}{w\ell} \end{pmatrix} = (\mathbf{I} - \mathbf{A})^{-1} \begin{pmatrix} \frac{p_1 c_1 + p_1 g_1}{w\ell} \\ \frac{p_2 c_2 + p_2 g_2}{w\ell} \\ \vdots \\ \frac{p_n c_n + p_n g_n}{w\ell} \end{pmatrix} = (\mathbf{I} - \mathbf{A})^{-1} \begin{pmatrix} \frac{\beta_1 (w\ell - T) + p_1 g_1}{w\ell} \\ \frac{\beta_2 (w\ell - T) + p_2 g_2}{w\ell} \\ \vdots \\ \frac{\beta_n (w\ell - T) + p_n g_n}{w\ell} \end{pmatrix}.$$

While in the general case the expression in the right hand-side is not very illuminating, it simplifies when the government is absent. Let us introduce vector of consumption shares $\boldsymbol{\beta} = (\beta_1\ \beta_2\ \ldots\ \beta_n)$. Then in the case without government (or in the absence of demand shocks) we have

$$\frac{\mathbf{s}}{\boldsymbol{\alpha} \cdot \mathbf{s}} = (\mathbf{I} - \mathbf{A})^{-1} \boldsymbol{\beta}\,.$$

If we further assume that the labor elasticities are the same across the industries, i.e., when $\boldsymbol{\alpha} = \alpha \mathbf{1}$, we obtain the expression for the *relative* vector of sales

$$\frac{\mathbf{s}}{\mathbf{1} \cdot \mathbf{s}} = \begin{pmatrix} \frac{p_1 y_1}{\sum_{i=1}^{n} p_i y_i} \\ \frac{p_2 y_2}{\sum_{i=1}^{n} p_i y_i} \\ \vdots \\ \frac{p_n y_n}{\sum_{i=1}^{n} p_i y_i} \end{pmatrix} = \alpha (\mathbf{I} - \mathbf{A})^{-1} \boldsymbol{\beta}\,.$$

Thus vector of the relative sales in the economy depends on the technology and preferences. Note that the last equation implies that the sum of components of this vector is equal to one, i.e.,

$$\alpha \mathbf{1}^T (\mathbf{I} - \mathbf{A})^{-1} \boldsymbol{\beta} = 1\,. \tag{20}$$

If preferences are symmetric as in Acemoglu et al. (2012), i.e., $\beta_i = 1/n$ for every i, the vector of sales is $\mathbf{s} = \frac{\alpha}{n} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{1}$.

B Shock Propagation

Using the results about the competitive equilibrium discussed in Appendix A, we will now focus on the consequences of the shock to the values of endogenous variables.

Effects of shocks on equilibrium prices. Using Eq. (19), one can quantify the effect of a demand shock (change in the government spending) to the sales in different industries. However, such demand shock may also affect the prices in the competitive equilibrium. If this is the case, expression (19) is not very useful if one would like to predict changes on output level, y_j.

To evaluate the effect on real output, we will first take the *log* of the production function (1). We then have

$$\ln y_j = z_j + \alpha_j \ln \ell_j + \sum_{i=1}^{n} a_{ij} \ln x_{ij} \,.$$

Furthermore, the first order conditions of the firm imply:

$$\begin{aligned} x_{ij}: &\quad \ln x_{ij} = \ln a_{ij} + \ln p_j + \ln y_j - \ln p_i \\ \ell_j: &\quad \ln \ell_j = \ln \alpha_j + \ln p_j + \ln y_j - \ln w \end{aligned}$$

Substituting these last two equations into the previous formula and using (2), we can write

$$\ln y_j = z_j + \alpha_j \ln \alpha_j + \sum_{i=1}^{n} a_{ij} \ln a_{ij} + \ln p_j + \ln y_j - \alpha_j \ln w - \sum_{i=1}^{n} a_{ij} \ln p_i \,.$$

Let us define for industry j the constant $b_j = -\alpha_j \ln \alpha_j - \sum_{i=1}^{n} a_{ij} \ln a_{ij} > 0$, which does not depend on the shocks z_j in the production function and on the endogenous variables. Then from the previous equation we obtain

$$\ln p_j = -z_j + b_j + \alpha_j \ln w + \sum_{i=1}^{n} a_{ij} \ln p_i \,.$$

or, equivalently in the light of (2),

$$\ln \frac{p_j}{w} = -z_j + b_j + \sum_{i=1}^{n} a_{ij} \ln \frac{p_i}{w} \,. \tag{21}$$

This expression defines the linear system of equations with respect to the normalized prices. (For the sake of concreteness, we take $w = 1$.) In the matrix form this system reads

$$\left(\mathbf{I} - \mathbf{A}^T\right) \ln \mathbf{p} = -\mathbf{z} + \mathbf{b} \,,$$

where $\ln \mathbf{p}$, $\mathbf{z}$ and $\mathbf{b}$ are the column vectors whose components are $\ln p_j$, z_j and b_j, respectively. Solving the system we find

$$\ln \mathbf{p} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1}(-\mathbf{z} + \mathbf{b})\,. \tag{22}$$

This implies that the equilibrium price vector does not respond to the changes in the government spending. On the other hand, the response of the technological shocks $\mathbf{z}$ on prices is linear in shocks.

Effects of technological shocks. To study the effect of technological shocks on various variables we will totally differentiate (21) (and set $w = 1$ by normalization). We then have

$$d \ln p_j = -dz_j + \sum_{i=1}^{n} a_{ij} d \ln p_i\,.$$

We will assume that the government spendings are set to zero. Then from (16) we conclude that the equilibrium labor ℓ does not depend on the technological shocks. But then from (15) we have that $d \ln p_i = -d \ln c_i$ for every i. Therefore,

$$d \ln c_j = dz_j + \sum_{i=1}^{n} a_{ij} d \ln c_i,$$

which can be solved with respect to the vector $\mathbf{d} \ln \mathbf{c}$. We obtain that

$$\mathbf{d} \ln \mathbf{c} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{dz}.$$

Comparing it with (22), which implies $\mathbf{d} \ln \mathbf{p} = -\left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{dz}$ we observe that chocks have exactly opposite effects on price and consumed quantities. Indeed, under the Cobb-Douglas preferences the effect of prices and consumption cancel out leaving the total spending for a commodity unchanged.

Market clearing (14) and the first order condition of consumers imply that

$$\frac{y_j}{c_j} = 1 + \sum_{i=1}^{n} \frac{x_{ji}}{c_j} = 1 + \sum_{i=1}^{n} \frac{a_{ji} p_i y_i}{p_j c_j} = 1 + \sum_{i=1}^{n} \frac{\beta_i}{\beta_j} \frac{a_{ij} y_i}{c_i},$$

which means that in the equilibrium there is a unique vector of y_j/c_j that depends only on taste and production coefficients. This implies that $\mathbf{d} \ln \mathbf{y} = \mathbf{d} \ln \mathbf{c}$ and hence,

$$\mathbf{d} \ln \mathbf{y} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{dz}. \tag{23}$$

This is a consequence of the Cobb-Douglas production function: the effect of shock on prices are exactly balanced by the effect of shocks on quantities.

Equations (22) and (23) summarize the effect that the endogenous variables will have as a consequence of shocks.

Effect of technological shocks on GDP. To evaluate the effect of shock $\mathbf{z}$ on the GDP, which is given by $w\ell$ as in (2.1), we recall that the optimal ℓ does not depend on the shocks, and so all the effect will be accumulated in the w. We rewrite (21) as

$$\alpha_j \ln w = z_j - b_j + \ln p_j - \sum_{i=1}^{n} a_{ij} \ln p_i$$

or in the vector form

$$\ln w \, \boldsymbol{\alpha} = \mathbf{z} - \mathbf{b} + \left(\mathbf{I} - \mathbf{A}^T\right) \ln \mathbf{p}\,.$$

Recall from (10) that $\boldsymbol{\alpha} = (\alpha_1 \ \alpha_2 \ \ldots \ \alpha_n)^T = \left(\mathbf{I} - \mathbf{A}^T\right)\mathbf{1}$. Therefore, premultiplying this equality by $\left(\mathbf{I} - \mathbf{A}^T\right)^{-1}$ we obtain

$$\ln w \, \mathbf{1} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{z} - \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{b} + \ln \mathbf{p}\,.$$

Finally, to express $\ln w$, we sum all the equalities up and divide by their number, n. In other words, we premultiply the last equality by $\mathbf{1}^T/n$ in order to obtain

$$\ln w = \frac{1}{n}\mathbf{1}^T \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{z} - \frac{1}{n}\mathbf{1}^T \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \mathbf{b} + \frac{1}{n}\mathbf{1}^T \ln \mathbf{p}\,. \tag{24}$$

Let us analyze this decomposition. First of all we introduce the vector

$$\mathbf{v} = \frac{1}{n}\left(\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{1},$$

which is defined in (5) in the main text. The transpose of this vector, $\mathbf{v}^T$, premultiplies the vector of shocks in the right hand-side of (24). For this reason, following Acemoglu et al. (2012), we call vector $\mathbf{v}$ the influence vector. The second term in the right hand-side of (24) is $\mathbf{v}^T\mathbf{b}$, which depends only on the parameters of technology and the number of sector but not on endogenous variables and shocks. The last term

$$\bar{\boldsymbol{p}}_{\boldsymbol{n}} := \frac{1}{n}\mathbf{1}^T \ln \mathbf{p} = \frac{1}{n}\sum_{i=1}^{n} \ln p_i = \ln \prod_{i=1}^{n} p_i^{1/n}$$

is the average of the log prices, which in this procedure can be interpreted as a price index. Choosing the price normalization so that $\ln \ell - \mathbf{v}^T\mathbf{b} + \bar{\boldsymbol{p}}_{\boldsymbol{n}} = 0$, where ℓ is an optimal labor defined from (16), from (24) we obtain that

$$\ln \mathrm{GDP} = \ln w + \ln \ell = \mathbf{v}^T\mathbf{z}\,.$$

This result is essentially the same as in Acemoglu et al. (2012), except that it is obtained here with more general utility function and production process. In Acemoglu et al. (2012) it is assumed that $\boldsymbol{\alpha} = \alpha\mathbf{1}$.